

$$N = 111 = 3 \cdot 37$$

By the Ibukiyama-Kitayama dimension formula,
 $\dim(S_4(K(111))) = 33$

By the Skoruppa-Zagier dimension formula and Jacobi restriction,
the lift dimension of $S_4(K(111))^+$ is 18
the nonlift dimension of $S_4(K(111))^+$ is heuristically 14
 $\dim(S_4(K(111))^+)$ thus is heuristically 32
 $\dim(S_4(K(111))^-)$ is heuristically 1

The heuristic dimensions are correct by the spanning results to follow

$\dim(J_{\{2,111\}}^{\{\text{cusp}\}}) = 1$ (Skoruppa-Zagier), so need to span completely

$q = 5$ for TraceDown

After TD($\text{Grit}(J_{\{4,555\}}^{\{\text{cusp}\}})$) and $(\text{Grit}(J_{\{2,111\}}^{\{\text{cusp}\}}))^2$,
spanned rank in $S_4(K(111))^+$ is 32
spanned rank in $S_4(K(111))^-$ is 0

Hecke operators applied: $\{\{3, 2\}\}$

After Hecke spreading,
spanned rank in $S_4(K(111))^-$ is 1

Final spanned rank in $S_4(K(111))^+$ is 32

Final spanned rank in $S_4(K(111))^-$ is 1

$S_2(K(111))$ is determined by Jacobi restriction and the $H4Nd1(1)$ test
($\dim(H_4(111,1,1)) \leq 1$ and this is less than $\dim(J_{\{2,111\}}^{\{\text{cusp}\}})+1 = 2$)

So $S_2(K(111)) = \text{Grit}(J_{\{2,111\}}^{\{\text{cusp}\}})$ (dimension 1)