

$$N = 77 = 7 \cdot 11$$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(77))) = 15$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(77))^+$ is 12
the nonlift dimension of $S_4(K(77))^+$ is heuristically 3
 $\dim(S_4(K(77))^+)$ thus is heuristically 15
 $\dim(S_4(K(77))^-)$ is heuristically 0

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,77\}}^{\{\text{cusp}\}}) = 1$ (Skoruppa-Zagier), so need to span completely

$q = 5$ for TraceDown

After $\text{TD}(\text{Grit}(J_{\{4,385\}}^{\{\text{cusp}\}}))$ and $(\text{Grit}(J_{\{2,77\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(77))^+$ is 15
spanned rank in $S_4(K(77))^-$ is 0

Final spanned rank in $S_4(K(77))^+$ is 15

Final spanned rank in $S_4(K(77))^-$ is 0

 $S_2(K(77))$ is determined by Jacobi restriction and the $H4Ndl(1)$ test
($\dim(H_4(77,1,1)) \leq 1$ and this is less than $\dim(J_{\{2,77\}}^{\{\text{cusp}\}}) + 1 = 2$)

So $S_2(K(77)) = \text{Grit}(J_{\{2,77\}}^{\{\text{cusp}\}})$ (dimension 1)