

$$N = 170 = 2 \cdot 5 \cdot 17$$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(170))) = 70$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(170))^+$ is 28
the nonlift dimension of $S_4(K(170))^+$ is heuristically 31
 $\dim(S_4(K(170))^+)$ thus is heuristically 59
 $\dim(S_4(K(170))^-)$ is heuristically 11

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,170\}}^{\{\text{cusp}\}}) = 3$ (Skoruppa-Zagier), so need to span to within 2 dimensions

$q = 7$ for TraceDown

After TD($\text{Grit}(J_{\{4,1190\}}^{\{\text{cusp}\}})$) and $(\text{Grit}(J_{\{2,170\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(170))^+$ is 59
spanned rank in $S_4(K(170))^-$ is 0

Hecke operators applied: $\{\{2, 2\}, \{2, 2\}, \{2, 1\}\}, \{\{2, 2\}, \{3, 1\}\}$

After Hecke spreading,
spanned rank in $S_4(K(170))^-$ is 9

After Borcherds products,
spanned rank in $S_4(K(170))^-$ is 11

Final spanned rank in $S_4(K(170))^+$ is 59

Final spanned rank in $S_4(K(170))^-$ is 11

$S_2(K(170))$ is determined by Jacobi restriction and the $H4Ndd(2,+)$ test
($H_4(170,2,2)^+ = 0$)

So $S_2(K(170)) = \text{Grit}(J_{\{2,170\}}^{\{\text{cusp}\}})$ (dimension 3)