

$$N = 82 = 2 \cdot 41$$

By the Ibukiyama-Kitayama dimension formula,  
 $\dim(S_4(K(82))) = 23$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,  
the lift dimension of  $S_4(K(82))^+$  is 14  
the nonlift dimension of  $S_4(K(82))^+$  is heuristically 8  
 $\dim(S_4(K(82))^+)$  thus is heuristically 22  
 $\dim(S_4(K(82))^-)$  is heuristically 1

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,82\}}^{\{\text{cusp}\}}) = 1$  (Skoruppa-Zagier), so need to span completely

$q = 5$  for TraceDown

After TD( $\text{Grit}(J_{\{4,410\}}^{\{\text{cusp}\}})$ ) and  $(\text{Grit}(J_{\{2,82\}}^{\{\text{cusp}\}}))^2$ ,  
spanned rank in  $S_4(K(82))^+$  is 22  
spanned rank in  $S_4(K(82))^-$  is 0

Hecke operators applied:  $\{\{2, 2\}\}$

After Hecke spreading,  
spanned rank in  $S_4(K(82))^-$  is 1

Final spanned rank in  $S_4(K(82))^+$  is 22

Final spanned rank in  $S_4(K(82))^-$  is 1

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$S_2(K(82))$  is determined by Jacobi restriction and the  $H4Nd1(1)$  test  
( $\dim(H_4(82,1,1)) \leq 1$  and this is less than  $\dim(J_{\{2,82\}}^{\{\text{cusp}\}}) + 1 = 2$ )

So  $S_2(K(82)) = \text{Grit}(J_{\{2,82\}}^{\{\text{cusp}\}})$  (dimension 1)