

$$N = 226 = 2 \cdot 113$$

By the Ibukiyama-Kitayama dimension formula,  
 $\dim(S_4(K(226))) = 134$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,  
the lift dimension of  $S_4(K(226))^+$  is 43  
the nonlift dimension of  $S_4(K(226))^+$  is heuristically 75  
 $\dim(S_4(K(226))^+)$  thus is heuristically 118  
 $\dim(S_4(K(226))^-)$  is heuristically 16

$\dim(J_{\{2,226\}}^{\{\text{cusp}\}}) = 6$  (Skoruppa-Zagier), so need to span to within 5 dimensions

$q = 7$  for TraceDown

After TD( $\text{Grit}(J_{\{4,1582\}}^{\{\text{cusp}\}})$ ) and  $(\text{Grit}(J_{\{2,226\}}^{\{\text{cusp}\}}))^2$ ,  
spanned rank in  $S_4(K(226))^+$  is 117  
spanned rank in  $S_4(K(226))^-$  is 0

Hecke operators applied:  $\{\{2, 2\}\}$

After Hecke spreading,  
spanned rank in  $S_4(K(226))^-$  is 12

Final spanned rank in  $S_4(K(226))^+$  is 117

Final spanned rank in  $S_4(K(226))^-$  is 12

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 $S_2(K(226))$  is determined by Jacobi restriction and the  $H4Nd1(3)$  test  
( $\dim(H_4(226,3,1)) \leq 5$  and this is less than  $\dim(J_{\{2,226\}}^{\{\text{cusp}\}})+1 = 7$ )

So  $S_2(K(226)) = \text{Grit}(J_{\{2,226\}}^{\{\text{cusp}\}})$  (dimension 6)