

$$N = 161 = 7 \cdot 23$$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(161))) = 57$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(161))^+$ is 27
the nonlift dimension of $S_4(K(161))^+$ is heuristically 28
 $\dim(S_4(K(161))^+)$ thus is heuristically 55
 $\dim(S_4(K(161))^-)$ is heuristically 2

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,161\}}^{\{\text{cusp}\}}) = 2$ (Skoruppa-Zagier), so need to span to within 1 dimension

$q = 5$ for TraceDown

After TD($\text{Grit}(J_{\{4,805\}}^{\{\text{cusp}\}})$) and $(\text{Grit}(J_{\{2,161\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(161))^+$ is 55
spanned rank in $S_4(K(161))^-$ is 0

After Borcherds products,
spanned rank in $S_4(K(161))^-$ is 2

Final spanned rank in $S_4(K(161))^+$ is 55

Final spanned rank in $S_4(K(161))^-$ is 2

$S_2(K(161))$ is determined by Jacobi restriction and the $H4Ndd(2,+)$ test
($H_4(161,2,2)^+ = 0$)

So $S_2(K(161)) = \text{Grit}(J_{\{2,161\}}^{\{\text{cusp}\}})$ (dimension 2)