

$$N = 253 = 11 \cdot 23$$

By the Ibukiyama-Kitayama dimension formula,  
 $\dim(S_4(K(253))) = 141$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,  
the lift dimension of  $S_4(K(253))^+$  is 49  
the nonlift dimension of  $S_4(K(253))^+$  is heuristically 82  
 $\dim(S_4(K(253))^+)$  thus is heuristically 131  
 $\dim(S_4(K(253))^-)$  is heuristically 10

$\dim(J_{\{2,253\}}^{\{\text{cusp}\}}) = 8$  (Skoruppa-Zagier), so need to span to within 7 dimensions

$q = 5$  for TraceDown

After TD( $\text{Grit}(J_{\{4,1265\}}^{\{\text{cusp}\}})$ ) and  $(\text{Grit}(J_{\{2,253\}}^{\{\text{cusp}\}}))^2$ ,  
spanned rank in  $S_4(K(253))^+$  is 131  
spanned rank in  $S_4(K(253))^-$  is 0

After Borcherds products,  
spanned rank in  $S_4(K(253))^-$  is 3

Final spanned rank in  $S_4(K(253))^+$  is 131

Final spanned rank in  $S_4(K(253))^-$  is 3

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 $S_2(K(253))^+$  is determined by Jacobi restriction and the  $H4Ndl(4,+)$  test  
( $\dim(H_4(253,4,1))^+ \leq 7$  and this is less than  $\dim(J_{\{2,253\}}^{\{\text{cusp}\}})+1 = 9$ )  
 $S_2(K(253))^- = 0$  by Jacobi restriction and the  $H4Ndl(1,-)$  test  
( $\dim(H_4(253,1,1))^- \leq 7$  and this is less than  $\dim(J_{\{2,253\}}^{\{\text{cusp}\}}) = 8$ )

So  $S_2(K(253)) = \text{Grit}(J_{\{2,253\}}^{\{\text{cusp}\}})$  (dimension 8)