

$$N = 278 = 2 \cdot 139$$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(278))) = 185$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(278))^+$ is 51
the nonlift dimension of $S_4(K(278))^+$ is heuristically 91
 $\dim(S_4(K(278))^+)$ thus is heuristically 142
 $\dim(S_4(K(278))^-)$ is heuristically 43

$\dim(J_{\{2,278\}}^{\{\text{cusp}\}}) = 6$ (Skoruppa-Zagier), so need to span to within 5 dimensions

$q = 7$ for TraceDown

After TD($\text{Grit}(J_{\{4,1946\}}^{\{\text{cusp}\}})$) and $(\text{Grit}(J_{\{2,278\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(278))^+$ is 140
spanned rank in $S_4(K(278))^-$ is 0

Hecke operators applied: $\{\{2, 2\}, \{2, 2\}, \{2, 1\}\}$

After Hecke spreading,
spanned rank in $S_4(K(278))^-$ is 25

After Borcherds products,
spanned rank in $S_4(K(278))^-$ is 41

Final spanned rank in $S_4(K(278))^+$ is 140

Final spanned rank in $S_4(K(278))^-$ is 41

$S_2(K(278))$ is determined by Jacobi restriction and the $H4Ndl(3)$ test
($\dim(H_4(278,3,1)) \leq 4$ and this is less than $\dim(J_{\{2,278\}}^{\{\text{cusp}\}})+1 = 7$)

So $S_2(K(278)) = \text{Grit}(J_{\{2,278\}}^{\{\text{cusp}\}})$ (dimension 6)