

$$N = 122 = 2 \cdot 61$$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(122))) = 41$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(122))^+$ is 21
the nonlift dimension of $S_4(K(122))^+$ is heuristically 14
 $\dim(S_4(K(122))^+)$ thus is heuristically 35
 $\dim(S_4(K(122))^-)$ is heuristically 6

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,122\}}^{\{\text{cusp}\}}) = 2$ (Skoruppa-Zagier), so need to span to within 1 dimension

$q = 3$ for TraceDown

After TD($\text{Grit}(J_{\{4,366\}}^{\{\text{cusp}\}})$) and $(\text{Grit}(J_{\{2,122\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(122))^+$ is 35
spanned rank in $S_4(K(122))^-$ is 0

Hecke operators applied: $\{\{2, 2\}, \{2, 2\}, \{2, 1\}\}$

After Hecke spreading,
spanned rank in $S_4(K(122))^-$ is 5

After Borcherds products,
spanned rank in $S_4(K(122))^-$ is 6

Final spanned rank in $S_4(K(122))^+$ is 35

Final spanned rank in $S_4(K(122))^-$ is 6

$S_2(K(122))$ is determined by Jacobi restriction and the $H4Ndd(2,+)$ test
($H_4(122,2,2)^+ = 0$)

So $S_2(K(122)) = \text{Grit}(J_{\{2,122\}}^{\{\text{cusp}\}})$ (dimension 2)