

$$N = 87 = 3 \cdot 29$$

By the Ibukiyama-Kitayama dimension formula,  
 $\dim(S_4(K(87))) = 20$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,  
the lift dimension of  $S_4(K(87))^+$  is 13  
the nonlift dimension of  $S_4(K(87))^+$  is heuristically 6  
 $\dim(S_4(K(87))^+)$  thus is heuristically 19  
 $\dim(S_4(K(87))^-)$  is heuristically 1

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,87\}}^{\{\text{cusp}\}}) = 0$  (Skoruppa-Zagier), so need to span completely

$q = 5$  for TraceDown

After TD( $\text{Grit}(J_{\{4,435\}}^{\{\text{cusp}\}})$ ) and  $(\text{Grit}(J_{\{2,87\}}^{\{\text{cusp}\}}))^2$ ,  
spanned rank in  $S_4(K(87))^+$  is 19  
spanned rank in  $S_4(K(87))^-$  is 0

After Borcherds products,  
spanned rank in  $S_4(K(87))^-$  is 1

Final spanned rank in  $S_4(K(87))^+$  is 19

Final spanned rank in  $S_4(K(87))^-$  is 1

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$S_2(K(87))$  is determined by Jacobi restriction and the  $H4Ndd(1,+)$  test  
 $(H_4(87,1,1))^+ = 0$

So  $S_2(K(87)) = \text{Grit}(J_{\{2,87\}}^{\{\text{cusp}\}})$  (dimension 0)