

$$N = 66 = 2 \cdot 3 \cdot 11$$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(66))) = 11$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(66))^+$ is 8
the nonlift dimension of $S_4(K(66))^+$ is heuristically 3
 $\dim(S_4(K(66))^+)$ thus is heuristically 11
 $\dim(S_4(K(66))^-)$ is heuristically 0

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,66\}}^{\{\text{cusp}\}}) = 0$ (Skoruppa-Zagier), so need to span completely

$q = 13$ for TraceDown

After $\text{TD}(\text{Grit}(J_{\{4,858\}}^{\{\text{cusp}\}}))$ and $(\text{Grit}(J_{\{2,66\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(66))^+$ is 11
spanned rank in $S_4(K(66))^-$ is 0

Final spanned rank in $S_4(K(66))^+$ is 11

Final spanned rank in $S_4(K(66))^-$ is 0

 $S_2(K(66))$ is determined by Jacobi restriction and the $H4Ndd(1,+)$ test
($H_4(66,1,1)^+ = 0$)

So $S_2(K(66)) = \text{Grit}(J_{\{2,66\}}^{\{\text{cusp}\}})$ (dimension 0)