

$$N = 290 = 2 \cdot 5 \cdot 29$$

By the Ibukiyama-Kitayama dimension formula,  
 $\dim(S_4(K(290))) = 198$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,  
the lift dimension of  $S_4(K(290))^+$  is 50  
the nonlift dimension of  $S_4(K(290))^+$  is heuristically 103  
 $\dim(S_4(K(290))^+)$  thus is heuristically 153  
 $\dim(S_4(K(290))^-)$  is heuristically 45

$\dim(J_{\{2,290\}}^{\{\text{cusp}\}}) = 5$  (Skoruppa-Zagier), so need to span to within 4 dimensions

$q = 7$  for TraceDown

After TD( $\text{Grit}(J_{\{4,2030\}}^{\{\text{cusp}\}})$ ) and  $(\text{Grit}(J_{\{2,290\}}^{\{\text{cusp}\}}))^2$ ,  
spanned rank in  $S_4(K(290))^+$  is 152  
spanned rank in  $S_4(K(290))^-$  is 0

Hecke operators applied:  $\{\{2, 2\}, \{2, 2\}, \{2, 1\}\}, \{\{2, 2\}, \{3, 1\}\}$   
After Hecke spreading,  
spanned rank in  $S_4(K(290))^-$  is 30

After Borcherds products,  
spanned rank in  $S_4(K(290))^-$  is 43

Final spanned rank in  $S_4(K(290))^+$  is 152

Final spanned rank in  $S_4(K(290))^-$  is 43

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 $S_2(K(290))^+$  is determined by Jacobi restriction and the  $H4Ndl(2,+)$  test  
( $\dim(H_4(290,2,1)^+) \leq 5$  and this is less than  $\dim(J_{\{2,290\}}^{\{\text{cusp}\}})+1 = 6$ )  
 $S_2(K(290))^- = 0$  by Jacobi restriction and the  $H4Ndl(2,-)$  test  
( $\dim(H_4(290,2,1)^-) \leq 3$  and this is less than  $\dim(J_{\{2,290\}}^{\{\text{cusp}\}}) = 5$ )

So  $S_2(K(290)) = \text{Grit}(J_{\{2,290\}}^{\{\text{cusp}\}})$  (dimension 5)