

$$N = 258 = 2 \cdot 3 \cdot 43$$

By the Ibukiyama-Kitayama dimension formula,  
 $\dim(S_4(K(258))) = 163$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,  
the lift dimension of  $S_4(K(258))^+$  is 44  
the nonlift dimension of  $S_4(K(258))^+$  is heuristically 84  
 $\dim(S_4(K(258))^+)$  thus is heuristically 128  
 $\dim(S_4(K(258))^-)$  is heuristically 35

$\dim(J_{\{2,258\}}^{\{\text{cusp}\}}) = 4$  (Skoruppa-Zagier), so need to span to within 3 dimensions

$q = 7$  for TraceDown

After TD( $\text{Grit}(J_{\{4,1806\}}^{\{\text{cusp}\}})$ ) and  $(\text{Grit}(J_{\{2,258\}}^{\{\text{cusp}\}}))^2$ ,  
spanned rank in  $S_4(K(258))^+$  is 127  
spanned rank in  $S_4(K(258))^-$  is 0

Hecke operators applied:  $\{\{2, 2\}, \{2, 2\}, \{2, 1\}\}, \{\{3, 2\}, \{2, 2\}, \{3, 1\}\}$   
After Hecke spreading,  
spanned rank in  $S_4(K(258))^-$  is 24

After Borcherds products,  
spanned rank in  $S_4(K(258))^-$  is 34

Final spanned rank in  $S_4(K(258))^+$  is 127

Final spanned rank in  $S_4(K(258))^-$  is 34

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 $S_2(K(258))^+$  is determined by Jacobi restriction and the  $H4Ndl(2,+)$  test  
( $\dim(H_4(258,2,1)^+) \leq 3$  and this is less than  $\dim(J_{\{2,258\}}^{\{\text{cusp}\}})+1 = 5$ )  
 $S_2(K(258))^- = 0$  by Jacobi restriction and the  $H4Ndl(3,-)$  test  
( $\dim(H_4(258,3,1)^-) \leq 3$  and this is less than  $\dim(J_{\{2,258\}}^{\{\text{cusp}\}}) = 4$ )

So  $S_2(K(258)) = \text{Grit}(J_{\{2,258\}}^{\{\text{cusp}\}})$  (dimension 4)