

$N = 166 = 2 \cdot 83$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(166))) = 75$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(166))^+$ is 29
the nonlift dimension of $S_4(K(166))^+$ is heuristically 40
 $\dim(S_4(K(166))^+)$ thus is heuristically 69
 $\dim(S_4(K(166))^-)$ is heuristically 6

$\dim(J_{\{2,166\}}^{\{\text{cusp}\}}) = 2$ (Skoruppa-Zagier), so need to span to within 1 dimension

$q = 7$ for TraceDown

After TD($\text{Grit}(J_{\{4,1162\}}^{\{\text{cusp}\}})$) and $(\text{Grit}(J_{\{2,166\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(166))^+$ is 68
spanned rank in $S_4(K(166))^-$ is 0

Hecke operators applied: $\{\{2, 2\}, \{2, 2\}, \{2, 1\}\}$
After Hecke spreading,
spanned rank in $S_4(K(166))^-$ is 4

After Borcherds products,
spanned rank in $S_4(K(166))^-$ is 6

Final spanned rank in $S_4(K(166))^+$ is 68

Final spanned rank in $S_4(K(166))^-$ is 6

 $S_2(K(166))^+$ is determined by Jacobi restriction and the $H4Ndl(2,+)$ test
($\dim(H_4(166,2,1)^+) \leq 2$ and this is less than $\dim(J_{\{2,166\}}^{\{\text{cusp}\}})+1 = 3$)
 $S_2(K(166))^- = 0$ by Jacobi restriction and the $H4Ndl(1,-)$ test
($\dim(H_4(166,1,1)^-) \leq 1$ and this is less than $\dim(J_{\{2,166\}}^{\{\text{cusp}\}}) = 2$)

So $S_2(K(166)) = \text{Grit}(J_{\{2,166\}}^{\{\text{cusp}\}})$ (dimension 2)