```
By the Ibukiyama-Kitayama dimension formula,
```

dim  $(S_4(K(217))) = 113$ 

N = 217 = 7 \* 31

By the Skoruppa-Zagier dimension formula and Jacobi restriction, the lift dimension of  $S_-4\left(K\left(217\right)\right)^++$  is 41 the nonlift dimension of  $S_-4\left(K\left(217\right)\right)^++$  is heuristically 66  $dim\left(S_-4\left(K\left(217\right)\right)^++\right)$  thus is heuristically 107  $dim\left(S_-4\left(K\left(217\right)\right)^--\right)$  is heuristically 6

The heuristic dimensions are correct by the spanning results to follow

```
\label{eq:dim_space} \begin{split} &\text{dim}(J_{\{2,217\}}^{\text{cusp}}) = 6 \text{ (Skoruppa-Zagier), so need to span to within 5 dimensions} \\ &q = 5 \text{ for TraceDown} \\ &\text{After TD}(\text{Grit}(J_{\{4,1085\}}^{\text{cusp}})) \text{ and } (\text{Grit}(J_{\{2,217\}}^{\text{cusp}}))^2, \end{split}
```

spanned rank in  $S_4(K(217))^-$  is 0 After Borcherds products,

spanned rank in S  $4(K(217))^+$  is 107

spanned rank in  $S_4(K(217))^-$  is 6

```
Final spanned rank in S_4(K(217))^+ is 107 Final spanned rank in S_4(K(217))^- is 6
```

 $S_2(K(217))$  is determined by Jacobi restriction and the  ${\tt H4Ndd}(2,+)$  test  $({\tt H_4}(217,2,2)^+ = 0)$ 

```
So S_2(K(217)) = Grit(J_{2,217}^{cusp}) (dimension 6)
```