

$$N = 177 = 3 \cdot 59$$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(177))) = 74$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(177))^+$ is 32
the nonlift dimension of $S_4(K(177))^+$ is heuristically 37
 $\dim(S_4(K(177))^+)$ thus is heuristically 69
 $\dim(S_4(K(177))^-)$ is heuristically 5

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,177\}}^{\{\text{cusp}\}}) = 4$ (Skoruppa-Zagier), so need to span to within 3 dimensions

$q = 7$ for TraceDown

After TD($\text{Grit}(J_{\{4,1239\}}^{\{\text{cusp}\}})$) and $(\text{Grit}(J_{\{2,177\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(177))^+$ is 69
spanned rank in $S_4(K(177))^-$ is 0

Hecke operators applied: $\{\{3, 2\}\}$

After Hecke spreading,
spanned rank in $S_4(K(177))^-$ is 2

After Borcherds products,
spanned rank in $S_4(K(177))^-$ is 5

Final spanned rank in $S_4(K(177))^+$ is 69

Final spanned rank in $S_4(K(177))^-$ is 5

$S_2(K(177))$ is determined by Jacobi restriction and the $H4Ndd(2,+)$ test
($H_4(177,2,2)^+ = 0$)

So $S_2(K(177)) = \text{Grit}(J_{\{2,177\}}^{\{\text{cusp}\}})$ (dimension 4)