

$$N = 218 = 2 \cdot 109$$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(218))) = 119$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(218))^+$ is 41
the nonlift dimension of $S_4(K(218))^+$ is heuristically 55
 $\dim(S_4(K(218))^+)$ thus is heuristically 96
 $\dim(S_4(K(218))^-)$ is heuristically 23

$\dim(J_{\{2,218\}}^{\{\text{cusp}\}}) = 6$ (Skoruppa-Zagier), so need to span to within 5 dimensions

$q = 5$ for TraceDown

After TD($\text{Grit}(J_{\{4,1090\}}^{\{\text{cusp}\}})$) and $(\text{Grit}(J_{\{2,218\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(218))^+$ is 96
spanned rank in $S_4(K(218))^-$ is 0

Hecke operators applied: $\{\{2, 2\}, \{2, 2\}, \{2, 1\}\}$
After Hecke spreading,
spanned rank in $S_4(K(218))^-$ is 17

After Borcherds products,
spanned rank in $S_4(K(218))^-$ is 22

Final spanned rank in $S_4(K(218))^+$ is 96

Final spanned rank in $S_4(K(218))^-$ is 22

 $S_2(K(218))^+$ is determined by Jacobi restriction and the $H4Ndl(2,+)$ test
($\dim(H_4(218,2,1)^+) \leq 3$ and this is less than $\dim(J_{\{2,218\}}^{\{\text{cusp}\}})+1 = 7$)
 $S_2(K(218))^- = 0$ by Jacobi restriction and the $H4Ndl(1,-)$ test
($\dim(H_4(218,1,1)^-) \leq 3$ and this is less than $\dim(J_{\{2,218\}}^{\{\text{cusp}\}}) = 6$)

So $S_2(K(218)) = \text{Grit}(J_{\{2,218\}}^{\{\text{cusp}\}})$ (dimension 6)