

$$N = 246 = 2 \cdot 3 \cdot 41$$

By the Ibukiyama-Kitayama dimension formula,  
 $\dim(S_4(K(246))) = 149$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,  
the lift dimension of  $S_4(K(246))^+$  is 42  
the nonlift dimension of  $S_4(K(246))^+$  is heuristically 82  
 $\dim(S_4(K(246))^+)$  thus is heuristically 124  
 $\dim(S_4(K(246))^-)$  is heuristically 25

$\dim(J_{\{2,246\}}^{\{\text{cusp}\}}) = 4$  (Skoruppa-Zagier), so need to span to within 3 dimensions

$q = 7$  for TraceDown

After TD( $\text{Grit}(J_{\{4,1722\}}^{\{\text{cusp}\}})$ ) and  $(\text{Grit}(J_{\{2,246\}}^{\{\text{cusp}\}}))^2$ ,  
spanned rank in  $S_4(K(246))^+$  is 121  
spanned rank in  $S_4(K(246))^-$  is 0

Hecke operators applied:  $\{\{2, 2\}\}, \{2, 2\}, \{2, 1\}\}, \{\{3, 2\}\}, \{2, 2\}, \{3, 1\}\}$   
After Hecke spreading,  
spanned rank in  $S_4(K(246))^-$  is 19

After Borcherds products,  
spanned rank in  $S_4(K(246))^-$  is 25

Final spanned rank in  $S_4(K(246))^+$  is 121  
Final spanned rank in  $S_4(K(246))^-$  is 25

---

$S_2(K(246))$  is determined by Jacobi restriction and the  $H_4Ndl(2)$  test  
( $\dim(H_4(246,2,1)) \leq 3$  and this is less than  $\dim(J_{\{2,246\}}^{\{\text{cusp}\}})+1 = 5$ )

So  $S_2(K(246)) = \text{Grit}(J_{\{2,246\}}^{\{\text{cusp}\}})$  (dimension 4)