

$$N = 221 = 13 \cdot 17$$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(221))) = 104$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(221))^+$ is 40
the nonlift dimension of $S_4(K(221))^+$ is heuristically 55
 $\dim(S_4(K(221))^+)$ thus is heuristically 95
 $\dim(S_4(K(221))^-)$ is heuristically 9

$\dim(J_{\{2,221\}}^{\{\text{cusp}\}}) = 5$ (Skoruppa-Zagier), so need to span to within 4 dimensions

$q = 5$ for TraceDown

After TD($\text{Grit}(J_{\{4,1105\}}^{\{\text{cusp}\}})$) and $(\text{Grit}(J_{\{2,221\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(221))^+$ is 95
spanned rank in $S_4(K(221))^-$ is 0

After Borcherds products,
spanned rank in $S_4(K(221))^-$ is 6

Final spanned rank in $S_4(K(221))^+$ is 95

Final spanned rank in $S_4(K(221))^-$ is 6

 $S_2(K(221))^+$ is determined by Jacobi restriction and the $H4Ndl(3,+)$ test
($\dim(H_4(221,3,1))^+ \leq 4$ and this is less than $\dim(J_{\{2,221\}}^{\{\text{cusp}\}})+1 = 6$)
 $S_2(K(221))^- = 0$ by Jacobi restriction and the $H4Ndl(1,-)$ test
($\dim(H_4(221,1,1))^- \leq 3$ and this is less than $\dim(J_{\{2,221\}}^{\{\text{cusp}\}}) = 5$)

So $S_2(K(221)) = \text{Grit}(J_{\{2,221\}}^{\{\text{cusp}\}})$ (dimension 5)