

$$N = 186 = 2 \cdot 3 \cdot 31$$

By the Ibukiyama-Kitayama dimension formula,  
 $\dim(S_4(K(186))) = 85$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,  
the lift dimension of  $S_4(K(186))^+$  is 30  
the nonlift dimension of  $S_4(K(186))^+$  is heuristically 43  
 $\dim(S_4(K(186))^+)$  thus is heuristically 73  
 $\dim(S_4(K(186))^-)$  is heuristically 12

$\dim(J_{\{2,186\}}^{\{\text{cusp}\}}) = 2$  (Skoruppa-Zagier), so need to span to within 1 dimension

$q = 7$  for TraceDown

After TD( $\text{Grit}(J_{\{4,1302\}}^{\{\text{cusp}\}})$ ) and  $(\text{Grit}(J_{\{2,186\}}^{\{\text{cusp}\}}))^2$ ,  
spanned rank in  $S_4(K(186))^+$  is 72  
spanned rank in  $S_4(K(186))^-$  is 0

Hecke operators applied:  $\{\{2, 2\}, \{2, 2\}, \{2, 1\}, \{3, 2\}\}$   
After Hecke spreading,  
spanned rank in  $S_4(K(186))^-$  is 7

After Borcherds products,  
spanned rank in  $S_4(K(186))^-$  is 12

Final spanned rank in  $S_4(K(186))^+$  is 72

Final spanned rank in  $S_4(K(186))^-$  is 12

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$S_2(K(186))^+$  is determined by Jacobi restriction and the  $H4Ndl(2,+)$  test  
( $\dim(H_4(186,2,1)^+) \leq 2$  and this is less than  $\dim(J_{\{2,186\}}^{\{\text{cusp}\}})+1 = 3$ )  
 $S_2(K(186))^- = 0$  by Jacobi restriction and the  $H4Ndl(1,-)$  test  
( $\dim(H_4(186,1,1)^-) \leq 1$  and this is less than  $\dim(J_{\{2,186\}}^{\{\text{cusp}\}}) = 2$ )

So  $S_2(K(186)) = \text{Grit}(J_{\{2,186\}}^{\{\text{cusp}\}})$  (dimension 2)