

$$N = 222 = 2 \cdot 3 \cdot 37$$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(222))) = 120$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(222))^+$ is 35
the nonlift dimension of $S_4(K(222))^+$ is heuristically 61
 $\dim(S_4(K(222))^+)$ thus is heuristically 96
 $\dim(S_4(K(222))^-)$ is heuristically 24

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,222\}}^{\{\text{cusp}\}}) = 1$ (Skoruppa-Zagier), so need to span completely

$q = 7$ for TraceDown

After TD($\text{Grit}(J_{\{4,1554\}}^{\{\text{cusp}\}})$) and $(\text{Grit}(J_{\{2,222\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(222))^+$ is 95
spanned rank in $S_4(K(222))^-$ is 0

After Borcherds products,

spanned rank in $S_4(K(222))^+$ is 96
spanned rank in $S_4(K(222))^-$ is 24

Final spanned rank in $S_4(K(222))^+$ is 96

Final spanned rank in $S_4(K(222))^-$ is 24

$S_2(K(222))$ is determined by Jacobi restriction and the $H4Ndd(3,+)$ test
 $(H_4(222,3,3))^+ = 0$

So $S_2(K(222)) = \text{Grit}(J_{\{2,222\}}^{\{\text{cusp}\}})$ (dimension 1)