

$$N = 238 = 2 \cdot 7 \cdot 17$$

By the Ibukiyama-Kitayama dimension formula,  
 $\dim(S_4(K(238))) = 134$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,  
the lift dimension of  $S_4(K(238))^+$  is 39  
the nonlift dimension of  $S_4(K(238))^+$  is heuristically 75  
 $\dim(S_4(K(238))^+)$  thus is heuristically 114  
 $\dim(S_4(K(238))^-)$  is heuristically 20

$\dim(J_{\{2,238\}}^{\text{cusp}}) = 2$  (Skoruppa-Zagier), so need to span to within 1 dimension

$q = 3$  for TraceDown

After TD( $\text{Grit}(J_{\{4,714\}}^{\text{cusp}})$ ) and ( $\text{Grit}(J_{\{2,238\}}^{\text{cusp}})$ )<sup>2</sup>,  
spanned rank in  $S_4(K(238))^+$  is 85  
spanned rank in  $S_4(K(238))^-$  is 0

Hecke operators applied:  $\{\{2, 2\}\}, \{2, 2\}, \{2, 1\}\}, \{2, 2\}, \{3, 1\}\}$

After Hecke spreading,  
spanned rank in  $S_4(K(238))^+$  is 89  
spanned rank in  $S_4(K(238))^-$  is 5

Merged in the plus basis attempt for  
 $q=5$ , raising the spanned rank in  $S_4(K(238))^+$  to 113

After Borcherds products,  
spanned rank in  $S_4(K(238))^-$  is 20

Final spanned rank in  $S_4(K(238))^+$  is 113  
Final spanned rank in  $S_4(K(238))^-$  is 20

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 $S_2(K(238))^+$  is determined by Jacobi restriction and the  $H4Ndl(2,+)$  test  
( $\dim(H_4(238,2,1))^+ \leq 1$  and this is less than  $\dim(J_{\{2,238\}}^{\text{cusp}})+1 = 3$ )  
 $S_2(K(238))^- = 0$  by Jacobi restriction and the  $H4Ndl(1,-)$  test  
( $\dim(H_4(238,1,1))^- \leq 1$  and this is less than  $\dim(J_{\{2,238\}}^{\text{cusp}}) = 2$ )

So  $S_2(K(238)) = \text{Grit}(J_{\{2,238\}}^{\text{cusp}})$  (dimension 2)