

$$N = 85 = 5 \cdot 17$$

By the Ibukiyama-Kitayama dimension formula,  
 $\dim(S_4(K(85))) = 24$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,  
the lift dimension of  $S_4(K(85))^+$  is 15  
the nonlift dimension of  $S_4(K(85))^+$  is heuristically 9  
 $\dim(S_4(K(85))^+)$  thus is heuristically 24  
 $\dim(S_4(K(85))^-)$  is heuristically 0

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,85\}}^{\{\text{cusp}\}}) = 2$  (Skoruppa-Zagier), so need to span to within 1 dimension

$q = 7$  for TraceDown

After  $\text{TD}(\text{Grit}(J_{\{4,595\}}^{\{\text{cusp}\}}))$  and  $(\text{Grit}(J_{\{2,85\}}^{\{\text{cusp}\}}))^2$ ,  
spanned rank in  $S_4(K(85))^+$  is 24  
spanned rank in  $S_4(K(85))^-$  is 0

Final spanned rank in  $S_4(K(85))^+$  is 24

Final spanned rank in  $S_4(K(85))^-$  is 0

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 $S_2(K(85))$  is determined by Jacobi restriction and the  $H4Ndd(2,+)$  test  
( $H_4(85,2,2)^+ = 0$ )

So  $S_2(K(85)) = \text{Grit}(J_{\{2,85\}}^{\{\text{cusp}\}})$  (dimension 2)