

$$N = 62 = 2 \cdot 31$$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(62))) = 12$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(62))^+$ is 9
the nonlift dimension of $S_4(K(62))^+$ is heuristically 2
 $\dim(S_4(K(62))^+)$ thus is heuristically 11
 $\dim(S_4(K(62))^-)$ is heuristically 1

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,62\}}^{\{\text{cusp}\}}) = 0$ (Skoruppa-Zagier), so need to span completely

$q = 5$ for TraceDown

After TD($\text{Grit}(J_{\{4,310\}}^{\{\text{cusp}\}})$) and $(\text{Grit}(J_{\{2,62\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(62))^+$ is 11
spanned rank in $S_4(K(62))^-$ is 0

After Borcherds products,
spanned rank in $S_4(K(62))^-$ is 1

Final spanned rank in $S_4(K(62))^+$ is 11

Final spanned rank in $S_4(K(62))^-$ is 1

$S_2(K(62))$ is determined by Jacobi restriction and the $H4Ndd(1,+)$ test
 $(H_4(62,1,1))^+ = 0$

So $S_2(K(62)) = \text{Grit}(J_{\{2,62\}}^{\{\text{cusp}\}})$ (dimension 0)