

$$N = 249 = 3 \cdot 83$$

By the Ibukiyama-Kitayama dimension formula,  
 $\dim(S_4(K(249))) = 139$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,  
the lift dimension of  $S_4(K(249))^+$  is 45  
the nonlift dimension of  $S_4(K(249))^+$  is heuristically 82  
 $\dim(S_4(K(249))^+)$  thus is heuristically 127  
 $\dim(S_4(K(249))^-)$  is heuristically 12

$\dim(J_{\{2,249\}}^{\{\text{cusp}\}}) = 5$  (Skoruppa-Zagier), so need to span to within 4 dimensions

$q = 7$  for TraceDown

After TD( $\text{Grit}(J_{\{4,1743\}}^{\{\text{cusp}\}})$ ) and  $(\text{Grit}(J_{\{2,249\}}^{\{\text{cusp}\}}))^2$ ,  
spanned rank in  $S_4(K(249))^+$  is 126  
spanned rank in  $S_4(K(249))^-$  is 0

Hecke operators applied:  $\{\{3, 2\}\}$   
After Hecke spreading,  
spanned rank in  $S_4(K(249))^-$  is 4

After Borcherds products,  
spanned rank in  $S_4(K(249))^-$  is 10

Final spanned rank in  $S_4(K(249))^+$  is 126

Final spanned rank in  $S_4(K(249))^-$  is 10

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 $\dim(S_2(K(249))^+)$  is bounded by Jacobi restriction and the  $H4Ndl(3,+)$  test  
( $\dim(H_4(249,3,1))^+ \leq 3$  and this is less than  $\dim(J_{\{2,249\}}^{\{\text{cusp}\}})+1 = 6$ )  
 $S_2(K(249))^- = 0$  by Jacobi restriction and the  $H4Ndl(1,-)$  test  
( $\dim(H_4(249,1,1))^- \leq 3$  and this is less than  $\dim(J_{\{2,249\}}^{\{\text{cusp}\}}) = 5$ )

So  $\dim(S_2(K(249))^+) \leq \dim(J_{\{2,249\}}^{\{\text{cusp}\}})+1 = 6$  and  $S_2(K(249))^- = 0$