

$$N = 86 = 2 \cdot 43$$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(86))) = 21$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(86))^+$ is 14
the nonlift dimension of $S_4(K(86))^+$ is heuristically 6
 $\dim(S_4(K(86))^+)$ thus is heuristically 20
 $\dim(S_4(K(86))^-)$ is heuristically 1

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,86\}}^{\{\text{cusp}\}}) = 1$ (Skoruppa-Zagier), so need to span completely

$q = 5$ for TraceDown

After TD($\text{Grit}(J_{\{4,430\}}^{\{\text{cusp}\}})$) and $(\text{Grit}(J_{\{2,86\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(86))^+$ is 20
spanned rank in $S_4(K(86))^-$ is 0

Hecke operators applied: $\{\{2, 2\}\}$

After Hecke spreading,
spanned rank in $S_4(K(86))^-$ is 1

Final spanned rank in $S_4(K(86))^+$ is 20

Final spanned rank in $S_4(K(86))^-$ is 1

$S_2(K(86))$ is determined by Jacobi restriction and the $H4Nd1(1)$ test
($\dim(H_4(86,1,1)) \leq 1$ and this is less than $\dim(J_{\{2,86\}}^{\{\text{cusp}\}}) + 1 = 2$)

So $S_2(K(86)) = \text{Grit}(J_{\{2,86\}}^{\{\text{cusp}\}})$ (dimension 1)