

$$N = 266 = 2 \cdot 7 \cdot 19$$

By the Ibukiyama-Kitayama dimension formula,  
 $\dim(S_4(K(266))) = 158$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,  
the lift dimension of  $S_4(K(266))^+$  is 45  
the nonlift dimension of  $S_4(K(266))^+$  is heuristically 81  
 $\dim(S_4(K(266))^+)$  thus is heuristically 126  
 $\dim(S_4(K(266))^-)$  is heuristically 32

$\dim(J_{\{2,266\}}^{\{\text{cusp}\}}) = 4$  (Skoruppa-Zagier), so need to span to within 3 dimensions

$q = 5$  for TraceDown

After TD(Grit( $J_{\{4,1330\}}^{\{\text{cusp}\}}$ )) and ( $\text{Grit}(J_{\{2,266\}}^{\{\text{cusp}\}})^2$ ),  
spanned rank in  $S_4(K(266))^+$  is 125  
spanned rank in  $S_4(K(266))^-$  is 0

Hecke operators applied:  $\{\{2, 2\}, \{2, 2\}, \{2, 1\}\}, \{\{2, 2\}, \{3, 1\}\}$   
After Hecke spreading,  
spanned rank in  $S_4(K(266))^-$  is 23

After Borcherds products,  
spanned rank in  $S_4(K(266))^-$  is 29

Final spanned rank in  $S_4(K(266))^+$  is 125  
Final spanned rank in  $S_4(K(266))^-$  is 29

-----  
 $S_2(K(266))$  is determined by Jacobi restriction and the  $H4Ndl(3)$  test  
( $\dim(H_4(266,3,1)) \leq 4$  and this is less than  $\dim(J_{\{2,266\}}^{\{\text{cusp}\}})+1 = 5$ )

So  $S_2(K(266)) = \text{Grit}(J_{\{2,266\}}^{\{\text{cusp}\}})$  (dimension 4)