

$$N = 129 = 3 \cdot 43$$

By the Ibukiyama-Kitayama dimension formula,  
 $\dim(S_4(K(129))) = 44$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,  
the lift dimension of  $S_4(K(129))^+$  is 22  
the nonlift dimension of  $S_4(K(129))^+$  is heuristically 21  
 $\dim(S_4(K(129))^+)$  thus is heuristically 43  
 $\dim(S_4(K(129))^-)$  is heuristically 1

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,129\}}^{\{\text{cusp}\}}) = 2$  (Skoruppa-Zagier), so need to span to within 1 dimension

$q = 5$  for TraceDown

After TD( $\text{Grit}(J_{\{4,645\}}^{\{\text{cusp}\}})$ ) and  $(\text{Grit}(J_{\{2,129\}}^{\{\text{cusp}\}}))^2$ ,  
spanned rank in  $S_4(K(129))^+$  is 43  
spanned rank in  $S_4(K(129))^-$  is 0

Hecke operators applied:  $\{\{3, 2\}\}$

After Hecke spreading,  
spanned rank in  $S_4(K(129))^-$  is 1

Final spanned rank in  $S_4(K(129))^+$  is 43

Final spanned rank in  $S_4(K(129))^-$  is 1

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 $S_2(K(129))$  is determined by Jacobi restriction and the  $H4Ndd(2,+)$  test  
 $(H_4(129,2,2))^+ = 0$

So  $S_2(K(129)) = \text{Grit}(J_{\{2,129\}}^{\{\text{cusp}\}})$  (dimension 2)