

$$N = 146 = 2 \cdot 73$$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(146))) = 57$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(146))^+$ is 25
the nonlift dimension of $S_4(K(146))^+$ is heuristically 24
 $\dim(S_4(K(146))^+)$ thus is heuristically 49
 $\dim(S_4(K(146))^-)$ is heuristically 8

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,146\}}^{\{\text{cusp}\}}) = 2$ (Skoruppa-Zagier), so need to span to within 1 dimension

$q = 7$ for TraceDown

After TD($\text{Grit}(J_{\{4,1022\}}^{\{\text{cusp}\}})$) and $(\text{Grit}(J_{\{2,146\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(146))^+$ is 49
spanned rank in $S_4(K(146))^-$ is 0

Hecke operators applied: $\{\{2, 2\}\}, \{\{2, 2\}, \{2, 1\}\}, \{\{2, 2\}, \{3, 1\}\}$
After Hecke spreading,
spanned rank in $S_4(K(146))^-$ is 7

After Borcherds products,
spanned rank in $S_4(K(146))^-$ is 8

Final spanned rank in $S_4(K(146))^+$ is 49
Final spanned rank in $S_4(K(146))^-$ is 8

$S_2(K(146))$ is determined by Jacobi restriction and the $H4Ndd(2,+)$ test
($H_4(146,2,2)^+ = 0$)

So $S_2(K(146)) = \text{Grit}(J_{\{2,146\}}^{\{\text{cusp}\}})$ (dimension 2)