

$N = 138 = 2 \cdot 3 \cdot 23$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(138))) = 48$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(138))^+$ is 21
the nonlift dimension of $S_4(K(138))^+$ is heuristically 20
 $\dim(S_4(K(138))^+)$ thus is heuristically 41
 $\dim(S_4(K(138))^-)$ is heuristically 7

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,138\}}^{\{\text{cusp}\}}) = 1$ (Skoruppa-Zagier), so need to span completely

$q = 7$ for TraceDown

After $\text{TD}(\text{Grit}(J_{\{4,966\}}^{\{\text{cusp}\}}))$ and $(\text{Grit}(J_{\{2,138\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(138))^+$ is 41
spanned rank in $S_4(K(138))^-$ is 0

Hecke operators applied: $\{\{3, 2\}\}$

After Hecke spreading,
spanned rank in $S_4(K(138))^-$ is 1

After Borcherds products,
spanned rank in $S_4(K(138))^-$ is 7

Final spanned rank in $S_4(K(138))^+$ is 41

Final spanned rank in $S_4(K(138))^-$ is 7

 $S_2(K(138))^+$ is determined by Jacobi restriction and the $H4Nd1(1,+)$ test
($\dim(H_4(138,1,1)^+) \leq 1$ and this is less than $\dim(J_{\{2,138\}}^{\{\text{cusp}\}})+1 = 2$)
 $S_2(K(138))^- = 0$ by Jacobi restriction and the $H4Nd1(1,-)$ test
($\dim(H_4(138,1,1)^-) \leq 0$ and this is less than $\dim(J_{\{2,138\}}^{\{\text{cusp}\}}) = 1$)

So $S_2(K(138)) = \text{Grit}(J_{\{2,138\}}^{\{\text{cusp}\}})$ (dimension 1)