

$$N = 70 = 2 \cdot 5 \cdot 7$$

By the Ibukiyama-Kitayama dimension formula,  
 $\dim(S_4(K(70))) = 14$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,  
the lift dimension of  $S_4(K(70))^+$  is 9  
the nonlift dimension of  $S_4(K(70))^+$  is heuristically 5  
 $\dim(S_4(K(70))^+)$  thus is heuristically 14  
 $\dim(S_4(K(70))^-)$  is heuristically 0

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,70\}}^{\{\text{cusp}\}}) = 0$  (Skoruppa-Zagier), so need to span completely

$q = 11$  for TraceDown

After  $\text{TD}(\text{Grit}(J_{\{4,770\}}^{\{\text{cusp}\}}))$  and  $(\text{Grit}(J_{\{2,70\}}^{\{\text{cusp}\}}))^2$ ,  
spanned rank in  $S_4(K(70))^+$  is 14  
spanned rank in  $S_4(K(70))^-$  is 0

Final spanned rank in  $S_4(K(70))^+$  is 14

Final spanned rank in  $S_4(K(70))^-$  is 0

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 $S_2(K(70))$  is determined by Jacobi restriction and the  $H4Ndd(1,+)$  test  
( $H_4(70,1,1)^+ = 0$ )

So  $S_2(K(70)) = \text{Grit}(J_{\{2,70\}}^{\{\text{cusp}\}})$  (dimension 0)