

$$N = 123 = 3 \cdot 41$$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(123))) = 37$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(123))^+$ is 21
the nonlift dimension of $S_4(K(123))^+$ is heuristically 13
 $\dim(S_4(K(123))^+)$ thus is heuristically 34
 $\dim(S_4(K(123))^-)$ is heuristically 3

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,123\}}^{\{\text{cusp}\}}) = 2$ (Skoruppa-Zagier), so need to span to within 1 dimension

$q = 5$ for TraceDown

After TD($\text{Grit}(J_{\{4,615\}}^{\{\text{cusp}\}})$) and $(\text{Grit}(J_{\{2,123\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(123))^+$ is 34
spanned rank in $S_4(K(123))^-$ is 0

Hecke operators applied: $\{\{3, 2\}\}$

After Hecke spreading,
spanned rank in $S_4(K(123))^-$ is 1

After Borcherds products,
spanned rank in $S_4(K(123))^-$ is 3

Final spanned rank in $S_4(K(123))^+$ is 34

Final spanned rank in $S_4(K(123))^-$ is 3

$S_2(K(123))$ is determined by Jacobi restriction and the $H4Ndd(2,+)$ test
($H_4(123,2,2)^+ = 0$)

So $S_2(K(123)) = \text{Grit}(J_{\{2,123\}}^{\{\text{cusp}\}})$ (dimension 2)