

$$N = 182 = 2 \cdot 7 \cdot 13$$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(182))) = 74$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(182))^+$ is 29
the nonlift dimension of $S_4(K(182))^+$ is heuristically 32
 $\dim(S_4(K(182))^+)$ thus is heuristically 61
 $\dim(S_4(K(182))^-)$ is heuristically 13

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,182\}}^{\{\text{cusp}\}}) = 2$ (Skoruppa-Zagier), so need to span to within 1 dimension

$q = 5$ for TraceDown

After TD($\text{Grit}(J_{\{4,910\}}^{\{\text{cusp}\}})$) and $(\text{Grit}(J_{\{2,182\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(182))^+$ is 61
spanned rank in $S_4(K(182))^-$ is 0

Hecke operators applied: $\{\{2, 2\}, \{2, 2\}, \{2, 1\}\}, \{\{2, 2\}, \{3, 1\}\}$
After Hecke spreading,
spanned rank in $S_4(K(182))^-$ is 7

After Borcherds products,
spanned rank in $S_4(K(182))^-$ is 13

Final spanned rank in $S_4(K(182))^+$ is 61
Final spanned rank in $S_4(K(182))^-$ is 13

$S_2(K(182))$ is determined by Jacobi restriction and the $H4Ndd(2,+)$ test
($H_4(182,2,2)^+ = 0$)

So $S_2(K(182)) = \text{Grit}(J_{\{2,182\}}^{\{\text{cusp}\}})$ (dimension 2)