

$$N = 78 = 2 \cdot 3 \cdot 13$$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(78))) = 15$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(78))^+$ is 10
the nonlift dimension of $S_4(K(78))^+$ is heuristically 4
 $\dim(S_4(K(78))^+)$ thus is heuristically 14
 $\dim(S_4(K(78))^-)$ is heuristically 1

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,78\}}^{\{\text{cusp}\}}) = 0$ (Skoruppa-Zagier), so need to span completely

$q = 11$ for TraceDown

After $\text{TD}(\text{Grit}(J_{\{4,858\}}^{\{\text{cusp}\}}))$ and $(\text{Grit}(J_{\{2,78\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(78))^+$ is 14
spanned rank in $S_4(K(78))^-$ is 0

After Borcherds products,
spanned rank in $S_4(K(78))^-$ is 1

Final spanned rank in $S_4(K(78))^+$ is 14

Final spanned rank in $S_4(K(78))^-$ is 1

$S_2(K(78))$ is determined by Jacobi restriction and the $H4Ndd(1,+)$ test
 $(H_4(78,1,1))^+ = 0$

So $S_2(K(78)) = \text{Grit}(J_{\{2,78\}}^{\{\text{cusp}\}})$ (dimension 0)