N = 62

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}

lift dimension of S_4(K(62))^+ : 9

nonlift dimension of S_4(K(62))^+ : 2

dimension of S_4(K(62))^- : 1

Ibukiyama-Kitayama dimension of S_4(K(62)) : 12

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(62))^+ = 11

Rank of minus basis attempt = 0 and dimension of S_4(K(62))^- = 1

Initial short vector length: 24

Have vectors of length 29: long enough

Determinant shell containing the vectors of length 24 : 48

Short vector length is 29, and the corresponding determinant is 48

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-62-50.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,62}^{cusp}), which is 9

Rank of plus basis attempt = 9 and dimension of S_4(K(62))^+ = 11

Rank of minus basis attempt = 0 and dimension of S_4(K(62))^- = 1

STEP 2: TRACE DOWN

N = 62 q = 5 Nq = 310

29-th determinant is 48

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 15

Trace down to try to hit S4(K(N)), of dimension 12

Plus lifts, plus nonlifts, minus dimension: {9, 2, 1}

Target plus rank, target minus rank: 11 1

Dimensions of savedTargetMats: {29, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-310-1200.ma, Getting it

 ... done making G2. Dimensions of Grits: {5, 5688}

Determining indices: {1, 2, 5, 6, 9}

```
Atkin-Lehner truncation: 14

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 3}, {{1, -1, -1}, 1}, {{-1, 1, -1}, 1}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-310-1200-to-62-48.ma exists, reading it
Found file Grits-2-310-1200.ma, Getting it

Grit dimensions: {5, 5688}

Viable file found: GG-Grits-2-310-1200.ma-1200-mod-12347.ma

New best file: GG-Grits-2-310-1200.ma-1200-mod-12347.ma

Dimensions of G2G2: {15, 5688}

Dimensions of fDown = {15, 29}

Rank of fDown mod pp = 11

Dimensions and rank of plus space fDown = {15, 29}  11

Dimensions and rank of minus space fDown = {15, 29}  0

Rank of plus basis attempt = 11 and dimension of S_4(K(62))^+ = 11

Rank of minus basis attempt = 0 and dimension of S_4(K(62))^- = 1
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 0
```

STEP 4: HECKE SPREAD

```
J2=0, or already have plus space basis and no extra Hecke spreads were specified

Hecke spreads: 0
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-62-100.ma}

Dimensions of BPcoeffMatProven: {1, 89}

Dimensions of join: {1, 29}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 11 and dimension of S_4(K(62))^+ = 11

Rank of minus basis attempt = 1 and dimension of S_4(K(62))^- = 1
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}
```

Found a plus basis of dimension 11

Found minus basis of dimension 1

N = 65

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}

lift dimension of S_4(K(65))^+ : 10

nonlift dimension of S_4(K(65))^+ : 3

dimension of S_4(K(65))^- : 0

Ibukiyama-Kitayama dimension of S_4(K(65)) : 13

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(65))^+ = 13

Rank of minus basis attempt = 0 and dimension of S_4(K(65))^- = 0

Initial short vector length: 25

Have vectors of length 36: long enough

Determinant shell containing the vectors of length 25 : 55

Short vector length is 28, and the corresponding determinant is 55

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-65-60.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,65}^{cusp}), which is 10

Rank of plus basis attempt = 10 and dimension of S_4(K(65))^+ = 13

Rank of minus basis attempt = 0 and dimension of S_4(K(65))^- = 0

STEP 2: TRACE DOWN

N = 65 q = 7 Nq = 455

28-th determinant is 55

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 15

Trace down to try to hit S4(K(N)), of dimension 13

Plus lifts, plus nonlifts, minus dimension: {10, 3, 0}

Target plus rank, target minus rank: 13 0

Dimensions of savedTargetMats: {28, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

```
Found file Grits-2-455-2695.ma, Getting it
 ... done making G2.  Dimensions of Grits: {5, 15234}
Determining indices: {1, 3, 4, 7, 15}
Atkin-Lehner truncation: 22
Beginning ALSignaturesAndDims...
 ... done with ALSignaturesAndDims.
ALspacesDims = {{{1, 1, 1}, 3}, {{1, -1, -1}, 1}, {{-1, 1, -1}, 1}}
Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-455-2695-to-65-55.ma exists, reading it
Found file Grits-2-455-2695.ma, Getting it
Grit dimensions: {5, 15234}
Viable file found: GG-Grits-2-455-2695.ma-2695-mod-12347.ma
New best file: GG-Grits-2-455-2695.ma-2695-mod-12347.ma
Dimensions of G2G2: {15, 15234}
Dimensions of fDown = {15, 28}
Rank of fDown mod pp = 10
Dimensions and rank of plus space fDown = {15, 28}  10
Dimensions and rank of minus space fDown = {15, 28}  0
Rank of plus basis attempt = 13 and dimension of S_4(K(65))^+ = 13
Rank of minus basis attempt = 0 and dimension of S_4(K(65))^- = 0

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS
Pairs from dim(J_{2,N}^{cusp}: 1
Found file Grits-2-65-55.ma, Getting it
Viable file found: GG-Grits-2-65-1024.ma-1024-mod-12347.ma
New best file: GG-Grits-2-65-1024.ma-1024-mod-12347.ma
Viable file found: GG-Grits-2-65-5184.ma-5184-mod-12347.ma
Viable file found: GG-Grits-2-65-6399.ma-6399-mod-12347.ma
Dimensions of G2G2: {1, 28}
Rank of plus basis attempt = 13 and dimension of S_4(K(65))^+ = 13
Rank of minus basis attempt = 0 and dimension of S_4(K(65))^- = 0

STEP 4: HECKE SPREAD
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {}

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 13

Found minus basis of dimension 0

N = 66

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {13, 0, 0}

lift dimension of S_4(K(66))^+ : 8

nonlift dimension of S_4(K(66))^+ : 3

dimension of S_4(K(66))^- : 0

Ibukiyama-Kitayama dimension of S_4(K(66)) : 11

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(66))^+ = 11

Rank of minus basis attempt = 0 and dimension of S_4(K(66))^- = 0

Initial short vector length: 23

Have vectors of length 28: long enough

Determinant shell containing the vectors of length 23 : 44

Short vector length is 28, and the corresponding determinant is 44

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-66-50.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,66}^{cusp}), which is 8

Rank of plus basis attempt = 8 and dimension of S_4(K(66))^+ = 11

Rank of minus basis attempt = 0 and dimension of S_4(K(66))^- = 0

STEP 2: TRACE DOWN

N = 66 q = 13 Nq = 858

28-th determinant is 44

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 136

Trace down to try to hit S4(K(N)), of dimension 11

```
Plus lifts, plus nonlifts, minus dimension: {8, 3, 0}

Target plus rank, target minus rank: 11  0

Dimensions of savedTargetMats: {28, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-858-7436.ma, Getting it

 ... done making G2.  Dimensions of Grits: {16, 118 060}

Determining indices: {1, 2, 9, 10, 11, 12, 25, 26, 27, 49, 50, 51, 52, 113, 213, 229}

Atkin-Lehner truncation: 260

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1, 1}, 6}, {{1, 1, -1, -1}, 3},
  {{1, -1, 1, -1}, 2}, {{1, -1, -1, 1}, 2}, {{-1, 1, 1, -1}, 2}, {{-1, 1, -1, 1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-858-7436-to-66-44.ma exists, reading it

Found file Grits-2-858-7436.ma, Getting it

Grit dimensions: {16, 118 060}

Viable file found: GG-Grits-2-858-7436.ma-7436-mod-12347.ma

New best file: GG-Grits-2-858-7436.ma-7436-mod-12347.ma

Dimensions of G2G2: {136, 118 060}

Dimensions of fDown = {136, 28}

Rank of fDown mod pp = 11

Dimensions and rank of plus space fDown = {136, 28}  11

Dimensions and rank of minus space fDown = {136, 28}  0

Rank of plus basis attempt = 11 and dimension of S_4(K(66))^+ = 11

Rank of minus basis attempt = 0 and dimension of S_4(K(66))^- = 0


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 0


STEP 4: HECKE SPREAD

J2=0, or already have plus space basis and no extra Hecke spreads were specified

Hecke spreads: 0


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
```

```
Found files: {}
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Found a plus basis of dimension 11

Found minus basis of dimension 0
```

N = 69

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}

lift dimension of S_4(K(69))^+ : 10

nonlift dimension of S_4(K(69))^+ : 4

dimension of S_4(K(69))^- : 0

Ibukiyama-Kitayama dimension of S_4(K(69)) : 14
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(69))^+ = 14

Rank of minus basis attempt = 0 and dimension of S_4(K(69))^- = 0

Initial short vector length: 22

Have vectors of length 33: long enough

Determinant shell containing the vectors of length 22 : 56

Short vector length is 29, and the corresponding determinant is 56

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-69-60.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,69}^{cusp}), which is 10

Rank of plus basis attempt = 10 and dimension of S_4(K(69))^+ = 14

Rank of minus basis attempt = 0 and dimension of S_4(K(69))^- = 0
```

STEP 2: TRACE DOWN

```
N = 69  q = 5  Nq = 345

29-th determinant is 56

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 36

Trace down to try to hit S4(K(N)), of dimension 14

Plus lifts, plus nonlifts, minus dimension: {10, 4, 0}
```

```
Target plus rank, target minus rank: 14 0

Dimensions of savedTargetMats: {29, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-345-1400.ma, Getting it

 ... done making G2.  Dimensions of Grits: {8, 6414}

Determining indices: {1, 2, 3, 4, 5, 7, 11, 12}

Atkin-Lehner truncation: 26

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 4}, {{1, -1, -1}, 2}, {{-1, 1, -1}, 1}, {{-1, -1, 1}, 1}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-345-1400-to-69-56.ma exists, reading it

Found file Grits-2-345-1400.ma, Getting it

Grit dimensions: {8, 6414}

Viable file found: GG-Grits-2-345-1400.ma-1400-mod-12347.ma

New best file: GG-Grits-2-345-1400.ma-1400-mod-12347.ma

Dimensions of G2G2: {36, 6414}

Dimensions of fDown = {36, 29}

Rank of fDown mod pp = 14

Dimensions and rank of plus space fDown = {36, 29}  14

Dimensions and rank of minus space fDown = {36, 29}  0

Rank of plus basis attempt = 14 and dimension of S_4(K(69))^+ = 14

Rank of minus basis attempt = 0 and dimension of S_4(K(69))^- = 0


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 0


STEP 4: HECKE SPREAD

J2=0, or already have plus space basis and no extra Hecke spreads were specified

Hecke spreads: 0


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {}
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 14

Found minus basis of dimension 0

N = 70

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {11, 0, 0}

lift dimension of S_4(K(70))^+ : 9

nonlift dimension of S_4(K(70))^+ : 5

dimension of S_4(K(70))^- : 0

Ibukiyama-Kitayama dimension of S_4(K(70)) : 14

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(70))^+ = 14

Rank of minus basis attempt = 0 and dimension of S_4(K(70))^- = 0

Initial short vector length: 21

Have vectors of length 32: long enough

Determinant shell containing the vectors of length 21 : 55

Short vector length is 28, and the corresponding determinant is 55

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-70-60.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,70}^{cusp}), which is 9

Rank of plus basis attempt = 9 and dimension of S_4(K(70))^+ = 14

Rank of minus basis attempt = 0 and dimension of S_4(K(70))^- = 0

STEP 2: TRACE DOWN

N = 70 q = 11 Nq = 770

28-th determinant is 55

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 91

Trace down to try to hit S4(K(N)), of dimension 14

Plus lifts, plus nonlifts, minus dimension: {9, 5, 0}

Target plus rank, target minus rank: 14 0

Dimensions of savedTargetMats: {28, 3}

```
Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-770-6655.ma, Getting it

 ... done making G2.  Dimensions of Grits: {13, 93528}

Determining indices: {1, 2, 3, 4, 9, 10, 13, 14, 21, 22, 23, 24, 45}

Atkin-Lehner truncation: 52

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1, 1}, 5}, {{1, 1, -1, -1}, 3},
  {{1, -1, 1, -1}, 2}, {{1, -1, -1, 1}, 2}, {{-1, 1, -1, 1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-770-6655-to-70-55.ma exists, reading it

Found file Grits-2-770-6655.ma, Getting it

Grit dimensions: {13, 93528}

Viable file found: GG-Grits-2-770-6655.ma-6655-mod-12347.ma

New best file: GG-Grits-2-770-6655.ma-6655-mod-12347.ma

Dimensions of G2G2: {91, 93528}

Dimensions of fDown = {91, 28}

Rank of fDown mod pp = 14

Dimensions and rank of plus space fDown = {91, 28}  14

Dimensions and rank of minus space fDown = {91, 28}  0

Rank of plus basis attempt = 14 and dimension of S_4(K(70))^+ = 14

Rank of minus basis attempt = 0 and dimension of S_4(K(70))^- = 0


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 0


STEP 4: HECKE SPREAD

J2=0, or already have plus space basis and no extra Hecke spreads were specified

Hecke spreads: 0


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {}


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
```

```
Found files: {}

Found a plus basis of dimension 14

Found minus basis of dimension 0

N = 74

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 0}

lift dimension of S_4(K(74))^+ : 12

nonlift dimension of S_4(K(74))^+ : 4

dimension of S_4(K(74))^- : 1

Ibukiyama-Kitayama dimension of S_4(K(74)) : 17

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(74))^+ = 16

Rank of minus basis attempt = 0 and dimension of S_4(K(74))^- = 1

Initial short vector length: 35

Have vectors of length 39: long enough

Determinant shell containing the vectors of length 35 : 48

Short vector length is 39, and the corresponding determinant is 48

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-74-50.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,74}^{cusp}), which is 12

Rank of plus basis attempt = 12 and dimension of S_4(K(74))^+ = 16

Rank of minus basis attempt = 0 and dimension of S_4(K(74))^- = 1

STEP 2: TRACE DOWN

N = 74  q = 7  Nq = 518

39-th determinant is 48

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 78

Trace down to try to hit S4(K(N)), of dimension 17

Plus lifts, plus nonlifts, minus dimension: {12, 4, 1}

Target plus rank, target minus rank: 16 1

Dimensions of savedTargetMats: {39, 3}

Making ordered good sigs for source space S_4(K(Nq))...
```

```
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
Found file Grits-2-518-2352.ma, Getting it
 ... done making G2.  Dimensions of Grits: {12, 15558}
Determining indices: {1, 2, 3, 4, 7, 11, 12, 15, 16, 35, 36, 47}
Atkin-Lehner truncation: 56
Beginning ALSignaturesAndDims...
 ... done with ALSignaturesAndDims.
ALspacesDims = {{{1, 1, 1}, 7}, {{1, -1, -1}, 3}, {{-1, 1, -1}, 2}}
Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-518-2352-to-74-48.ma exists, reading it
Found file Grits-2-518-2352.ma, Getting it
Grit dimensions: {12, 15558}
Viable file found: GG-Grits-2-518-2352.ma-2352-mod-12347.ma
New best file: GG-Grits-2-518-2352.ma-2352-mod-12347.ma
Dimensions of G2G2: {78, 15558}
Dimensions of fDown = {78, 39}
Rank of fDown mod pp = 15
Dimensions and rank of plus space fDown = {78, 39}  15
Dimensions and rank of minus space fDown = {78, 39}  0
Rank of plus basis attempt = 16 and dimension of S_4(K(74))^+ = 16
Rank of minus basis attempt = 0 and dimension of S_4(K(74))^- = 1

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS
Pairs from dim(J_{2,N}^{cusp}: 1
Found file Grits-2-74-48.ma, Getting it
Viable file found: GG-Grits-2-74-10000.ma-10000-mod-12347.ma
New best file: GG-Grits-2-74-10000.ma-10000-mod-12347.ma
Dimensions of G2G2: {1, 39}
Rank of plus basis attempt = 16 and dimension of S_4(K(74))^+ = 16
Rank of minus basis attempt = 0 and dimension of S_4(K(74))^- = 1

STEP 4: HECKE SPREAD
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified
Found custom table of Hecke spreads
Preparing for 1 Hecke spreads, the last of which dilates determinants by 16
```

```
Need to multiply the minimal max det 48 by the largest det contraction factor
 16 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 768

Hecke spreads: 1

Pre-Hecke expansions will have length 2539

Found file Grits-2-74-768.ma, Getting it

Viable file found: GG-Grits-2-74-10000.ma-10000-mod-12347.ma

New best file: GG-Grits-2-74-10000.ma-10000-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 129 510

Called ShortenVecs with G2G2T[{}]

 input dimensions {1, 129 510}, output dimensions {1, 2539}

Length, max det of input vectors to ParaHeckeOp: 2539, 768 (check: 2539, 768)

Desired length, max det of output vectors: 39, 48

Quotient of the max dets should be 16

Smaller max det is the minimal max det 48
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 39, 48

Rank of T[G2G2]^+/- = (1,1)

Rank of plus basis attempt = 16 and dimension of S_4(K(74))^+ = 16

Rank of minus basis attempt = 1 and dimension of S_4(K(74))^- = 1
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-74-100.ma}

Dimensions of BPcoeffMatProven: {1, 93}

Dimensions of join: {2, 39}

Minus space span attempt may contain non-numerical entries
```

Rank of plus basis attempt = 16 and dimension of S_4(K(74))^+ = 16

Rank of minus basis attempt = 1 and dimension of S_4(K(74))^- = 1

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 16

Found minus basis of dimension 1

N = 77

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}

lift dimension of S_4(K(77))^+ : 12

nonlift dimension of S_4(K(77))^+ : 3

dimension of S_4(K(77))^- : 0

Ibukiyama-Kitayama dimension of S_4(K(77)) : 15

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(77))^+ = 15

Rank of minus basis attempt = 0 and dimension of S_4(K(77))^- = 0

Initial short vector length: 20

Have vectors of length 23: long enough

Determinant shell containing the vectors of length 20 : 55

Short vector length is 23, and the corresponding determinant is 55

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-77-60.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,77}^{cusp}), which is 12

Rank of plus basis attempt = 12 and dimension of S_4(K(77))^+ = 15

Rank of minus basis attempt = 0 and dimension of S_4(K(77))^- = 0

STEP 2: TRACE DOWN

N = 77 q = 5 Nq = 385

23-th determinant is 55

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 45

Trace down to try to hit S4(K(N)), of dimension 15

```
Plus lifts, plus nonlifts, minus dimension: {12, 3, 0}

Target plus rank, target minus rank: 15 0

Dimensions of savedTargetMats: {23, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-385-1375.ma, Getting it

 ... done making G2.  Dimensions of Grits: {9, 5398}

Determining indices: {1, 2, 3, 4, 5, 6, 13, 15, 16}

Atkin-Lehner truncation: 18

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 4}, {{1, -1, -1}, 3}, {{-1, 1, -1}, 1}, {{-1, -1, 1}, 1}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-385-1375-to-77-55.ma exists, reading it

Found file Grits-2-385-1375.ma, Getting it

Grit dimensions: {9, 5398}

Viable file found: GG-Grits-2-385-1375.ma-1375-mod-12347.ma

New best file: GG-Grits-2-385-1375.ma-1375-mod-12347.ma

Dimensions of G2G2: {45, 5398}

Dimensions of fDown = {45, 23}

Rank of fDown mod pp = 14

Dimensions and rank of plus space fDown = {45, 23}  14

Dimensions and rank of minus space fDown = {45, 23}  0

Rank of plus basis attempt = 15 and dimension of S_4(K(77))^+ = 15

Rank of minus basis attempt = 0 and dimension of S_4(K(77))^- = 0
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 1

Found file Grits-2-77-55.ma, Getting it

Viable file found: GG-Grits-2-77-1328.ma-1328-mod-12347.ma

New best file: GG-Grits-2-77-1328.ma-1328-mod-12347.ma

Viable file found: GG-Grits-2-77-5312.ma-5312-mod-12347.ma

Viable file found: GG-Grits-2-77-6723.ma-6723-mod-12347.ma

Viable file found: GG-Grits-2-77-7047.ma-7047-mod-12347.ma

Dimensions of G2G2: {1, 23}
```

Rank of plus basis attempt = 15 and dimension of S_4(K(77))^+ = 15

Rank of minus basis attempt = 0 and dimension of S_4(K(77))^- = 0

STEP 4: HECKE SPREAD

J2=0, or already have plus space basis and no extra Hecke spreads were specified

Hecke spreads: 0

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {}

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 15

Found minus basis of dimension 0

N = 78

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {11, 0, 0}

lift dimension of S_4(K(78))^+ : 10

nonlift dimension of S_4(K(78))^+ : 4

dimension of S_4(K(78))^- : 1

Ibukiyama-Kitayama dimension of S_4(K(78)) : 15

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(78))^+ = 14

Rank of minus basis attempt = 0 and dimension of S_4(K(78))^- = 1

Initial short vector length: 32

Have vectors of length 36: long enough

Determinant shell containing the vectors of length 32 : 56

Short vector length is 36, and the corresponding determinant is 56

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-78-60.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,78}^{cusp}), which is 10

Rank of plus basis attempt = 10 and dimension of S_4(K(78))^+ = 14

```
Rank of minus basis attempt = 0 and dimension of S_4(K(78))^- = 1
```

STEP 2: TRACE DOWN

```
N = 78  q = 11  Nq = 858

36-th determinant is 56

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 136

Trace down to try to hit S4(K(N)), of dimension 15

Plus lifts, plus nonlifts, minus dimension: {10, 4, 1}

Target plus rank, target minus rank: 14 1

Dimensions of savedTargetMats: {36, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-858-6776.ma, Getting it

 ... done making G2.  Dimensions of Grits: {16, 100 988}

Determining indices: {1, 2, 9, 10, 11, 12, 25, 26, 27, 49, 50, 51, 52, 113, 213, 229}

Atkin-Lehner truncation: 260

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1, 1}, 6}, {{1, 1, -1, -1}, 3},
  {{1, -1, 1, -1}, 2}, {{1, -1, -1, 1}, 2}, {{-1, 1, 1, -1}, 2}, {{-1, 1, -1, 1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-858-6776-to-78-56.ma exists, reading it

Found file Grits-2-858-6776.ma, Getting it

Grit dimensions: {16, 100 988}

Viable file found: GG-Grits-2-858-7436.ma-7436-mod-12347.ma

New best file: GG-Grits-2-858-7436.ma-7436-mod-12347.ma

Dimensions of G2G2: {136, 118 060}

Dimensions of fDown = {136, 36}

Rank of fDown mod pp = 14

Dimensions and rank of plus space fDown = {136, 36}  14

Dimensions and rank of minus space fDown = {136, 36}  0

Rank of plus basis attempt = 14 and dimension of S_4(K(78))^+ = 14

Rank of minus basis attempt = 0 and dimension of S_4(K(78))^- = 1
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 0
```

STEP 4: HECKE SPREAD

```
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-78-100.ma}
Dimensions of BPcoeffMatProven: {1, 112}
Dimensions of join: {1, 36}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 14 and dimension of S_4(K(78))^+ = 14
Rank of minus basis attempt = 1 and dimension of S_4(K(78))^- = 1
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Found a plus basis of dimension 14
Found minus basis of dimension 1
```

N = 82

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}
lift dimension of S_4(K(82))^+ : 14
nonlift dimension of S_4(K(82))^+ : 8
dimension of S_4(K(82))^- : 1
Ibukiyama-Kitayama dimension of S_4(K(82)) : 23
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(82))^+ = 22
Rank of minus basis attempt = 0 and dimension of S_4(K(82))^- = 1
Initial short vector length: 39
Have vectors of length 44: long enough
Determinant shell containing the vectors of length 39 : 64
Short vector length is 44, and the corresponding determinant is 64
Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-82-70.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,82}^{cusp}), which is 14
Rank of plus basis attempt = 14 and dimension of S_4(K(82))^+ = 22
Rank of minus basis attempt = 0 and dimension of S_4(K(82))^- = 1
```

STEP 2: TRACE DOWN

```
N = 82  q = 5  Nq = 410
44-th determinant is 64
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 55
Trace down to try to hit S4(K(N)), of dimension 23
Plus lifts, plus nonlifts, minus dimension: {14, 8, 1}
Target plus rank, target minus rank: 22 1
Dimensions of savedTargetMats: {44, 3}
Making ordered good sigs for source space S_4(K(Nq))...
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
Found file Grits-2-410-1600.ma, Getting it
 ... done making G2.  Dimensions of Grits: {10, 8818}
Determining indices: {1, 2, 3, 4, 9, 11, 12, 13, 23, 24}
Atkin-Lehner truncation: 28
Beginning ALSignaturesAndDims...
 ... done with ALSignaturesAndDims.
ALspacesDims = {{{1, 1, 1}, 5}, {{1, -1, -1}, 4}, {{-1, 1, -1}, 1}}
Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-410-1600-to-82-64.ma exists, reading it
Found file Grits-2-410-1600.ma, Getting it
Grit dimensions: {10, 8818}
Viable file found: GG-Grits-2-410-1600.ma-1600-mod-12347.ma
New best file: GG-Grits-2-410-1600.ma-1600-mod-12347.ma
Dimensions of G2G2: {55, 8818}
Dimensions of fDown = {55, 44}
Rank of fDown mod pp = 21
Dimensions and rank of plus space fDown = {55, 44}  21
Dimensions and rank of minus space fDown = {55, 44}  0
Rank of plus basis attempt = 22 and dimension of S_4(K(82))^+ = 22
```

```
Rank of minus basis attempt = 0 and dimension of S_4(K(82))^- = 1
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 1

Found file Grits-2-82-64.ma, Getting it

Viable file found: GG-Grits-2-82-12528.ma-12528-mod-12347.ma

New best file: GG-Grits-2-82-12528.ma-12528-mod-12347.ma

Viable file found: GG-Grits-2-82-14400.ma-14400-mod-12347.ma

Viable file found: GG-Grits-2-82-8700.ma-8700-mod-12347.ma

New best file: GG-Grits-2-82-8700.ma-8700-mod-12347.ma

Dimensions of G2G2: {1, 44}

Rank of plus basis attempt = 22 and dimension of S_4(K(82))^+ = 22

Rank of minus basis attempt = 0 and dimension of S_4(K(82))^- = 1
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 16

Need to multiply the minimal max det 64 by the largest det contraction factor
 16 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 1024

Hecke spreads: 1

Pre-Hecke expansions will have length 4092

Found file Grits-2-82-1024.ma, Getting it

Viable file found: GG-Grits-2-82-12528.ma-12528-mod-12347.ma

New best file: GG-Grits-2-82-12528.ma-12528-mod-12347.ma

Viable file found: GG-Grits-2-82-14400.ma-14400-mod-12347.ma

Viable file found: GG-Grits-2-82-8700.ma-8700-mod-12347.ma

New best file: GG-Grits-2-82-8700.ma-8700-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 104 714
```

```
Called ShortenVecs with G2G2T[{}]

 input dimensions {1, 104 714}, output dimensions {1, 4092}

Length, max det of input vectors to ParaHeckeOp: 4092, 1024 (check: 4092, 1024)

Desired length, max det of output vectors: 44, 64

Quotient of the max dets should be 16

Smaller max det is the minimal max det 64
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 44, 64

Rank of T[G2G2]^+/- = (1,1)

Rank of plus basis attempt = 22 and dimension of S_4(K(82))^+ = 22

Rank of minus basis attempt = 1 and dimension of S_4(K(82))^- = 1


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {}


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Found a plus basis of dimension 22

Found minus basis of dimension 1


N = 85

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}

lift dimension of S_4(K(85))^+ : 15

nonlift dimension of S_4(K(85))^+ : 9

dimension of S_4(K(85))^- : 0

Ibukiyama-Kitayama dimension of S_4(K(85)) : 24


STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(85))^+ = 24

Rank of minus basis attempt = 0 and dimension of S_4(K(85))^- = 0

Initial short vector length: 32
```

```
Have vectors of length 34: long enough

Determinant shell containing the vectors of length 32 : 60

Short vector length is 34, and the corresponding determinant is 60

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-85-60.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,85}^{cusp}), which is 15

Rank of plus basis attempt = 15 and dimension of S_4(K(85))^+ = 24

Rank of minus basis attempt = 0 and dimension of S_4(K(85))^- = 0
```

STEP 2: TRACE DOWN

```
N = 85  q = 7  Nq = 595

34-th determinant is 60

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 105

Trace down to try to hit S4(K(N)), of dimension 24

Plus lifts, plus nonlifts, minus dimension: {15, 9, 0}

Target plus rank, target minus rank: 24 0

Dimensions of savedTargetMats: {34, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-595-2940.ma, Getting it

 ... done making G2.  Dimensions of Grits: {14, 17710}

Determining indices: {1, 2, 3, 4, 5, 7, 8, 15, 17, 27, 28, 43, 51, 52}

Atkin-Lehner truncation: 74

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 5}, {{1, -1, -1}, 3}, {{-1, 1, -1}, 3}, {{-1, -1, 1}, 3}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-595-2940-to-85-60.ma exists, reading it

Found file Grits-2-595-2940.ma, Getting it

Grit dimensions: {14, 17710}

Viable file found: GG-Grits-2-595-2940.ma-2940-mod-12347.ma

New best file: GG-Grits-2-595-2940.ma-2940-mod-12347.ma

Dimensions of G2G2: {105, 17710}
```

```
Dimensions of fDown = {105, 34}
Rank of fDown mod pp = 22
Dimensions and rank of plus space fDown = {105, 34}  22
Dimensions and rank of minus space fDown = {105, 34}  0
Rank of plus basis attempt = 24 and dimension of S_4(K(85))^+ = 24
Rank of minus basis attempt = 0 and dimension of S_4(K(85))^- = 0
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 3
Found file Grits-2-85-60.ma, Getting it
Viable file found: GG-Grits-2-85-6656.ma-6656-mod-12347.ma
New best file: GG-Grits-2-85-6656.ma-6656-mod-12347.ma
Viable file found: GG-Grits-2-85-8424.ma-8424-mod-12347.ma
Viable file found: GG-Grits-2-85-9639.ma-9639-mod-12347.ma
Dimensions of G2G2: {3, 34}
Rank of plus basis attempt = 24 and dimension of S_4(K(85))^+ = 24
Rank of minus basis attempt = 0 and dimension of S_4(K(85))^- = 0
```

STEP 4: HECKE SPREAD

```
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {}
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Found a plus basis of dimension 24
Found minus basis of dimension 0
```

N = 86

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}
lift dimension of S_4(K(86))^+ : 14
nonlift dimension of S_4(K(86))^+ : 6
dimension of S_4(K(86))^- : 1
```

```
Ibukiyama-Kitayama dimension of S_4(K(86)) : 21

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(86))^+ = 20
Rank of minus basis attempt = 0 and dimension of S_4(K(86))^- = 1
Initial short vector length: 49
Have vectors of length 81: long enough
Determinant shell containing the vectors of length 49 : 71
Short vector length is 62, and the corresponding determinant is 71
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-86-80.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,86}^{cusp}), which is 14
Rank of plus basis attempt = 14 and dimension of S_4(K(86))^+ = 20
Rank of minus basis attempt = 0 and dimension of S_4(K(86))^- = 1

STEP 2: TRACE DOWN
N = 86  q = 5  Nq = 430
62-th determinant is 71
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 21
Trace down to try to hit S4(K(N)), of dimension 21
Plus lifts, plus nonlifts, minus dimension: {14, 6, 1}
Target plus rank, target minus rank: 20 1
Dimensions of savedTargetMats: {62, 3}
Making ordered good sigs for source space S_4(K(Nq))...
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
Found file Grits-2-430-1775.ma, Getting it
 ... done making G2.  Dimensions of Grits: {6, 10458}
Determining indices: {1, 3, 4, 5, 6, 19}
Atkin-Lehner truncation: 26
Beginning ALSignaturesAndDims...
 ... done with ALSignaturesAndDims.
ALspacesDims = {{{1, 1, 1}, 3}, {{1, -1, -1}, 1}, {{-1, 1, -1}, 1}, {{-1, -1, 1}, 1}}
Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}
```

```
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-430-1775-to-86-71.ma exists, reading it
Found file Grits-2-430-1775.ma, Getting it
Grit dimensions: {6, 10458}
Viable file found: GG-Grits-2-430-1775.ma-1775-mod-12347.ma
New best file: GG-Grits-2-430-1775.ma-1775-mod-12347.ma
Dimensions of G2G2: {21, 10458}
Dimensions of fDown = {21, 62}
Rank of fDown mod pp = 13
Dimensions and rank of plus space fDown = {21, 62}  13
Dimensions and rank of minus space fDown = {21, 62}  0
Rank of plus basis attempt = 20 and dimension of S_4(K(86))^+ = 20
Rank of minus basis attempt = 0 and dimension of S_4(K(86))^- = 1
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 1
Found file Grits-2-86-71.ma, Getting it
Viable file found: GG-Grits-2-86-5751.ma-5751-mod-12347.ma
New best file: GG-Grits-2-86-5751.ma-5751-mod-12347.ma
Viable file found: GG-Grits-2-86-8591.ma-8591-mod-12347.ma
Viable file found: GG-Grits-2-86-9680.ma-9680-mod-12347.ma
Dimensions of G2G2: {1, 62}
Rank of plus basis attempt = 20 and dimension of S_4(K(86))^+ = 20
Rank of minus basis attempt = 0 and dimension of S_4(K(86))^- = 1
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified
Found custom table of Hecke spreads
Preparing for 1 Hecke spreads, the last of which dilates determinants by 16
Need to multiply the minimal max det 71 by the largest det contraction factor
 16 under Hecke spreads to get the pre-Hecke max det
Need pre-Hecke expansions up to determinant 1136
Hecke spreads: 1
Pre-Hecke expansions will have length 4529
Found file Grits-2-86-1136.ma, Getting it
Viable file found: GG-Grits-2-86-5751.ma-5751-mod-12347.ma
New best file: GG-Grits-2-86-5751.ma-5751-mod-12347.ma
```

```
Viable file found: GG-Grits-2-86-8591.ma-8591-mod-12347.ma

Viable file found: GG-Grits-2-86-9680.ma-9680-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 55582

Called ShortenVecs with G2G2T[{}]

 input dimensions {1, 55582}, output dimensions {1, 4529}

Length, max det of input vectors to ParaHeckeOp: 4529, 1136 (check: 4529, 1136)

Desired length, max det of output vectors: 62, 71

Quotient of the max dets should be 16

Smaller max det is the minimal max det 71
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 62, 71

Rank of T[G2G2]^+/- = (1,1)

Rank of plus basis attempt = 20 and dimension of S_4(K(86))^+ = 20

Rank of minus basis attempt = 1 and dimension of S_4(K(86))^- = 1


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {}


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Found a plus basis of dimension 20

Found minus basis of dimension 1


N = 87
```

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}
lift dimension of S_4(K(87))^+ : 13
nonlift dimension of S_4(K(87))^+ : 6
dimension of S_4(K(87))^- : 1
Ibukiyama-Kitayama dimension of S_4(K(87)) : 20

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(87))^+ = 19
Rank of minus basis attempt = 0 and dimension of S_4(K(87))^- = 1
Initial short vector length: 37
Have vectors of length 62: long enough
Determinant shell containing the vectors of length 37 : 71
Short vector length is 50, and the corresponding determinant is 71
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-87-80.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,87}^{cusp}), which is 13
Rank of plus basis attempt = 13 and dimension of S_4(K(87))^+ = 19
Rank of minus basis attempt = 0 and dimension of S_4(K(87))^- = 1

STEP 2: TRACE DOWN
N = 87  q = 5  Nq = 435
50-th determinant is 71
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 45
Trace down to try to hit S4(K(N)), of dimension 20
Plus lifts, plus nonlifts, minus dimension: {13, 6, 1}
Target plus rank, target minus rank: 19 1
Dimensions of savedTargetMats: {50, 3}
Making ordered good sigs for source space S_4(K(Nq))...
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
Found file Grits-2-435-1775.ma, Getting it
 ... done making G2.  Dimensions of Grits: {9, 9160}
Determining indices: {1, 2, 5, 6, 9, 10, 13, 14, 21}
Atkin-Lehner truncation: 24
```

```
Beginning ALSignaturesAndDims...
 ... done with ALSignaturesAndDims.
ALspacesDims = {{{1, 1, 1}, 5}, {{1, -1, -1}, 2}, {{-1, 1, -1}, 2}}
Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-435-1775-to-87-71.ma exists, reading it
Found file Grits-2-435-1775.ma, Getting it
Grit dimensions: {9, 9160}
Viable file found: GG-Grits-2-435-1775.ma-1775-mod-12347.ma
New best file: GG-Grits-2-435-1775.ma-1775-mod-12347.ma
Dimensions of G2G2: {45, 9160}
Dimensions of fDown = {45, 50}
Rank of fDown mod pp = 19
Dimensions and rank of plus space fDown = {45, 50}  19
Dimensions and rank of minus space fDown = {45, 50}  0
Rank of plus basis attempt = 19 and dimension of S_4(K(87))^+ = 19
Rank of minus basis attempt = 0 and dimension of S_4(K(87))^- = 1

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS
Pairs from dim(J_{2,N}^{cusp}: 0

STEP 4: HECKE SPREAD
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-87-100.ma}
Dimensions of BPcoeffMatProven: {1, 90}
Dimensions of join: {1, 50}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 19 and dimension of S_4(K(87))^+ = 19
Rank of minus basis attempt = 1 and dimension of S_4(K(87))^- = 1

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Found a plus basis of dimension 19
```

```
Found minus basis of dimension 1
```

N = 91

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}
lift dimension of S_4(K(91))^+ : 16
nonlift dimension of S_4(K(91))^+ : 10
dimension of S_4(K(91))^- : 0
Ibukiyama-Kitayama dimension of S_4(K(91)) : 26
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(91))^+ = 26
Rank of minus basis attempt = 0 and dimension of S_4(K(91))^- = 0
Initial short vector length: 39
Have vectors of length 46: long enough
Determinant shell containing the vectors of length 39 : 68
Short vector length is 46, and the corresponding determinant is 68
Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-91-70.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,91}^{cusp}), which is 16
Rank of plus basis attempt = 16 and dimension of S_4(K(91))^+ = 26
Rank of minus basis attempt = 0 and dimension of S_4(K(91))^- = 0
```

STEP 2: TRACE DOWN

```
N = 91  q = 5  Nq = 455
46-th determinant is 68
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 15
Trace down to try to hit S4(K(N)), of dimension 26
Plus lifts, plus nonlifts, minus dimension: {16, 10, 0}
Target plus rank, target minus rank: 26 0
Dimensions of savedTargetMats: {46, 3}
Making ordered good sigs for source space S_4(K(Nq))...
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
Found file Grits-2-455-1700.ma, Getting it
```

```
 ... done making G2.  Dimensions of Grits: {5, 7594}

Determining indices: {1, 3, 4, 7, 15}

Atkin-Lehner truncation: 22

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 3}, {{1, -1, -1}, 1}, {{-1, 1, -1}, 1}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-455-1700-to-91-68.ma exists, reading it

Found file Grits-2-455-1700.ma, Getting it

Grit dimensions: {5, 7594}

Viable file found: GG-Grits-2-455-2695.ma-2695-mod-12347.ma

New best file: GG-Grits-2-455-2695.ma-2695-mod-12347.ma

Dimensions of G2G2: {15, 15234}

Dimensions of fDown = {15, 46}

Rank of fDown mod pp = 11

Dimensions and rank of plus space fDown = {15, 46}  11

Dimensions and rank of minus space fDown = {15, 46}  0

Rank of plus basis attempt = 23 and dimension of S_4(K(91))^+ = 26

Rank of minus basis attempt = 0 and dimension of S_4(K(91))^- = 0


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-91-68.ma, Getting it

Viable file found: GG-Grits-2-91-6656.ma-6656-mod-12347.ma

New best file: GG-Grits-2-91-6656.ma-6656-mod-12347.ma

Dimensions of G2G2: {3, 46}

Rank of plus basis attempt = 24 and dimension of S_4(K(91))^+ = 26

Rank of minus basis attempt = 0 and dimension of S_4(K(91))^- = 0


STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 2 Hecke spreads, the last of which dilates determinants by 9

Need to multiply the minimal max det 68 by the largest det contraction factor
 9 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 612
```

```
Hecke spreads: 2

Pre-Hecke expansions will have length 1334

Found file Grits-2-91-612.ma, Getting it

Viable file found: GG-Grits-2-91-6656.ma-6656-mod-12347.ma

New best file: GG-Grits-2-91-6656.ma-6656-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 1}} 1

{{3, 1}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 1}}] having G2G2T[{}] vectors to length 55 394

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 55 394}, output dimensions {3, 370}

Length, max det of input vectors to ParaHeckeOp: 370, 272 (check: 370, 272)

Desired length, max det of output vectors: 46, 68

Quotient of the max dets should be 4

Smaller max det is the minimal max det 68
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 1}}

About to compute G2G2T[{{2, 1}}]

Making abstract Hecke formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 46, 68

Rank of T[G2G2]^+/- = (3,0)

Rank of plus basis attempt = 25 and dimension of S_4(K(91))^+ = 26

Rank of minus basis attempt = 0 and dimension of S_4(K(91))^- = 0


Hecke spread 2 out of 2

Is Hecke spread 2 present?  False

DoOneHecke called {},{{3, 1}}

{tp,tpdel} = {3, 1}

Need to compute G2G2T[{{3, 1}}] having G2G2T[{}] vectors to length 55 394

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 55 394}, output dimensions {3, 1334}
```

```
Length, max det of input vectors to ParaHeckeOp: 1334, 612 (check: 1334, 612)

Desired length, max det of output vectors: 46, 68

Quotient of the max dets should be 9

Smaller max det is the minimal max det 68
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 1}}

About to compute G2G2T[{{3, 1}}]

Making abstract Hecke formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 46, 68

Rank of T[G2G2]^+/- = (3,0)

Rank of plus basis attempt = 26 and dimension of S_4(K(91))^+ = 26

Rank of minus basis attempt = 0 and dimension of S_4(K(91))^- = 0
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {}
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Found a plus basis of dimension 26

Found minus basis of dimension 0
```

N = 93

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}

lift dimension of S_4(K(93))^+ : 16

nonlift dimension of S_4(K(93))^+ : 8

dimension of S_4(K(93))^- : 1

Ibukiyama-Kitayama dimension of S_4(K(93)) : 25
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(93))^+ = 24

Rank of minus basis attempt = 0 and dimension of S_4(K(93))^- = 1

Initial short vector length: 44

Have vectors of length 45: long enough

Determinant shell containing the vectors of length 44 : 75

Short vector length is 45, and the corresponding determinant is 75
```

```
Vectors fill out a determinant shell, as they should


STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-93-80.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,93}^{cusp}), which is 16
Rank of plus basis attempt = 16 and dimension of S_4(K(93))^+ = 24
Rank of minus basis attempt = 0 and dimension of S_4(K(93))^- = 1


STEP 2: TRACE DOWN
N = 93  q = 5  Nq = 465
45-th determinant is 75
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 55
Trace down to try to hit S4(K(N)), of dimension 25
Plus lifts, plus nonlifts, minus dimension: {16, 8, 1}
Target plus rank, target minus rank: 24 1
Dimensions of savedTargetMats: {45, 3}
Making ordered good sigs for source space S_4(K(Nq))...
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
Found file Grits-2-465-1875.ma, Getting it
 ... done making G2.  Dimensions of Grits: {10, 10 124}
Determining indices: {1, 2, 3, 4, 5, 7, 8, 11, 12, 27}
Atkin-Lehner truncation: 30
Beginning ALSignaturesAndDims...
 ... done with ALSignaturesAndDims.
ALspacesDims = {{{1, 1, 1}, 5}, {{1, -1, -1}, 2}, {{-1, 1, -1}, 2}, {{-1, -1, 1}, 1}}
Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-465-1875-to-93-75.ma exists, reading it
Found file Grits-2-465-1875.ma, Getting it
Grit dimensions: {10, 10 124}
Viable file found: GG-Grits-2-465-1875.ma-1875-mod-12347.ma
New best file: GG-Grits-2-465-1875.ma-1875-mod-12347.ma
Dimensions of G2G2: {55, 10 124}
Dimensions of fDown = {55, 45}
Rank of fDown mod pp = 23
Dimensions and rank of plus space fDown = {55, 45}  23
```

```
Dimensions and rank of minus space fDown = {55, 45}  0

Rank of plus basis attempt = 24 and dimension of S_4(K(93))^+ = 24

Rank of minus basis attempt = 0 and dimension of S_4(K(93))^- = 1
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-93-75.ma, Getting it

Viable file found: GG-Grits-2-93-6144.ma-6144-mod-12347.ma

New best file: GG-Grits-2-93-6144.ma-6144-mod-12347.ma

Viable file found: GG-Grits-2-93-7776.ma-7776-mod-12347.ma

Viable file found: GG-Grits-2-93-8748.ma-8748-mod-12347.ma

Dimensions of G2G2: {3, 45}

Rank of plus basis attempt = 24 and dimension of S_4(K(93))^+ = 24

Rank of minus basis attempt = 0 and dimension of S_4(K(93))^- = 1
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 75 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 6075

Hecke spreads: 1

Pre-Hecke expansions will have length 54 405

Found file Grits-2-93-6075.ma, Getting it

Viable file found: GG-Grits-2-93-6144.ma-6144-mod-12347.ma

New best file: GG-Grits-2-93-6144.ma-6144-mod-12347.ma

Viable file found: GG-Grits-2-93-7776.ma-7776-mod-12347.ma

Viable file found: GG-Grits-2-93-8748.ma-8748-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 54 983
```

```
Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 54 983}, output dimensions {3, 54 405}

Length, max det of input vectors to ParaHeckeOp: 54 405, 6075 (check: 54 405, 6075)

Desired length, max det of output vectors: 45, 75

Quotient of the max dets should be 81

Smaller max det is the minimal max det 75
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 45, 75

Rank of T[G2G2]^+/- = (3,1)

Rank of plus basis attempt = 24 and dimension of S_4(K(93))^+ = 24

Rank of minus basis attempt = 1 and dimension of S_4(K(93))^- = 1


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {}


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Found a plus basis of dimension 24

Found minus basis of dimension 1


N = 94

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}

lift dimension of S_4(K(94))^+ : 15

nonlift dimension of S_4(K(94))^+ : 12

dimension of S_4(K(94))^- : 1

Ibukiyama-Kitayama dimension of S_4(K(94)) : 28


STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(94))^+ = 27

Rank of minus basis attempt = 0 and dimension of S_4(K(94))^- = 1

Initial short vector length: 50
```

```
Have vectors of length 65: long enough

Determinant shell containing the vectors of length 50 : 76

Short vector length is 55, and the corresponding determinant is 76

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-94-80.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,94}^{cusp}), which is 15

Rank of plus basis attempt = 15 and dimension of S_4(K(94))^+ = 27

Rank of minus basis attempt = 0 and dimension of S_4(K(94))^- = 1
```

STEP 2: TRACE DOWN

```
N = 94  q = 5  Nq = 470

55-th determinant is 76

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 55

Trace down to try to hit S4(K(N)), of dimension 28

Plus lifts, plus nonlifts, minus dimension: {15, 12, 1}

Target plus rank, target minus rank: 27 1

Dimensions of savedTargetMats: {55, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-470-1900.ma, Getting it

 ... done making G2.  Dimensions of Grits: {10, 11702}

Determining indices: {1, 2, 5, 7, 8, 9, 19, 20, 35, 49}

Atkin-Lehner truncation: 56

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 6}, {{1, -1, -1}, 2}, {{-1, 1, -1}, 1}, {{-1, -1, 1}, 1}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-470-1900-to-94-76.ma exists, reading it

Found file Grits-2-470-1900.ma, Getting it

Grit dimensions: {10, 11702}

Viable file found: GG-Grits-2-470-1900.ma-1900-mod-12347.ma

New best file: GG-Grits-2-470-1900.ma-1900-mod-12347.ma

Dimensions of G2G2: {55, 11702}
```

```
Dimensions of fDown = {55, 55}
Rank of fDown mod pp = 25
Dimensions and rank of plus space fDown = {55, 55}  25
Dimensions and rank of minus space fDown = {55, 55}  0
Rank of plus basis attempt = 27 and dimension of S_4(K(94))^+ = 27
Rank of minus basis attempt = 0 and dimension of S_4(K(94))^- = 1
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 0
```

STEP 4: HECKE SPREAD

```
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-94-100.ma}
Dimensions of BPcoeffMatProven: {1, 91}
Dimensions of join: {1, 55}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 27 and dimension of S_4(K(94))^+ = 27
Rank of minus basis attempt = 1 and dimension of S_4(K(94))^- = 1
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Found a plus basis of dimension 27
Found minus basis of dimension 1

N = 95
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}
lift dimension of S_4(K(95))^+ : 14
nonlift dimension of S_4(K(95))^+ : 7
dimension of S_4(K(95))^- : 1
Ibukiyama-Kitayama dimension of S_4(K(95)) : 22
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(95))^+ = 21

Rank of minus basis attempt = 0 and dimension of S_4(K(95))^- = 1

Initial short vector length: 36

Have vectors of length 65: long enough

Determinant shell containing the vectors of length 36 : 71

Short vector length is 47, and the corresponding determinant is 71

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-95-80.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,95}^{cusp}), which is 14

Rank of plus basis attempt = 14 and dimension of S_4(K(95))^+ = 21

Rank of minus basis attempt = 0 and dimension of S_4(K(95))^- = 1
```

STEP 2: TRACE DOWN

```
N = 95  q = 7  Nq = 665

47-th determinant is 71

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 136

Trace down to try to hit S4(K(N)), of dimension 22

Plus lifts, plus nonlifts, minus dimension: {14, 7, 1}

Target plus rank, target minus rank: 21 1

Dimensions of savedTargetMats: {47, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-665-3479.ma, Getting it

 ... done making G2.  Dimensions of Grits: {16, 22 838}

Determining indices: {1, 2, 3, 4, 5, 6, 15, 16, 19, 20, 27, 28, 29, 39, 47, 63}

Atkin-Lehner truncation: 66

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 8}, {{1, -1, -1}, 3}, {{-1, 1, -1}, 3}, {{-1, -1, 1}, 2}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-665-3479-to-95-71.ma exists, reading it

Found file Grits-2-665-3479.ma, Getting it

Grit dimensions: {16, 22 838}
```

```
Viable file found: GG-Grits-2-665-3479.ma-3479-mod-12347.ma
New best file: GG-Grits-2-665-3479.ma-3479-mod-12347.ma
Dimensions of G2G2: {136, 22 838}
Dimensions of fDown = {136, 47}
Rank of fDown mod pp = 21
Dimensions and rank of plus space fDown = {136, 47}  21
Dimensions and rank of minus space fDown = {136, 47}  0
Rank of plus basis attempt = 21 and dimension of S_4(K(95))^+ = 21
Rank of minus basis attempt = 0 and dimension of S_4(K(95))^- = 1

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS
Pairs from dim(J_{2,N}^{cusp}: 0

STEP 4: HECKE SPREAD
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-95-100.ma}
Dimensions of BPcoeffMatProven: {1, 82}
Dimensions of join: {1, 47}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 21 and dimension of S_4(K(95))^+ = 21
Rank of minus basis attempt = 1 and dimension of S_4(K(95))^- = 1

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Found a plus basis of dimension 21
Found minus basis of dimension 1

N = 102
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 0}
lift dimension of S_4(K(102))^+ : 15
nonlift dimension of S_4(K(102))^+ : 9
dimension of S_4(K(102))^- : 2
```

```
Ibukiyama-Kitayama dimension of S_4(K(102)) : 26

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(102))^+ = 24
Rank of minus basis attempt = 0 and dimension of S_4(K(102))^- = 2
Initial short vector length: 59
Have vectors of length 74: long enough
Determinant shell containing the vectors of length 59 : 84
Short vector length is 62, and the corresponding determinant is 84
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-102-90.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,102}^{cusp}), which is 15
Rank of plus basis attempt = 15 and dimension of S_4(K(102))^+ = 24
Rank of minus basis attempt = 0 and dimension of S_4(K(102))^- = 2

STEP 2: TRACE DOWN
N = 102  q = 7  Nq = 714
62-th determinant is 84
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 78
Trace down to try to hit S4(K(N)), of dimension 26
Plus lifts, plus nonlifts, minus dimension: {15, 9, 2}
Target plus rank, target minus rank: 24 2
Dimensions of savedTargetMats: {62, 3}
Making ordered good sigs for source space S_4(K(Nq))...
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
Found file Grits-2-714-4116.ma, Getting it
 ... done making G2.  Dimensions of Grits: {12, 48472}
Determining indices: {1, 2, 3, 4, 5, 7, 41, 49, 53, 54, 153, 161}
Atkin-Lehner truncation: 184
Beginning ALSignaturesAndDims...
 ... done with ALSignaturesAndDims.
ALspacesDims = {{{1, 1, 1, 1}, 5}, {{1, 1, -1, -1}, 2},
  {{1, -1, 1, -1}, 2}, {{1, -1, -1, 1}, 1}, {{-1, 1, 1, -1}, 1}, {{-1, 1, -1, 1}, 1}}
```

```
Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-714-4116-to-102-84.ma exists, reading it
Found file Grits-2-714-4116.ma, Getting it
Grit dimensions: {12, 48 472}
Viable file found: GG-Grits-2-714-4116.ma-4116-mod-12347.ma
New best file: GG-Grits-2-714-4116.ma-4116-mod-12347.ma
Viable file found: GG-Grits-2-714-4263.ma-4263-mod-12347.ma
Dimensions of G2G2: {78, 48 472}
Dimensions of fDown = {78, 62}
Rank of fDown mod pp = 24
Dimensions and rank of plus space fDown = {78, 62}  24
Dimensions and rank of minus space fDown = {78, 62}  0
Rank of plus basis attempt = 24 and dimension of S_4(K(102))^+ = 24
Rank of minus basis attempt = 0 and dimension of S_4(K(102))^- = 2
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 1
Found file Grits-2-102-84.ma, Getting it
Viable file found: GG-Grits-2-102-14196.ma-14196-mod-12347.ma
New best file: GG-Grits-2-102-14196.ma-14196-mod-12347.ma
Viable file found: GG-Grits-2-102-27216.ma-27216-mod-12347.ma
Viable file found: GG-Grits-2-102-5568.ma-5568-mod-12347.ma
New best file: GG-Grits-2-102-5568.ma-5568-mod-12347.ma
Dimensions of G2G2: {1, 62}
Rank of plus basis attempt = 24 and dimension of S_4(K(102))^+ = 24
Rank of minus basis attempt = 0 and dimension of S_4(K(102))^- = 2
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified
Found custom table of Hecke spreads
Preparing for 1 Hecke spreads, the last of which dilates determinants by 81
Need to multiply the minimal max det 84 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det
Need pre-Hecke expansions up to determinant 6804
Hecke spreads: 1
Pre-Hecke expansions will have length 97 306
```

```
Found file Grits-2-102-6804.ma, Getting it

Viable file found: GG-Grits-2-102-14196.ma-14196-mod-12347.ma

New best file: GG-Grits-2-102-14196.ma-14196-mod-12347.ma

Viable file found: GG-Grits-2-102-27216.ma-27216-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 298 356

Called ShortenVecs with G2G2T[{}]

 input dimensions {1, 298 356}, output dimensions {1, 97 306}

Length, max det of input vectors to ParaHeckeOp: 97 306, 6804 (check: 97 306, 6804)

Desired length, max det of output vectors: 62, 84

Quotient of the max dets should be 81

Smaller max det is the minimal max det 84
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 62, 84

Rank of T[G2G2]^+/- = (1,1)

Rank of plus basis attempt = 24 and dimension of S_4(K(102))^+ = 24

Rank of minus basis attempt = 1 and dimension of S_4(K(102))^- = 2
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-102-100.ma}

Dimensions of BPcoeffMatProven: {2, 74}

Dimensions of join: {3, 62}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 24 and dimension of S_4(K(102))^+ = 24

Rank of minus basis attempt = 2 and dimension of S_4(K(102))^- = 2
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 24

Found minus basis of dimension 2

N = 105

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {11, 0, 0}

lift dimension of S_4(K(105))^+ : 14

nonlift dimension of S_4(K(105))^+ : 10

dimension of S_4(K(105))^- : 0

Ibukiyama-Kitayama dimension of S_4(K(105)) : 24

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(105))^+ = 24

Rank of minus basis attempt = 0 and dimension of S_4(K(105))^- = 0

Initial short vector length: 51

Have vectors of length 62: long enough

Determinant shell containing the vectors of length 51 : 96

Short vector length is 62, and the corresponding determinant is 96

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-105-100.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,105}^{cusp}), which is 14

Rank of plus basis attempt = 14 and dimension of S_4(K(105))^+ = 24

Rank of minus basis attempt = 0 and dimension of S_4(K(105))^- = 0

STEP 2: TRACE DOWN

N = 105 q = 11 Nq = 1155

62-th determinant is 96

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 300

Trace down to try to hit S4(K(N)), of dimension 24

Plus lifts, plus nonlifts, minus dimension: {14, 10, 0}

Target plus rank, target minus rank: 24 0

Dimensions of savedTargetMats: {62, 3}

```
Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1155-11616.ma, Getting it

 ... done making G2.  Dimensions of Grits: {24, 193580}

Determining indices:
 {1, 2, 3, 4, 9, 10, 13, 14, 21, 22, 23, 24, 45, 46, 47, 48, 49, 50, 51, 52, 85, 101, 197, 198}

Atkin-Lehner truncation: 204

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims =
 {{{1, 1, 1, 1}, 8}, {{1, 1, -1, -1}, 4}, {{1, -1, 1, -1}, 4}, {{1, -1, -1, 1}, 3},
  {{-1, 1, 1, -1}, 2}, {{-1, 1, -1, 1}, 1}, {{-1, -1, 1, 1}, 1}, {{-1, -1, -1, -1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1155-11616-to-105-96.ma exists, reading it

Found file Grits-2-1155-11616.ma, Getting it

Grit dimensions: {24, 193580}

Viable file found: GG-Grits-2-1155-11616.ma-11616-mod-12347.ma

New best file: GG-Grits-2-1155-11616.ma-11616-mod-12347.ma

Dimensions of G2G2: {300, 193580}

Dimensions of fDown = {300, 62}

Rank of fDown mod pp = 24

Dimensions and rank of plus space fDown = {300, 62}  24

Dimensions and rank of minus space fDown = {300, 62}  0

Rank of plus basis attempt = 24 and dimension of S_4(K(105))^+ = 24

Rank of minus basis attempt = 0 and dimension of S_4(K(105))^- = 0


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 0


STEP 4: HECKE SPREAD

J2=0, or already have plus space basis and no extra Hecke spreads were specified

Hecke spreads: 0


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {}
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 24

Found minus basis of dimension 0

N = 106

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}

lift dimension of S_4(K(106))^+ : 19

nonlift dimension of S_4(K(106))^+ : 15

dimension of S_4(K(106))^- : 1

Ibukiyama-Kitayama dimension of S_4(K(106)) : 35

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(106))^+ = 34

Rank of minus basis attempt = 0 and dimension of S_4(K(106))^- = 1

Initial short vector length: 66

Have vectors of length 68: long enough

Determinant shell containing the vectors of length 66 : 68

Short vector length is 68, and the corresponding determinant is 68

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-106-70.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,106}^{cusp}), which is 19

Rank of plus basis attempt = 19 and dimension of S_4(K(106))^+ = 34

Rank of minus basis attempt = 0 and dimension of S_4(K(106))^- = 1

STEP 2: TRACE DOWN

N = 106 q = 5 Nq = 530

68-th determinant is 68

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 78

Trace down to try to hit S4(K(N)), of dimension 35

Plus lifts, plus nonlifts, minus dimension: {19, 15, 1}

Target plus rank, target minus rank: 34 1

```
Dimensions of savedTargetMats: {68, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-530-1700.ma, Getting it

 ... done making G2.  Dimensions of Grits: {12, 9594}

Determining indices: {1, 2, 3, 4, 7, 8, 13, 14, 17, 18, 23, 37}

Atkin-Lehner truncation: 44

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 7}, {{1, -1, -1}, 4}, {{-1, 1, -1}, 1}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-530-1700-to-106-68.ma exists, reading it

Found file Grits-2-530-1700.ma, Getting it

Grit dimensions: {12, 9594}

Viable file found: GG-Grits-2-530-1700.ma-1700-mod-12347.ma

New best file: GG-Grits-2-530-1700.ma-1700-mod-12347.ma

Viable file found: GG-Grits-2-530-2500.ma-2500-mod-12347.ma

Dimensions of G2G2: {78, 9594}

Dimensions of fDown = {78, 68}

Rank of fDown mod pp = 30

Dimensions and rank of plus space fDown = {78, 68}  30

Dimensions and rank of minus space fDown = {78, 68}  0

Rank of plus basis attempt = 34 and dimension of S_4(K(106))^+ = 34

Rank of minus basis attempt = 0 and dimension of S_4(K(106))^- = 1
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-106-68.ma, Getting it

Viable file found: GG-Grits-2-106-4032.ma-4032-mod-12347.ma

New best file: GG-Grits-2-106-4032.ma-4032-mod-12347.ma

Viable file found: GG-Grits-2-106-6800.ma-6800-mod-12347.ma

Dimensions of G2G2: {3, 68}

Rank of plus basis attempt = 34 and dimension of S_4(K(106))^+ = 34

Rank of minus basis attempt = 0 and dimension of S_4(K(106))^- = 1
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 16

Need to multiply the minimal max det 68 by the largest det contraction factor
 16 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 1088

Hecke spreads: 1

Pre-Hecke expansions will have length 4284

Found file Grits-2-106-1088.ma, Getting it

Viable file found: GG-Grits-2-106-4032.ma-4032-mod-12347.ma

New best file: GG-Grits-2-106-4032.ma-4032-mod-12347.ma

Viable file found: GG-Grits-2-106-6800.ma-6800-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 32 225

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 32 225}, output dimensions {3, 4284}

Length, max det of input vectors to ParaHeckeOp: 4284, 1088 (check: 4284, 1088)

Desired length, max det of output vectors: 68, 68

Quotient of the max dets should be 16

Smaller max det is the minimal max det 68
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 68, 68

Rank of T[G2G2]^+/- = (3,1)

Rank of plus basis attempt = 34 and dimension of S_4(K(106))^+ = 34

Rank of minus basis attempt = 1 and dimension of S_4(K(106))^- = 1
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {}

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 34

Found minus basis of dimension 1

N = 110

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {13, 0, 0}

lift dimension of S_4(K(110))^+ : 15

nonlift dimension of S_4(K(110))^+ : 10

dimension of S_4(K(110))^- : 3

Ibukiyama-Kitayama dimension of S_4(K(110)) : 28

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(110))^+ = 25

Rank of minus basis attempt = 0 and dimension of S_4(K(110))^- = 3

Initial short vector length: 65

Have vectors of length 72: long enough

Determinant shell containing the vectors of length 65 : 84

Short vector length is 72, and the corresponding determinant is 84

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-110-90.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,110}^{cusp}), which is 15

Rank of plus basis attempt = 15 and dimension of S_4(K(110))^+ = 25

Rank of minus basis attempt = 0 and dimension of S_4(K(110))^- = 3

STEP 2: TRACE DOWN

N = 110 q = 13 Nq = 1430

72-th determinant is 84

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 595

Trace down to try to hit S4(K(N)), of dimension 28

```
Plus lifts, plus nonlifts, minus dimension: {15, 10, 3}

Target plus rank, target minus rank: 25 3

Dimensions of savedTargetMats: {72, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1430-14196.ma, Getting it

 ... done making G2.  Dimensions of Grits: {34, 284 664}

Determining indices: {1, 2, 3, 4, 17, 18, 21, 22, 29, 30, 31, 32, 33, 34, 35, 36, 69,
  71, 101, 102, 103, 104, 125, 126, 133, 134, 157, 158, 189, 190, 209, 210, 257, 321}

Atkin-Lehner truncation: 344

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims =
 {{{1, 1, 1, 1}, 12}, {{1, 1, -1, -1}, 7}, {{1, -1, 1, -1}, 4}, {{1, -1, -1, 1}, 4},
  {{-1, 1, 1, -1}, 2}, {{-1, 1, -1, 1}, 2}, {{-1, -1, 1, 1}, 2}, {{-1, -1, -1, -1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1430-14196-to-110-84.ma exists, reading it

Found file Grits-2-1430-14196.ma, Getting it

Grit dimensions: {34, 284 664}

Viable file found: GG-Grits-2-1430-14196.ma-14196-mod-12347.ma

New best file: GG-Grits-2-1430-14196.ma-14196-mod-12347.ma

Dimensions of G2G2: {595, 284 664}

Dimensions of fDown = {595, 72}

Rank of fDown mod pp = 25

Dimensions and rank of plus space fDown = {595, 72}  25

Dimensions and rank of minus space fDown = {595, 72}  0

Rank of plus basis attempt = 25 and dimension of S_4(K(110))^+ = 25

Rank of minus basis attempt = 0 and dimension of S_4(K(110))^- = 3


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 0


STEP 4: HECKE SPREAD

J2=0, or already have plus space basis and no extra Hecke spreads were specified

Hecke spreads: 0
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-110-150.ma}

Dimensions of BPcoeffMatProven: {4, 136}

Dimensions of join: {4, 72}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 25 and dimension of S_4(K(110))^+ = 25

Rank of minus basis attempt = 3 and dimension of S_4(K(110))^- = 3

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 25

Found minus basis of dimension 3

N = 111

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}

lift dimension of S_4(K(111))^+ : 18

nonlift dimension of S_4(K(111))^+ : 14

dimension of S_4(K(111))^- : 1

Ibukiyama-Kitayama dimension of S_4(K(111)) : 33

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(111))^+ = 32

Rank of minus basis attempt = 0 and dimension of S_4(K(111))^- = 1

Initial short vector length: 67

Have vectors of length 69: long enough

Determinant shell containing the vectors of length 67 : 84

Short vector length is 69, and the corresponding determinant is 84

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-111-90.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,111}^{cusp}), which is 18

Rank of plus basis attempt = 18 and dimension of S_4(K(111))^+ = 32

Rank of minus basis attempt = 0 and dimension of S_4(K(111))^- = 1

STEP 2: TRACE DOWN

N = 111 q = 5 Nq = 555

69-th determinant is 84

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 78

Trace down to try to hit S4(K(N)), of dimension 33

Plus lifts, plus nonlifts, minus dimension: {18, 14, 1}

Target plus rank, target minus rank: 32 1

Dimensions of savedTargetMats: {69, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-555-2100.ma, Getting it

 ... done making G2. Dimensions of Grits: {12, 11948}

Determining indices: {1, 2, 3, 4, 5, 6, 13, 14, 15, 16, 29, 69}

Atkin-Lehner truncation: 84

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 5}, {{1, -1, -1}, 3}, {{-1, 1, -1}, 2}, {{-1, -1, 1}, 2}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-555-2100-to-111-84.ma exists, reading it

Found file Grits-2-555-2100.ma, Getting it

Grit dimensions: {12, 11948}

Viable file found: GG-Grits-2-555-2100.ma-2100-mod-12347.ma

New best file: GG-Grits-2-555-2100.ma-2100-mod-12347.ma

Dimensions of G2G2: {78, 11948}

Dimensions of fDown = {78, 69}

Rank of fDown mod pp = 30

Dimensions and rank of plus space fDown = {78, 69} 30

Dimensions and rank of minus space fDown = {78, 69} 0

Rank of plus basis attempt = 32 and dimension of S_4(K(111))^+ = 32

Rank of minus basis attempt = 0 and dimension of S_4(K(111))^- = 1

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 1

Found file Grits-2-111-84.ma, Getting it

```
Viable file found: GG-Grits-2-111-14976.ma-14976-mod-12347.ma

New best file: GG-Grits-2-111-14976.ma-14976-mod-12347.ma

Viable file found: GG-Grits-2-111-21504.ma-21504-mod-12347.ma

Dimensions of G2G2: {1, 69}

Rank of plus basis attempt = 32 and dimension of S_4(K(111))^+ = 32

Rank of minus basis attempt = 0 and dimension of S_4(K(111))^- = 1


STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 84 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 6804

Hecke spreads: 1

Pre-Hecke expansions will have length 63 846

Found file Grits-2-111-6804.ma, Getting it

Viable file found: GG-Grits-2-111-14976.ma-14976-mod-12347.ma

New best file: GG-Grits-2-111-14976.ma-14976-mod-12347.ma

Viable file found: GG-Grits-2-111-21504.ma-21504-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 212 073

Called ShortenVecs with G2G2T[{}]

 input dimensions {1, 212 073}, output dimensions {1, 63 846}

Length, max det of input vectors to ParaHeckeOp: 63 846, 6804 (check: 63 846, 6804)

Desired length, max det of output vectors: 69, 84

Quotient of the max dets should be 81

Smaller max det is the minimal max det 84
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime
```

```
Using Ralf's formula
Substituting given paramodular forms in abstract formula
Hecke spread 1 length and max det should be as desired: 69, 84
Rank of T[G2G2]^+/- = (1,1)
Rank of plus basis attempt = 32 and dimension of S_4(K(111))^+ = 32
Rank of minus basis attempt = 1 and dimension of S_4(K(111))^- = 1

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {}

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Found a plus basis of dimension 32
Found minus basis of dimension 1

N = 114
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 0}
lift dimension of S_4(K(114))^+ : 17
nonlift dimension of S_4(K(114))^+ : 13
dimension of S_4(K(114))^- : 2
Ibukiyama-Kitayama dimension of S_4(K(114)) : 32

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(114))^+ = 30
Rank of minus basis attempt = 0 and dimension of S_4(K(114))^- = 2
Initial short vector length: 76
Have vectors of length 78: long enough
Determinant shell containing the vectors of length 76 : 84
Short vector length is 78, and the corresponding determinant is 84
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-114-90.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,114}^{cusp}), which is 17
Rank of plus basis attempt = 17 and dimension of S_4(K(114))^+ = 30
```

```
Rank of minus basis attempt = 0 and dimension of S_4(K(114))^- = 2
```

STEP 2: TRACE DOWN

```
N = 114  q = 7  Nq = 798

78-th determinant is 84

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 78

Trace down to try to hit S4(K(N)), of dimension 32

Plus lifts, plus nonlifts, minus dimension: {17, 13, 2}

Target plus rank, target minus rank: 30 2

Dimensions of savedTargetMats: {78, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-798-4116.ma, Getting it

 ... done making G2.  Dimensions of Grits: {12, 47764}

Determining indices: {1, 2, 5, 6, 17, 18, 25, 29, 30, 65, 66, 67}

Atkin-Lehner truncation: 144

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1, 1}, 5}, {{1, 1, -1, -1}, 3},
  {{1, -1, 1, -1}, 1}, {{1, -1, -1, 1}, 1}, {{-1, 1, 1, -1}, 1}, {{-1, -1, 1, 1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-798-4116-to-114-84.ma exists, reading it

Found file Grits-2-798-4116.ma, Getting it

Grit dimensions: {12, 47764}

Viable file found: GG-Grits-2-798-4116.ma-4116-mod-12347.ma

New best file: GG-Grits-2-798-4116.ma-4116-mod-12347.ma

Dimensions of G2G2: {78, 47764}

Dimensions of fDown = {78, 78}

Rank of fDown mod pp = 28

Dimensions and rank of plus space fDown = {78, 78}  28

Dimensions and rank of minus space fDown = {78, 78}  0

Rank of plus basis attempt = 30 and dimension of S_4(K(114))^+ = 30

Rank of minus basis attempt = 0 and dimension of S_4(K(114))^- = 2
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 1

Found file Grits-2-114-84.ma, Getting it

Viable file found: GG-Grits-2-114-11495.ma-11495-mod-12347.ma

New best file: GG-Grits-2-114-11495.ma-11495-mod-12347.ma

Viable file found: GG-Grits-2-114-16464.ma-16464-mod-12347.ma

Dimensions of G2G2: {1, 78}

Rank of plus basis attempt = 30 and dimension of S_4(K(114))^+ = 30

Rank of minus basis attempt = 0 and dimension of S_4(K(114))^- = 2
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 2 Hecke spreads, the last of which dilates determinants by 64

Need to multiply the minimal max det 84 by the largest det contraction factor
 64 under Hecke spreads to get the pre-Hecke max det
Need pre-Hecke expansions up to determinant 5376

Hecke spreads: 2

Pre-Hecke expansions will have length 67216

Found file Grits-2-114-5376.ma, Getting it

Viable file found: GG-Grits-2-114-11495.ma-11495-mod-12347.ma

New best file: GG-Grits-2-114-11495.ma-11495-mod-12347.ma

Viable file found: GG-Grits-2-114-16464.ma-16464-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another
{{2, 2}} 4

{{2, 2}, {2, 1}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 215476

Called ShortenVecs with G2G2T[{}]

 input dimensions {1, 215476}, output dimensions {1, 67216}

Length, max det of input vectors to ParaHeckeOp: 67216, 5376 (check: 67216, 5376)

Desired length, max det of output vectors: 840, 336

Quotient of the max dets should be 16
```

```
Smaller max det is the minimal max det 84
  times the biggest postpended det inverse dilation factor
 4 for the Hecke operator{{2, 2}}
About to compute G2G2T[{{2, 2}}]
Making abstract Hecke formula at a squared bad prime
Using Ralf's formula
Substituting given paramodular forms in abstract formula
Hecke spread 1 length and max det should be as desired: 840, 336
Rank of T[G2G2]^+/- = (1,1)
Rank of plus basis attempt = 30 and dimension of S_4(K(114))^+ = 30
Rank of minus basis attempt = 1 and dimension of S_4(K(114))^- = 2


Hecke spread 2 out of 2
Is Hecke spread 2 present?  False
DoOneHecke called {},{{2, 2}, {2, 1}}
{tp,tpdel} = {2, 2}
DoOneHecke called {{2, 2}},{{2, 1}}
{tp,tpdel} = {2, 1}
Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 840
Called ShortenVecs with G2G2T[{{2, 2}}]
 input dimensions {1, 840}, output dimensions {1, 840}
Length, max det of input vectors to ParaHeckeOp: 840, 336 (check: 840, 336)
Desired length, max det of output vectors: 78, 84
Quotient of the max dets should be 4
Smaller max det is the minimal max det 84
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}
About to compute G2G2T[{{2, 2}, {2, 1}}]
Making abstract Hecke formula at a bad prime
Using Ralf's formula
Substituting given paramodular forms in abstract formula
Hecke spread 2 length and max det should be as desired: 78, 84
Rank of T[G2G2]^+/- = (1,1)
Rank of plus basis attempt = 30 and dimension of S_4(K(114))^+ = 30
Rank of minus basis attempt = 2 and dimension of S_4(K(114))^- = 2


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {}
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 30

Found minus basis of dimension 2

N = 115

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}

lift dimension of S_4(K(115))^+ : 20

nonlift dimension of S_4(K(115))^+ : 15

dimension of S_4(K(115))^- : 1

Ibukiyama-Kitayama dimension of S_4(K(115)) : 36

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(115))^+ = 35

Rank of minus basis attempt = 0 and dimension of S_4(K(115))^- = 1

Initial short vector length: 59

Have vectors of length 66: long enough

Determinant shell containing the vectors of length 59 : 84

Short vector length is 66, and the corresponding determinant is 84

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-115-90.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,115}^{cusp}), which is 20

Rank of plus basis attempt = 20 and dimension of S_4(K(115))^+ = 35

Rank of minus basis attempt = 0 and dimension of S_4(K(115))^- = 1

STEP 2: TRACE DOWN

N = 115 q = 7 Nq = 805

66-th determinant is 84

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 253

Trace down to try to hit S4(K(N)), of dimension 36

Plus lifts, plus nonlifts, minus dimension: {20, 15, 1}

Target plus rank, target minus rank: 35 1

```
Dimensions of savedTargetMats: {66, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-805-4116.ma, Getting it

 ... done making G2.  Dimensions of Grits: {22, 29098}

Determining indices:
 {1, 2, 3, 4, 5, 6, 9, 10, 13, 14, 21, 22, 23, 24, 49, 50, 57, 58, 61, 62, 95, 96}

Atkin-Lehner truncation: 110

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 8}, {{1, -1, -1}, 5}, {{-1, 1, -1}, 5}, {{-1, -1, 1}, 4}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-805-4116-to-115-84.ma exists, reading it

Found file Grits-2-805-4116.ma, Getting it

Grit dimensions: {22, 29098}

Viable file found: GG-Grits-2-805-4116.ma-4116-mod-12347.ma

New best file: GG-Grits-2-805-4116.ma-4116-mod-12347.ma

Dimensions of G2G2: {253, 29098}

Dimensions of fDown = {253, 66}

Rank of fDown mod pp = 35

Dimensions and rank of plus space fDown = {253, 66}  35

Dimensions and rank of minus space fDown = {253, 66}  0

Rank of plus basis attempt = 35 and dimension of S_4(K(115))^+ = 35

Rank of minus basis attempt = 0 and dimension of S_4(K(115))^- = 1


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-115-84.ma, Getting it

Viable file found: GG-Grits-2-115-4860.ma-4860-mod-12347.ma

New best file: GG-Grits-2-115-4860.ma-4860-mod-12347.ma

Viable file found: GG-Grits-2-115-52500.ma-52500-mod-12347.ma

Dimensions of G2G2: {3, 66}

Rank of plus basis attempt = 35 and dimension of S_4(K(115))^+ = 35

Rank of minus basis attempt = 0 and dimension of S_4(K(115))^- = 1
```

STEP 4: HECKE SPREAD

J2=0, or already have plus space basis and no extra Hecke spreads were specified

Hecke spreads: 0

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-115-150.ma}

Dimensions of BPcoeffMatProven: {1, 114}

Dimensions of join: {1, 66}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 35 and dimension of S_4(K(115))^+ = 35

Rank of minus basis attempt = 1 and dimension of S_4(K(115))^- = 1

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 35

Found minus basis of dimension 1

N = 118

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}

lift dimension of S_4(K(118))^+ : 20

nonlift dimension of S_4(K(118))^+ : 18

dimension of S_4(K(118))^- : 3

Ibukiyama-Kitayama dimension of S_4(K(118)) : 41

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(118))^+ = 38

Rank of minus basis attempt = 0 and dimension of S_4(K(118))^- = 3

Initial short vector length: 76

Have vectors of length 85: long enough

Determinant shell containing the vectors of length 76 : 96

Short vector length is 85, and the corresponding determinant is 96

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-118-100.ma, Getting it

```
Rank of plus basis attempt should match dim(J_{4,118}^{cusp}), which is 20

Rank of plus basis attempt = 20 and dimension of S_4(K(118))^+ = 38

Rank of minus basis attempt = 0 and dimension of S_4(K(118))^- = 3
```

STEP 2: TRACE DOWN

```
N = 118  q = 5  Nq = 590

85-th determinant is 96

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 91

Trace down to try to hit S4(K(N)), of dimension 41

Plus lifts, plus nonlifts, minus dimension: {20, 18, 3}

Target plus rank, target minus rank: 38 3

Dimensions of savedTargetMats: {85, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-590-2400.ma, Getting it

 ... done making G2.  Dimensions of Grits: {13, 16886}

Determining indices: {1, 2, 5, 6, 7, 8, 17, 18, 19, 33, 35, 36, 47}

Atkin-Lehner truncation: 54

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 6}, {{1, -1, -1}, 4}, {{-1, 1, -1}, 2}, {{-1, -1, 1}, 1}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-590-2400-to-118-96.ma exists, reading it

Found file Grits-2-590-2400.ma, Getting it

Grit dimensions: {13, 16886}

Viable file found: GG-Grits-2-590-2400.ma-2400-mod-12347.ma

New best file: GG-Grits-2-590-2400.ma-2400-mod-12347.ma

Dimensions of G2G2: {91, 16886}

Dimensions of fDown = {91, 85}

Rank of fDown mod pp = 37

Dimensions and rank of plus space fDown = {91, 85}  37

Dimensions and rank of minus space fDown = {91, 85}  0

Rank of plus basis attempt = 38 and dimension of S_4(K(118))^+ = 38

Rank of minus basis attempt = 0 and dimension of S_4(K(118))^- = 3
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 1

Found file Grits-2-118-96.ma, Getting it

Viable file found: GG-Grits-2-118-27000.ma-27000-mod-12347.ma

New best file: GG-Grits-2-118-27000.ma-27000-mod-12347.ma

Dimensions of G2G2: {1, 85}

Rank of plus basis attempt = 38 and dimension of S_4(K(118))^+ = 38

Rank of minus basis attempt = 0 and dimension of S_4(K(118))^- = 3

STEP 4: HECKE SPREAD

J2=0, or already have plus space basis and no extra Hecke spreads were specified

Hecke spreads: 0

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-118-150.ma}

Dimensions of BPcoeffMatProven: {4, 150}

Dimensions of join: {4, 85}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 38 and dimension of S_4(K(118))^+ = 38

Rank of minus basis attempt = 3 and dimension of S_4(K(118))^- = 3

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 38

Found minus basis of dimension 3

N = 119

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}

lift dimension of S_4(K(119))^+ : 18

nonlift dimension of S_4(K(119))^+ : 13

dimension of S_4(K(119))^- : 1

Ibukiyama-Kitayama dimension of S_4(K(119)) : 32

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(119))^+ = 31

```
Rank of minus basis attempt = 0 and dimension of S_4(K(119))^- = 1

Initial short vector length: 67

Have vectors of length 88: long enough

Determinant shell containing the vectors of length 67 : 103

Short vector length is 76, and the corresponding determinant is 103

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-119-110.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,119}^{cusp}), which is 18

Rank of plus basis attempt = 18 and dimension of S_4(K(119))^+ = 31

Rank of minus basis attempt = 0 and dimension of S_4(K(119))^- = 1
```

STEP 2: TRACE DOWN

```
N = 119  q = 5  Nq = 595

76-th determinant is 103

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 105

Trace down to try to hit S4(K(N)), of dimension 32

Plus lifts, plus nonlifts, minus dimension: {18, 13, 1}

Target plus rank, target minus rank: 31 1

Dimensions of savedTargetMats: {76, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-595-2575.ma, Getting it

 ... done making G2.  Dimensions of Grits: {14, 13908}

Determining indices: {1, 2, 3, 4, 5, 7, 8, 15, 17, 27, 28, 43, 51, 52}

Atkin-Lehner truncation: 74

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 5}, {{1, -1, -1}, 3}, {{-1, 1, -1}, 3}, {{-1, -1, 1}, 3}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-595-2575-to-119-103.ma exists, reading it

Found file Grits-2-595-2575.ma, Getting it

Grit dimensions: {14, 13908}

Viable file found: GG-Grits-2-595-2940.ma-2940-mod-12347.ma
```

```
New best file: GG-Grits-2-595-2940.ma-2940-mod-12347.ma
Dimensions of G2G2: {105, 17710}
Dimensions of fDown = {105, 76}
Rank of fDown mod pp = 31
Dimensions and rank of plus space fDown = {105, 76}  31
Dimensions and rank of minus space fDown = {105, 76}  0
Rank of plus basis attempt = 31 and dimension of S_4(K(119))^+ = 31
Rank of minus basis attempt = 0 and dimension of S_4(K(119))^- = 1

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS
Pairs from dim(J_{2,N}^{cusp}: 0

STEP 4: HECKE SPREAD
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-119-150.ma}
Dimensions of BPcoeffMatProven: {1, 134}
Dimensions of join: {1, 76}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 31 and dimension of S_4(K(119))^+ = 31
Rank of minus basis attempt = 1 and dimension of S_4(K(119))^- = 1

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Found a plus basis of dimension 31
Found minus basis of dimension 1

N = 122
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {3, 1, 0}
lift dimension of S_4(K(122))^+ : 21
nonlift dimension of S_4(K(122))^+ : 14
dimension of S_4(K(122))^- : 6
Ibukiyama-Kitayama dimension of S_4(K(122)) : 41
```

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(122))^+ = 35

Rank of minus basis attempt = 0 and dimension of S_4(K(122))^- = 6

Initial short vector length: 80

Have vectors of length 97: long enough

Determinant shell containing the vectors of length 80 : 95

Short vector length is 94, and the corresponding determinant is 95

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-122-100.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,122}^{cusp}), which is 21

Rank of plus basis attempt = 21 and dimension of S_4(K(122))^+ = 35

Rank of minus basis attempt = 0 and dimension of S_4(K(122))^- = 6

STEP 2: TRACE DOWN

N = 122 q = 3 Nq = 366

94-th determinant is 95

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 21

Trace down to try to hit S4(K(N)), of dimension 41

Plus lifts, plus nonlifts, minus dimension: {21, 14, 6}

Target plus rank, target minus rank: 35 6

Dimensions of savedTargetMats: {94, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-366-855.ma, Getting it

 ... done making G2. Dimensions of Grits: {6, 3592}

Determining indices: {1, 3, 4, 7, 8, 11}

Atkin-Lehner truncation: 14

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 4}, {{1, -1, -1}, 1}, {{-1, 1, -1}, 1}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-366-855-to-122-95.ma exists, reading it

```
Found file Grits-2-366-855.ma, Getting it

Grit dimensions: {6, 3592}

Viable file found: GG-Grits-2-366-855.ma-855-mod-12347.ma

New best file: GG-Grits-2-366-855.ma-855-mod-12347.ma

Dimensions of G2G2: {21, 3592}

Dimensions of fDown = {21, 94}

Rank of fDown mod pp = 16

Dimensions and rank of plus space fDown = {21, 94}  16

Dimensions and rank of minus space fDown = {21, 94}  0

Rank of plus basis attempt = 35 and dimension of S_4(K(122))^+ = 35

Rank of minus basis attempt = 0 and dimension of S_4(K(122))^- = 6
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-122-95.ma, Getting it

Viable file found: GG-Grits-2-122-24320.ma-24320-mod-12347.ma

New best file: GG-Grits-2-122-24320.ma-24320-mod-12347.ma

Viable file found: GG-Grits-2-122-3708.ma-3708-mod-12347.ma

New best file: GG-Grits-2-122-3708.ma-3708-mod-12347.ma

Viable file found: GG-Grits-2-122-6080.ma-6080-mod-12347.ma

Dimensions of G2G2: {3, 94}

Rank of plus basis attempt = 35 and dimension of S_4(K(122))^+ = 35

Rank of minus basis attempt = 0 and dimension of S_4(K(122))^- = 6
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 2 Hecke spreads, the last of which dilates determinants by 64

Need to multiply the minimal max det 95 by the largest det contraction factor
 64 under Hecke spreads to get the pre-Hecke max det
Need pre-Hecke expansions up to determinant 6080

Hecke spreads: 2

Pre-Hecke expansions will have length 60 320

Found file Grits-2-122-6080.ma, Getting it

Viable file found: GG-Grits-2-122-24320.ma-24320-mod-12347.ma

New best file: GG-Grits-2-122-24320.ma-24320-mod-12347.ma

Viable file found: GG-Grits-2-122-6080.ma-6080-mod-12347.ma
```

```
New best file: GG-Grits-2-122-6080.ma-6080-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 4

{{2, 2}, {2, 1}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 60 320

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 60 320}, output dimensions {3, 60 320}

Length, max det of input vectors to ParaHeckeOp: 60 320, 6080 (check: 60 320, 6080)

Desired length, max det of output vectors: 793, 380

Quotient of the max dets should be 16

Smaller max det is the minimal max det 95
  times the biggest postpended det inverse dilation factor
 4 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 793, 380

Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 35 and dimension of S_4(K(122))^+ = 35

Rank of minus basis attempt = 3 and dimension of S_4(K(122))^- = 6


Hecke spread 2 out of 2

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 793

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {3, 793}, output dimensions {3, 793}

Length, max det of input vectors to ParaHeckeOp: 793, 380 (check: 793, 380)
```

```
Desired length, max det of output vectors: 94, 95

Quotient of the max dets should be 4

Smaller max det is the minimal max det 95
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}
About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 94, 95

Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 35 and dimension of S_4(K(122))^+ = 35

Rank of minus basis attempt = 5 and dimension of S_4(K(122))^- = 6
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-122-150.ma}

Dimensions of BPcoeffMatProven: {11, 178}

Dimensions of join: {16, 94}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 35 and dimension of S_4(K(122))^+ = 35

Rank of minus basis attempt = 6 and dimension of S_4(K(122))^- = 6
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Found a plus basis of dimension 35

Found minus basis of dimension 6
```

N = 123

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}

lift dimension of S_4(K(123))^+ : 21

nonlift dimension of S_4(K(123))^+ : 13

dimension of S_4(K(123))^- : 3

Ibukiyama-Kitayama dimension of S_4(K(123)) : 37
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(123))^+ = 34
```

```
Rank of minus basis attempt = 0 and dimension of S_4(K(123))^- = 3
Initial short vector length: 71
Have vectors of length 80: long enough
Determinant shell containing the vectors of length 71 : 92
Short vector length is 80, and the corresponding determinant is 92
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-123-100.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,123}^{cusp}), which is 21
Rank of plus basis attempt = 21 and dimension of S_4(K(123))^+ = 34
Rank of minus basis attempt = 0 and dimension of S_4(K(123))^- = 3

STEP 2: TRACE DOWN
N = 123  q = 5  Nq = 615
80-th determinant is 92
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 66
Trace down to try to hit S4(K(N)), of dimension 37
Plus lifts, plus nonlifts, minus dimension: {21, 13, 3}
Target plus rank, target minus rank: 34 3
Dimensions of savedTargetMats: {80, 3}
Making ordered good sigs for source space S_4(K(Nq))...
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
Found file Grits-2-615-2300.ma, Getting it
 ... done making G2.  Dimensions of Grits: {11, 13632}
Determining indices: {1, 2, 5, 6, 13, 14, 21, 22, 25, 26, 37}
Atkin-Lehner truncation: 48
Beginning ALSignaturesAndDims...
 ... done with ALSignaturesAndDims.
ALspacesDims = {{{1, 1, 1}, 6}, {{1, -1, -1}, 3}, {{-1, 1, -1}, 2}}
Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-615-2300-to-123-92.ma exists, reading it
Found file Grits-2-615-2300.ma, Getting it
Grit dimensions: {11, 13632}
Viable file found: GG-Grits-2-615-2300.ma-2300-mod-12347.ma
```

```
New best file: GG-Grits-2-615-2300.ma-2300-mod-12347.ma

Dimensions of G2G2: {66, 13 632}

Dimensions of fDown = {66, 80}

Rank of fDown mod pp = 30

Dimensions and rank of plus space fDown = {66, 80}  30

Dimensions and rank of minus space fDown = {66, 80}  0

Rank of plus basis attempt = 34 and dimension of S_4(K(123))^+ = 34

Rank of minus basis attempt = 0 and dimension of S_4(K(123))^- = 3
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-123-92.ma, Getting it

Viable file found: GG-Grits-2-123-11900.ma-11900-mod-12347.ma

New best file: GG-Grits-2-123-11900.ma-11900-mod-12347.ma

Viable file found: GG-Grits-2-123-29808.ma-29808-mod-12347.ma

Viable file found: GG-Grits-2-123-4284.ma-4284-mod-12347.ma

New best file: GG-Grits-2-123-4284.ma-4284-mod-12347.ma

Dimensions of G2G2: {3, 80}

Rank of plus basis attempt = 34 and dimension of S_4(K(123))^+ = 34

Rank of minus basis attempt = 0 and dimension of S_4(K(123))^- = 3
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 92 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 7452

Hecke spreads: 1

Pre-Hecke expansions will have length 73 124

Found file Grits-2-123-7452.ma, Getting it

Viable file found: GG-Grits-2-123-11900.ma-11900-mod-12347.ma

New best file: GG-Grits-2-123-11900.ma-11900-mod-12347.ma

Viable file found: GG-Grits-2-123-29808.ma-29808-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1
```

```
Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 149 706

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 149 706}, output dimensions {3, 73 124}

Length, max det of input vectors to ParaHeckeOp: 73 124, 7452 (check: 73 124, 7452)

Desired length, max det of output vectors: 80, 92

Quotient of the max dets should be 81

Smaller max det is the minimal max det 92
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}
About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 80, 92

Rank of T[G2G2]^+/- = (3,1)

Rank of plus basis attempt = 34 and dimension of S_4(K(123))^+ = 34

Rank of minus basis attempt = 1 and dimension of S_4(K(123))^- = 3


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-123-150.ma}

Dimensions of BPcoeffMatProven: {4, 168}

Dimensions of join: {5, 80}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 34 and dimension of S_4(K(123))^+ = 34

Rank of minus basis attempt = 3 and dimension of S_4(K(123))^- = 3


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Found a plus basis of dimension 34

Found minus basis of dimension 3


N = 129
```

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}
lift dimension of S_4(K(129))^+ : 22
nonlift dimension of S_4(K(129))^+ : 21
dimension of S_4(K(129))^- : 1
Ibukiyama-Kitayama dimension of S_4(K(129)) : 44

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(129))^+ = 43
Rank of minus basis attempt = 0 and dimension of S_4(K(129))^- = 1
Initial short vector length: 86
Have vectors of length 93: long enough
Determinant shell containing the vectors of length 86 : 108
Short vector length is 93, and the corresponding determinant is 108
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-129-110.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,129}^{cusp}), which is 22
Rank of plus basis attempt = 22 and dimension of S_4(K(129))^+ = 43
Rank of minus basis attempt = 0 and dimension of S_4(K(129))^- = 1

STEP 2: TRACE DOWN
N = 129  q = 5  Nq = 645
93-th determinant is 108
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 91
Trace down to try to hit S4(K(N)), of dimension 44
Plus lifts, plus nonlifts, minus dimension: {22, 21, 1}
Target plus rank, target minus rank: 43 1
Dimensions of savedTargetMats: {93, 3}
Making ordered good sigs for source space S_4(K(Nq))...
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
Found file Grits-2-645-2700.ma, Getting it
 ... done making G2.  Dimensions of Grits: {13, 17488}
Determining indices: {1, 2, 5, 6, 13, 14, 17, 18, 19, 49, 85, 86, 87}
Atkin-Lehner truncation: 108
```

```
Beginning ALSignaturesAndDims...
 ... done with ALSignaturesAndDims.
ALspacesDims = {{{1, 1, 1}, 5}, {{1, -1, -1}, 3}, {{-1, 1, -1}, 3}, {{-1, -1, 1}, 2}}
Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-645-2700-to-129-108.ma exists, reading it
Found file Grits-2-645-2700.ma, Getting it
Grit dimensions: {13, 17488}
Viable file found: GG-Grits-2-645-2700.ma-2700-mod-12347.ma
New best file: GG-Grits-2-645-2700.ma-2700-mod-12347.ma
Dimensions of G2G2: {91, 17488}
Dimensions of fDown = {91, 93}
Rank of fDown mod pp = 39
Dimensions and rank of plus space fDown = {91, 93}  39
Dimensions and rank of minus space fDown = {91, 93}  0
Rank of plus basis attempt = 43 and dimension of S_4(K(129))^+ = 43
Rank of minus basis attempt = 0 and dimension of S_4(K(129))^- = 1

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS
Pairs from dim(J_{2,N}^{cusp}: 3
Found file Grits-2-129-108.ma, Getting it
Viable file found: GG-Grits-2-129-7680.ma-7680-mod-12347.ma
New best file: GG-Grits-2-129-7680.ma-7680-mod-12347.ma
Viable file found: GG-Grits-2-129-8748.ma-8748-mod-12347.ma
Dimensions of G2G2: {3, 93}
Rank of plus basis attempt = 43 and dimension of S_4(K(129))^+ = 43
Rank of minus basis attempt = 0 and dimension of S_4(K(129))^- = 1

STEP 4: HECKE SPREAD
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified
Found custom table of Hecke spreads
Preparing for 1 Hecke spreads, the last of which dilates determinants by 81
Need to multiply the minimal max det 108 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det
Need pre-Hecke expansions up to determinant 8748
Hecke spreads: 1
Pre-Hecke expansions will have length 93019
```

```
Found file Grits-2-129-8748.ma, Getting it

Viable file found: GG-Grits-2-129-8748.ma-8748-mod-12347.ma

New best file: GG-Grits-2-129-8748.ma-8748-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 93 019

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 93 019}, output dimensions {3, 93 019}

Length, max det of input vectors to ParaHeckeOp: 93 019, 8748 (check: 93 019, 8748)

Desired length, max det of output vectors: 93, 108

Quotient of the max dets should be 81

Smaller max det is the minimal max det 108
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 93, 108

Rank of T[G2G2]^+/- = (3,1)

Rank of plus basis attempt = 43 and dimension of S_4(K(129))^+ = 43

Rank of minus basis attempt = 1 and dimension of S_4(K(129))^- = 1


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {}


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Found a plus basis of dimension 43

Found minus basis of dimension 1
```

N = 130

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 0}

lift dimension of S_4(K(130))^+ : 21

nonlift dimension of S_4(K(130))^+ : 22

dimension of S_4(K(130))^- : 3

Ibukiyama-Kitayama dimension of S_4(K(130)) : 46

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(130))^+ = 43

Rank of minus basis attempt = 0 and dimension of S_4(K(130))^- = 3

Initial short vector length: 91

Have vectors of length 96: long enough

Determinant shell containing the vectors of length 91 : 100

Short vector length is 96, and the corresponding determinant is 100

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-130-100.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,130}^{cusp}), which is 21

Rank of plus basis attempt = 21 and dimension of S_4(K(130))^+ = 43

Rank of minus basis attempt = 0 and dimension of S_4(K(130))^- = 3

STEP 2: TRACE DOWN

N = 130 q = 7 Nq = 910

96-th determinant is 100

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 120

Trace down to try to hit S4(K(N)), of dimension 46

Plus lifts, plus nonlifts, minus dimension: {21, 22, 3}

Target plus rank, target minus rank: 43 3

Dimensions of savedTargetMats: {96, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-910-4900.ma, Getting it

 ... done making G2. Dimensions of Grits: {15, 56020}

Determining indices: {1, 2, 5, 6, 7, 8, 21, 22, 29, 30, 41, 53, 54, 55, 57}

```
Atkin-Lehner truncation: 132

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1, 1}, 5}, {{1, 1, -1, -1}, 3}, {{1, -1, 1, -1}, 2},
  {{1, -1, -1, 1}, 1}, {{-1, 1, 1, -1}, 2}, {{-1, 1, -1, 1}, 1}, {{-1, -1, 1, 1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-910-4900-to-130-100.ma exists, reading it

Found file Grits-2-910-4900.ma, Getting it

Grit dimensions: {15, 56 020}

Viable file found: GG-Grits-2-910-4900.ma-4900-mod-12347.ma

New best file: GG-Grits-2-910-4900.ma-4900-mod-12347.ma

Dimensions of G2G2: {120, 56 020}

Dimensions of fDown = {120, 96}

Rank of fDown mod pp = 41

Dimensions and rank of plus space fDown = {120, 96}  41

Dimensions and rank of minus space fDown = {120, 96}  0

Rank of plus basis attempt = 43 and dimension of S_4(K(130))^+ = 43

Rank of minus basis attempt = 0 and dimension of S_4(K(130))^- = 3


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-130-100.ma, Getting it

Viable file found: GG-Grits-2-130-8960.ma-8960-mod-12347.ma

New best file: GG-Grits-2-130-8960.ma-8960-mod-12347.ma

Dimensions of G2G2: {3, 96}

Rank of plus basis attempt = 43 and dimension of S_4(K(130))^+ = 43

Rank of minus basis attempt = 0 and dimension of S_4(K(130))^- = 3


STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 2 Hecke spreads, the last of which dilates determinants by 64

Need to multiply the minimal max det 100 by the largest det contraction factor
 64 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 6400

Hecke spreads: 2
```

```
Pre-Hecke expansions will have length 80 682

Found file Grits-2-130-6400.ma, Getting it

Viable file found: GG-Grits-2-130-8960.ma-8960-mod-12347.ma

New best file: GG-Grits-2-130-8960.ma-8960-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 4

{{2, 2}, {2, 1}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 134 898

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 134 898}, output dimensions {3, 80 682}

Length, max det of input vectors to ParaHeckeOp: 80 682, 6400 (check: 80 682, 6400)

Desired length, max det of output vectors: 1006, 400

Quotient of the max dets should be 16

Smaller max det is the minimal max det 100
  times the biggest postpended det inverse dilation factor
 4 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 1006, 400

Rank of T[G2G2]^+/- = (3,2)

Rank of plus basis attempt = 43 and dimension of S_4(K(130))^+ = 43

Rank of minus basis attempt = 2 and dimension of S_4(K(130))^- = 3


Hecke spread 2 out of 2

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 1006
```

```
Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {3, 1006}, output dimensions {3, 1006}

Length, max det of input vectors to ParaHeckeOp: 1006, 400 (check: 1006, 400)

Desired length, max det of output vectors: 96, 100

Quotient of the max dets should be 4

Smaller max det is the minimal max det 100
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 96, 100

Rank of T[G2G2]^+/- = (3,2)

Rank of plus basis attempt = 43 and dimension of S_4(K(130))^+ = 43

Rank of minus basis attempt = 3 and dimension of S_4(K(130))^- = 3


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {}


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Found a plus basis of dimension 43

Found minus basis of dimension 3


N = 133

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}

lift dimension of S_4(K(133))^+ : 25

nonlift dimension of S_4(K(133))^+ : 23

dimension of S_4(K(133))^- : 1

Ibukiyama-Kitayama dimension of S_4(K(133)) : 49


STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(133))^+ = 48

Rank of minus basis attempt = 0 and dimension of S_4(K(133))^- = 1

Initial short vector length: 64
```

```
Have vectors of length 81: long enough

Determinant shell containing the vectors of length 64 : 103

Short vector length is 69, and the corresponding determinant is 103

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-133-110.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,133}^{cusp}), which is 25

Rank of plus basis attempt = 25 and dimension of S_4(K(133))^+ = 48

Rank of minus basis attempt = 0 and dimension of S_4(K(133))^- = 1
```

STEP 2: TRACE DOWN

```
N = 133  q = 5  Nq = 665

69-th determinant is 103

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 136

Trace down to try to hit S4(K(N)), of dimension 49

Plus lifts, plus nonlifts, minus dimension: {25, 23, 1}

Target plus rank, target minus rank: 48 1

Dimensions of savedTargetMats: {69, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-665-2575.ma, Getting it

 ... done making G2.  Dimensions of Grits: {16, 14 000}

Determining indices: {1, 2, 3, 4, 5, 6, 15, 16, 19, 20, 27, 28, 29, 39, 47, 63}

Atkin-Lehner truncation: 66

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 8}, {{1, -1, -1}, 3}, {{-1, 1, -1}, 3}, {{-1, -1, 1}, 2}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-665-2575-to-133-103.ma exists, reading it

Found file Grits-2-665-2575.ma, Getting it

Grit dimensions: {16, 14 000}

Viable file found: GG-Grits-2-665-3479.ma-3479-mod-12347.ma

New best file: GG-Grits-2-665-3479.ma-3479-mod-12347.ma

Dimensions of G2G2: {136, 22 838}
```

```
Dimensions of fDown = {136, 69}
Rank of fDown mod pp = 44
Dimensions and rank of plus space fDown = {136, 69}  44
Dimensions and rank of minus space fDown = {136, 69}  0
Rank of plus basis attempt = 48 and dimension of S_4(K(133))^+ = 48
Rank of minus basis attempt = 0 and dimension of S_4(K(133))^- = 1
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 10
Found file Grits-2-133-103.ma, Getting it
Viable file found: GG-Grits-2-133-2560.ma-2560-mod-12347.ma
New best file: GG-Grits-2-133-2560.ma-2560-mod-12347.ma
Dimensions of G2G2: {10, 69}
Rank of plus basis attempt = 48 and dimension of S_4(K(133))^+ = 48
Rank of minus basis attempt = 0 and dimension of S_4(K(133))^- = 1
```

STEP 4: HECKE SPREAD

```
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-133-150.ma}
Dimensions of BPcoeffMatProven: {1, 135}
Dimensions of join: {1, 69}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 48 and dimension of S_4(K(133))^+ = 48
Rank of minus basis attempt = 1 and dimension of S_4(K(133))^- = 1
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Found a plus basis of dimension 48
Found minus basis of dimension 1
```

N = 134

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}
```

```
lift dimension of S_4(K(134))^+ : 23

nonlift dimension of S_4(K(134))^+ : 19

dimension of S_4(K(134))^- : 5

Ibukiyama-Kitayama dimension of S_4(K(134)) : 47
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(134))^+ = 42

Rank of minus basis attempt = 0 and dimension of S_4(K(134))^- = 5

Initial short vector length: 122

Have vectors of length 153: long enough

Determinant shell containing the vectors of length 122 : 112

Short vector length is 129, and the corresponding determinant is 112

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-134-120.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,134}^{cusp}), which is 23

Rank of plus basis attempt = 23 and dimension of S_4(K(134))^+ = 42

Rank of minus basis attempt = 0 and dimension of S_4(K(134))^- = 5
```

STEP 2: TRACE DOWN

```
N = 134  q = 5  Nq = 670

129-th determinant is 112

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 91

Trace down to try to hit S4(K(N)), of dimension 47

Plus lifts, plus nonlifts, minus dimension: {23, 19, 5}

Target plus rank, target minus rank: 42 5

Dimensions of savedTargetMats: {129, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-670-2800.ma, Getting it

 ... done making G2.  Dimensions of Grits: {13, 21162}

Determining indices: {1, 3, 4, 5, 6, 15, 16, 27, 28, 29, 30, 47, 57}

Atkin-Lehner truncation: 72

Beginning ALSignaturesAndDims...
```

```
 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 6}, {{1, -1, -1}, 3}, {{-1, 1, -1}, 2}, {{-1, -1, 1}, 2}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-670-2800-to-134-112.ma exists, reading it

Found file Grits-2-670-2800.ma, Getting it

Grit dimensions: {13, 21162}

Viable file found: GG-Grits-2-670-2800.ma-2800-mod-12347.ma

New best file: GG-Grits-2-670-2800.ma-2800-mod-12347.ma

Dimensions of G2G2: {91, 21162}

Dimensions of fDown = {91, 129}

Rank of fDown mod pp = 38

Dimensions and rank of plus space fDown = {91, 129}  38

Dimensions and rank of minus space fDown = {91, 129}  0

Rank of plus basis attempt = 42 and dimension of S_4(K(134))^+ = 42

Rank of minus basis attempt = 0 and dimension of S_4(K(134))^- = 5
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-134-112.ma, Getting it

Viable file found: GG-Grits-2-134-16128.ma-16128-mod-12347.ma

New best file: GG-Grits-2-134-16128.ma-16128-mod-12347.ma

Viable file found: GG-Grits-2-134-28672.ma-28672-mod-12347.ma

Viable file found: GG-Grits-2-134-7168.ma-7168-mod-12347.ma

New best file: GG-Grits-2-134-7168.ma-7168-mod-12347.ma

Viable file found: GG-Grits-2-134-9072.ma-9072-mod-12347.ma

Dimensions of G2G2: {3, 129}

Rank of plus basis attempt = 42 and dimension of S_4(K(134))^+ = 42

Rank of minus basis attempt = 0 and dimension of S_4(K(134))^- = 5
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 2 Hecke spreads, the last of which dilates determinants by 64

Need to multiply the minimal max det 112 by the largest det contraction factor
 64 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 7168
```

```
Hecke spreads: 2

Pre-Hecke expansions will have length 76 999

Found file Grits-2-134-7168.ma, Getting it

Viable file found: GG-Grits-2-134-16128.ma-16128-mod-12347.ma

New best file: GG-Grits-2-134-16128.ma-16128-mod-12347.ma

Viable file found: GG-Grits-2-134-28672.ma-28672-mod-12347.ma

Viable file found: GG-Grits-2-134-7168.ma-7168-mod-12347.ma

New best file: GG-Grits-2-134-7168.ma-7168-mod-12347.ma

Viable file found: GG-Grits-2-134-9072.ma-9072-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 4

{{2, 2}, {2, 1}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 76 999

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 76 999}, output dimensions {3, 76 999}

Length, max det of input vectors to ParaHeckeOp: 76 999, 7168 (check: 76 999, 7168)

Desired length, max det of output vectors: 1037, 448

Quotient of the max dets should be 16

Smaller max det is the minimal max det 112
  times the biggest postpended det inverse dilation factor
 4 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 1037, 448

Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 42 and dimension of S_4(K(134))^+ = 42

Rank of minus basis attempt = 3 and dimension of S_4(K(134))^- = 5


Hecke spread 2 out of 2

Is Hecke spread 2 present?  False
```

```
DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 1037

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {3, 1037}, output dimensions {3, 1037}

Length, max det of input vectors to ParaHeckeOp: 1037, 448 (check: 1037, 448)

Desired length, max det of output vectors: 129, 112

Quotient of the max dets should be 4

Smaller max det is the minimal max det 112
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 129, 112

Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 42 and dimension of S_4(K(134))^+ = 42

Rank of minus basis attempt = 4 and dimension of S_4(K(134))^- = 5


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-134-150.ma}

Dimensions of BPcoeffMatProven: {7, 182}

Dimensions of join: {11, 129}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 42 and dimension of S_4(K(134))^+ = 42

Rank of minus basis attempt = 5 and dimension of S_4(K(134))^- = 5


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Found a plus basis of dimension 42

Found minus basis of dimension 5


N = 138
```

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 0}
lift dimension of S_4(K(138))^+ : 21
nonlift dimension of S_4(K(138))^+ : 20
dimension of S_4(K(138))^- : 7
Ibukiyama-Kitayama dimension of S_4(K(138)) : 48

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(138))^+ = 41
Rank of minus basis attempt = 0 and dimension of S_4(K(138))^- = 7
Initial short vector length: 107
Have vectors of length 122: long enough
Determinant shell containing the vectors of length 107 : 111
Short vector length is 118, and the corresponding determinant is 111
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-138-120.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,138}^{cusp}), which is 21
Rank of plus basis attempt = 21 and dimension of S_4(K(138))^+ = 41
Rank of minus basis attempt = 0 and dimension of S_4(K(138))^- = 7

STEP 2: TRACE DOWN
N = 138  q = 7  Nq = 966
118-th determinant is 111
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 210
Trace down to try to hit S4(K(N)), of dimension 48
Plus lifts, plus nonlifts, minus dimension: {21, 20, 7}
Target plus rank, target minus rank: 41 7
Dimensions of savedTargetMats: {118, 3}
Making ordered good sigs for source space S_4(K(Nq))...
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
Found file Grits-2-966-5439.ma, Getting it
 ... done making G2.  Dimensions of Grits: {20, 72156}
Determining indices:
 {1, 2, 3, 4, 9, 10, 17, 18, 33, 34, 35, 36, 49, 50, 89, 93, 94, 95, 125, 126}
Atkin-Lehner truncation: 132
```

```
Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1, 1}, 8}, {{1, 1, -1, -1}, 5},
  {{1, -1, 1, -1}, 3}, {{1, -1, -1, 1}, 2}, {{-1, 1, 1, -1}, 1}, {{-1, 1, -1, 1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-966-5439-to-138-111.ma exists, reading it

Found file Grits-2-966-5439.ma, Getting it

Grit dimensions: {20, 72156}

Viable file found: GG-Grits-2-966-5439.ma-5439-mod-12347.ma

New best file: GG-Grits-2-966-5439.ma-5439-mod-12347.ma

Dimensions of G2G2: {210, 72156}

Dimensions of fDown = {210, 118}

Rank of fDown mod pp = 41

Dimensions and rank of plus space fDown = {210, 118}  41

Dimensions and rank of minus space fDown = {210, 118}  0

Rank of plus basis attempt = 41 and dimension of S_4(K(138))^+ = 41

Rank of minus basis attempt = 0 and dimension of S_4(K(138))^- = 7


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 1

Found file Grits-2-138-111.ma, Getting it

Viable file found: GG-Grits-2-138-33792.ma-33792-mod-12347.ma

New best file: GG-Grits-2-138-33792.ma-33792-mod-12347.ma

Viable file found: GG-Grits-2-138-35964.ma-35964-mod-12347.ma

Dimensions of G2G2: {1, 118}

Rank of plus basis attempt = 41 and dimension of S_4(K(138))^+ = 41

Rank of minus basis attempt = 0 and dimension of S_4(K(138))^- = 7


STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 111 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 8991

Hecke spreads: 1
```

```
Pre-Hecke expansions will have length 146 694

Found file Grits-2-138-8991.ma, Getting it

Viable file found: GG-Grits-2-138-33792.ma-33792-mod-12347.ma

New best file: GG-Grits-2-138-33792.ma-33792-mod-12347.ma

Viable file found: GG-Grits-2-138-35964.ma-35964-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 1 098 074

Called ShortenVecs with G2G2T[{}]

 input dimensions {1, 1 098 074}, output dimensions {1, 146 694}

Length, max det of input vectors to ParaHeckeOp: 146 694, 8991 (check: 146 694, 8991)

Desired length, max det of output vectors: 118, 111

Quotient of the max dets should be 81

Smaller max det is the minimal max det 111
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 118, 111

Rank of T[G2G2]^+/- = (1,1)

Rank of plus basis attempt = 41 and dimension of S_4(K(138))^+ = 41

Rank of minus basis attempt = 1 and dimension of S_4(K(138))^- = 7
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-138-150.ma}

Dimensions of BPcoeffMatProven: {19, 194}

Dimensions of join: {20, 118}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 41 and dimension of S_4(K(138))^+ = 41

Rank of minus basis attempt = 7 and dimension of S_4(K(138))^- = 7
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 41

Found minus basis of dimension 7

N = 141

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}

lift dimension of S_4(K(141))^+ : 24

nonlift dimension of S_4(K(141))^+ : 23

dimension of S_4(K(141))^- : 2

Ibukiyama-Kitayama dimension of S_4(K(141)) : 49

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(141))^+ = 47

Rank of minus basis attempt = 0 and dimension of S_4(K(141))^- = 2

Initial short vector length: 94

Have vectors of length 109: long enough

Determinant shell containing the vectors of length 94 : 116

Short vector length is 105, and the corresponding determinant is 116

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-141-120.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,141}^{cusp}), which is 24

Rank of plus basis attempt = 24 and dimension of S_4(K(141))^+ = 47

Rank of minus basis attempt = 0 and dimension of S_4(K(141))^- = 2

STEP 2: TRACE DOWN

N = 141 q = 5 Nq = 705

105-th determinant is 116

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 171

Trace down to try to hit S4(K(N)), of dimension 49

Plus lifts, plus nonlifts, minus dimension: {24, 23, 2}

Target plus rank, target minus rank: 47 2

```
Dimensions of savedTargetMats: {105, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-705-2900.ma, Getting it

 ... done making G2.  Dimensions of Grits: {18, 19914}

Determining indices: {1, 2, 3, 4, 5, 7, 8, 11, 12, 15, 16, 23, 24, 25, 39, 43, 55, 56}

Atkin-Lehner truncation: 66

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 9}, {{1, -1, -1}, 4}, {{-1, 1, -1}, 3}, {{-1, -1, 1}, 2}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-705-2900-to-141-116.ma exists, reading it

Found file Grits-2-705-2900.ma, Getting it

Grit dimensions: {18, 19914}

Viable file found: GG-Grits-2-705-2900.ma-2900-mod-12347.ma

New best file: GG-Grits-2-705-2900.ma-2900-mod-12347.ma

Dimensions of G2G2: {171, 19914}

Dimensions of fDown = {171, 105}

Rank of fDown mod pp = 47

Dimensions and rank of plus space fDown = {171, 105}  47

Dimensions and rank of minus space fDown = {171, 105}  0

Rank of plus basis attempt = 47 and dimension of S_4(K(141))^+ = 47

Rank of minus basis attempt = 0 and dimension of S_4(K(141))^- = 2


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-141-116.ma, Getting it

Viable file found: GG-Grits-2-141-20160.ma-20160-mod-12347.ma

New best file: GG-Grits-2-141-20160.ma-20160-mod-12347.ma

Viable file found: GG-Grits-2-141-37584.ma-37584-mod-12347.ma

Viable file found: GG-Grits-2-141-8960.ma-8960-mod-12347.ma

New best file: GG-Grits-2-141-8960.ma-8960-mod-12347.ma

Dimensions of G2G2: {3, 105}

Rank of plus basis attempt = 47 and dimension of S_4(K(141))^+ = 47

Rank of minus basis attempt = 0 and dimension of S_4(K(141))^- = 2
```

STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 116 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 9396

Hecke spreads: 1

Pre-Hecke expansions will have length 103 871

Found file Grits-2-141-9396.ma, Getting it

Viable file found: GG-Grits-2-141-20160.ma-20160-mod-12347.ma

New best file: GG-Grits-2-141-20160.ma-20160-mod-12347.ma

Viable file found: GG-Grits-2-141-37584.ma-37584-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present? False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 330 854

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 330 854}, output dimensions {3, 103 871}

Length, max det of input vectors to ParaHeckeOp: 103 871, 9396 (check: 103 871, 9396)

Desired length, max det of output vectors: 105, 116

Quotient of the max dets should be 81

Smaller max det is the minimal max det 116
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 105, 116

Rank of T[G2G2]^+/- = (3,1)

Rank of plus basis attempt = 47 and dimension of S_4(K(141))^+ = 47

Rank of minus basis attempt = 1 and dimension of S_4(K(141))^- = 2

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-141-150.ma}

Dimensions of BPcoeffMatProven: {2, 145}

Dimensions of join: {3, 105}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 47 and dimension of S_4(K(141))^+ = 47

Rank of minus basis attempt = 2 and dimension of S_4(K(141))^- = 2

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 47

Found minus basis of dimension 2

N = 142

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}

lift dimension of S_4(K(142))^+ : 25

nonlift dimension of S_4(K(142))^+ : 27

dimension of S_4(K(142))^- : 5

Ibukiyama-Kitayama dimension of S_4(K(142)) : 57

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(142))^+ = 52

Rank of minus basis attempt = 0 and dimension of S_4(K(142))^- = 5

Initial short vector length: 102

Have vectors of length 109: long enough

Determinant shell containing the vectors of length 102 : 112

Short vector length is 109, and the corresponding determinant is 112

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-142-120.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,142}^{cusp}), which is 25

Rank of plus basis attempt = 25 and dimension of S_4(K(142))^+ = 52

Rank of minus basis attempt = 0 and dimension of S_4(K(142))^- = 5

STEP 2: TRACE DOWN

```
N = 142  q = 5  Nq = 710
109-th determinant is 112
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 120
Trace down to try to hit S4(K(N)), of dimension 57
Plus lifts, plus nonlifts, minus dimension: {25, 27, 5}
Target plus rank, target minus rank: 52 5
Dimensions of savedTargetMats: {109, 3}
Making ordered good sigs for source space S_4(K(Nq))...
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
Found file Grits-2-710-2800.ma, Getting it
 ... done making G2.  Dimensions of Grits: {15, 21176}
Determining indices: {1, 2, 3, 4, 13, 14, 15, 29, 30, 41, 49, 50, 57, 71, 72}
Atkin-Lehner truncation: 78
Beginning ALSignaturesAndDims...
 ... done with ALSignaturesAndDims.
ALspacesDims = {{{1, 1, 1}, 7}, {{1, -1, -1}, 5}, {{-1, 1, -1}, 2}, {{-1, -1, 1}, 1}}
Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-710-2800-to-142-112.ma exists, reading it
Found file Grits-2-710-2800.ma, Getting it
Grit dimensions: {15, 21176}
Viable file found: GG-Grits-2-710-2800.ma-2800-mod-12347.ma
New best file: GG-Grits-2-710-2800.ma-2800-mod-12347.ma
Dimensions of G2G2: {120, 21176}
Dimensions of fDown = {120, 109}
Rank of fDown mod pp = 50
Dimensions and rank of plus space fDown = {120, 109}  50
Dimensions and rank of minus space fDown = {120, 109}  0
Rank of plus basis attempt = 52 and dimension of S_4(K(142))^+ = 52
Rank of minus basis attempt = 0 and dimension of S_4(K(142))^- = 5
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 3
Found file Grits-2-142-112.ma, Getting it
```

```
Viable file found: GG-Grits-2-142-26364.ma-26364-mod-12347.ma

New best file: GG-Grits-2-142-26364.ma-26364-mod-12347.ma

Viable file found: GG-Grits-2-142-28672.ma-28672-mod-12347.ma

Viable file found: GG-Grits-2-142-9984.ma-9984-mod-12347.ma

New best file: GG-Grits-2-142-9984.ma-9984-mod-12347.ma

Dimensions of G2G2: {3, 109}

Rank of plus basis attempt = 52 and dimension of S_4(K(142))^+ = 52

Rank of minus basis attempt = 0 and dimension of S_4(K(142))^- = 5
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 3 Hecke spreads, the last of which dilates determinants by 144

Need to multiply the minimal max det 112 by the largest det contraction factor
 144 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 16 128

Hecke spreads: 3

Pre-Hecke expansions will have length 264 530

Found file Grits-2-142-16128.ma, Getting it

Viable file found: GG-Grits-2-142-26364.ma-26364-mod-12347.ma

New best file: GG-Grits-2-142-26364.ma-26364-mod-12347.ma

Viable file found: GG-Grits-2-142-28672.ma-28672-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 9

{{2, 2}, {2, 1}} 1

{{2, 2}, {3, 1}} 1

Hecke spread 1 out of 3

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 555 165

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 555 165}, output dimensions {3, 264 530}

Length, max det of input vectors to ParaHeckeOp: 264 530, 16 128 (check: 264 530, 16 128)

Desired length, max det of output vectors: 3766, 1008

Quotient of the max dets should be 16
```

```
Smaller max det is the minimal max det 112
  times the biggest postpended det inverse dilation factor
 9 for the Hecke operator{{2, 2}}
About to compute G2G2T[{{2, 2}}]
Making abstract Hecke formula at a squared bad prime
Using Ralf's formula
Substituting given paramodular forms in abstract formula
Hecke spread 1 length and max det should be as desired: 3766, 1008
Rank of T[G2G2]^+/- = (3,2)
Rank of plus basis attempt = 52 and dimension of S_4(K(142))^+ = 52
Rank of minus basis attempt = 2 and dimension of S_4(K(142))^- = 5


Hecke spread 2 out of 3
Is Hecke spread 2 present?  False
DoOneHecke called {},{{2, 2}, {2, 1}}
{tp,tpdel} = {2, 2}
DoOneHecke called {{2, 2}},{{2, 1}}
{tp,tpdel} = {2, 1}
Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 3766
Called ShortenVecs with G2G2T[{{2, 2}}]
 input dimensions {3, 3766}, output dimensions {3, 1052}
Length, max det of input vectors to ParaHeckeOp: 1052, 448 (check: 1052, 448)
Desired length, max det of output vectors: 109, 112
Quotient of the max dets should be 4
Smaller max det is the minimal max det 112
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}
About to compute G2G2T[{{2, 2}, {2, 1}}]
Making abstract Hecke formula at a bad prime
Using Ralf's formula
Substituting given paramodular forms in abstract formula
Hecke spread 2 length and max det should be as desired: 109, 112
Rank of T[G2G2]^+/- = (3,2)
Rank of plus basis attempt = 52 and dimension of S_4(K(142))^+ = 52
Rank of minus basis attempt = 3 and dimension of S_4(K(142))^- = 5


Hecke spread 3 out of 3
Is Hecke spread 3 present?  False
```

```
DoOneHecke called {},{{2, 2}, {3, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{3, 1}}

{tp,tpdel} = {3, 1}

Need to compute G2G2T[{{2, 2}, {3, 1}}] having G2G2T[{{2, 2}}] vectors to length 3766

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {3, 3766}, output dimensions {3, 3766}

Length, max det of input vectors to ParaHeckeOp: 3766, 1008 (check: 3766, 1008)

Desired length, max det of output vectors: 109, 112

Quotient of the max dets should be 9

Smaller max det is the minimal max det 112
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {3, 1}}
About to compute G2G2T[{{2, 2}, {3, 1}}]

Making abstract Hecke formula

Substituting given paramodular forms in abstract formula

Hecke spread 3 length and max det should be as desired: 109, 112

Rank of T[G2G2]^+/- = (3,2)

Rank of plus basis attempt = 52 and dimension of S_4(K(142))^+ = 52

Rank of minus basis attempt = 4 and dimension of S_4(K(142))^- = 5


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-142-150.ma}

Dimensions of BPcoeffMatProven: {9, 151}

Dimensions of join: {13, 109}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 52 and dimension of S_4(K(142))^+ = 52

Rank of minus basis attempt = 5 and dimension of S_4(K(142))^- = 5


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Found a plus basis of dimension 52

Found minus basis of dimension 5


N = 143

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}
```

```
lift dimension of S_4(K(143))^+ : 23

nonlift dimension of S_4(K(143))^+ : 17

dimension of S_4(K(143))^- : 4

Ibukiyama-Kitayama dimension of S_4(K(143)) : 44
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(143))^+ = 40

Rank of minus basis attempt = 0 and dimension of S_4(K(143))^- = 4

Initial short vector length: 148

Have vectors of length 166: long enough

Determinant shell containing the vectors of length 148 : 156

Short vector length is 154, and the corresponding determinant is 156

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-143-160.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,143}^{cusp}), which is 23

Rank of plus basis attempt = 23 and dimension of S_4(K(143))^+ = 40

Rank of minus basis attempt = 0 and dimension of S_4(K(143))^- = 4
```

STEP 2: TRACE DOWN

```
N = 143  q = 5  Nq = 715

154-th determinant is 156

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 171

Trace down to try to hit S4(K(N)), of dimension 44

Plus lifts, plus nonlifts, minus dimension: {23, 17, 4}

Target plus rank, target minus rank: 40 4

Dimensions of savedTargetMats: {154, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-715-3900.ma, Getting it

 ... done making G2.  Dimensions of Grits: {18, 25984}

Determining indices: {1, 2, 5, 6, 13, 14, 17, 18, 19, 20, 25, 29, 30, 31, 49, 50, 89, 90}

Atkin-Lehner truncation: 96

Beginning ALSignaturesAndDims...
```

```
 ... done with ALSignaturesAndDims.
ALspacesDims = {{{1, 1, 1}, 8}, {{1, -1, -1}, 6}, {{-1, 1, -1}, 2}, {{-1, -1, 1}, 2}}
Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-715-3900-to-143-156.ma exists, reading it
Found file Grits-2-715-3900.ma, Getting it
Grit dimensions: {18, 25984}
Viable file found: GG-Grits-2-715-3900.ma-3900-mod-12347.ma
New best file: GG-Grits-2-715-3900.ma-3900-mod-12347.ma
Dimensions of G2G2: {171, 25984}
Dimensions of fDown = {171, 154}
Rank of fDown mod pp = 40
Dimensions and rank of plus space fDown = {171, 154}  40
Dimensions and rank of minus space fDown = {171, 154}  0
Rank of plus basis attempt = 40 and dimension of S_4(K(143))^+ = 40
Rank of minus basis attempt = 0 and dimension of S_4(K(143))^- = 4

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS
Pairs from dim(J_{2,N}^{cusp}: 1
Found file Grits-2-143-156.ma, Getting it
Viable file found: GG-Grits-2-143-27244.ma-27244-mod-12347.ma
New best file: GG-Grits-2-143-27244.ma-27244-mod-12347.ma
Viable file found: GG-Grits-2-143-27440.ma-27440-mod-12347.ma
Viable file found: GG-Grits-2-143-31360.ma-31360-mod-12347.ma
Dimensions of G2G2: {1, 154}
Rank of plus basis attempt = 40 and dimension of S_4(K(143))^+ = 40
Rank of minus basis attempt = 0 and dimension of S_4(K(143))^- = 4

STEP 4: HECKE SPREAD
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-143-150.ma}
Dimensions of BPcoeffMatProven: {7, 146}
Borcherds product vectors are shorter than notebook space vectors
Dimensions of join: {7, 146}
```

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 40 and dimension of S_4(K(143))^+ = 40

Rank of minus basis attempt = 4 and dimension of S_4(K(143))^- = 4

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 40

Found minus basis of dimension 4

N = 145

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}

lift dimension of S_4(K(145))^+ : 26

nonlift dimension of S_4(K(145))^+ : 30

dimension of S_4(K(145))^- : 1

Ibukiyama-Kitayama dimension of S_4(K(145)) : 57

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(145))^+ = 56

Rank of minus basis attempt = 0 and dimension of S_4(K(145))^- = 1

Initial short vector length: 82

Have vectors of length 108: long enough

Determinant shell containing the vectors of length 82 : 111

Short vector length is 96, and the corresponding determinant is 111

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-145-120.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,145}^{cusp}), which is 26

Rank of plus basis attempt = 26 and dimension of S_4(K(145))^+ = 56

Rank of minus basis attempt = 0 and dimension of S_4(K(145))^- = 1

STEP 2: TRACE DOWN

N = 145 q = 7 Nq = 1015

96-th determinant is 111

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 378

```
Trace down to try to hit S4(K(N)), of dimension 57

Plus lifts, plus nonlifts, minus dimension: {26, 30, 1}

Target plus rank, target minus rank: 56 1

Dimensions of savedTargetMats: {96, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1015-5439.ma, Getting it

 ... done making G2.  Dimensions of Grits: {27, 43794}

Determining indices: {1, 2, 5, 6, 7, 8, 13, 15, 16, 17,
  18, 27, 28, 39, 40, 43, 44, 45, 46, 67, 68, 69, 99, 100, 103, 104, 131}

Atkin-Lehner truncation: 138

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 11}, {{1, -1, -1}, 8}, {{-1, 1, -1}, 5}, {{-1, -1, 1}, 3}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1015-5439-to-145-111.ma exists, reading it

Found file Grits-2-1015-5439.ma, Getting it

Grit dimensions: {27, 43794}

Viable file found: GG-Grits-2-1015-5439.ma-5439-mod-12347.ma

New best file: GG-Grits-2-1015-5439.ma-5439-mod-12347.ma

Dimensions of G2G2: {378, 43794}

Dimensions of fDown = {378, 96}

Rank of fDown mod pp = 54

Dimensions and rank of plus space fDown = {378, 96}  54

Dimensions and rank of minus space fDown = {378, 96}  0

Rank of plus basis attempt = 56 and dimension of S_4(K(145))^+ = 56

Rank of minus basis attempt = 0 and dimension of S_4(K(145))^- = 1


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 6

Found file Grits-2-145-111.ma, Getting it

Viable file found: GG-Grits-2-145-6300.ma-6300-mod-12347.ma

New best file: GG-Grits-2-145-6300.ma-6300-mod-12347.ma

Dimensions of G2G2: {6, 96}

Rank of plus basis attempt = 56 and dimension of S_4(K(145))^+ = 56
```

```
Rank of minus basis attempt = 0 and dimension of S_4(K(145))^- = 1
```

STEP 4: HECKE SPREAD

```
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-145-150.ma}
Dimensions of BPcoeffMatProven: {1, 146}
Dimensions of join: {1, 96}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 56 and dimension of S_4(K(145))^+ = 56
Rank of minus basis attempt = 1 and dimension of S_4(K(145))^- = 1
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Found a plus basis of dimension 56
Found minus basis of dimension 1
```

N = 146

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 0}
lift dimension of S_4(K(146))^+ : 25
nonlift dimension of S_4(K(146))^+ : 24
dimension of S_4(K(146))^- : 8
Ibukiyama-Kitayama dimension of S_4(K(146)) : 57
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(146))^+ = 49
Rank of minus basis attempt = 0 and dimension of S_4(K(146))^- = 8
Initial short vector length: 112
Have vectors of length 146: long enough
Determinant shell containing the vectors of length 112 : 111
Short vector length is 126, and the corresponding determinant is 111
Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-146-120.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,146}^{cusp}), which is 25

Rank of plus basis attempt = 25 and dimension of S_4(K(146))^+ = 49

Rank of minus basis attempt = 0 and dimension of S_4(K(146))^- = 8
```

STEP 2: TRACE DOWN

```
N = 146  q = 7  Nq = 1022

126-th determinant is 111

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 300

Trace down to try to hit S4(K(N)), of dimension 57

Plus lifts, plus nonlifts, minus dimension: {25, 24, 8}

Target plus rank, target minus rank: 49 8

Dimensions of savedTargetMats: {126, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1022-5439.ma, Getting it

 ... done making G2.  Dimensions of Grits: {24, 55 016}

Determining indices:
 {1, 2, 5, 6, 9, 10, 21, 22, 23, 24, 37, 38, 49, 50, 69, 70, 85, 86, 87, 88, 117, 137, 189, 357}

Atkin-Lehner truncation: 384

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 11}, {{1, -1, -1}, 8}, {{-1, 1, -1}, 2}, {{-1, -1, 1}, 3}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1022-5439-to-146-111.ma exists, reading it

Found file Grits-2-1022-5439.ma, Getting it

Grit dimensions: {24, 55 016}

Viable file found: GG-Grits-2-1022-5439.ma-5439-mod-12347.ma

New best file: GG-Grits-2-1022-5439.ma-5439-mod-12347.ma

Dimensions of G2G2: {300, 55 016}

Dimensions of fDown = {300, 126}

Rank of fDown mod pp = 47

Dimensions and rank of plus space fDown = {300, 126}  47

Dimensions and rank of minus space fDown = {300, 126}  0
```

```
Rank of plus basis attempt = 49 and dimension of S_4(K(146))^+ = 49

Rank of minus basis attempt = 0 and dimension of S_4(K(146))^- = 8
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-146-111.ma, Getting it

Viable file found: GG-Grits-2-146-15984.ma-15984-mod-12347.ma

New best file: GG-Grits-2-146-15984.ma-15984-mod-12347.ma

Viable file found: GG-Grits-2-146-28416.ma-28416-mod-12347.ma

Viable file found: GG-Grits-2-146-8128.ma-8128-mod-12347.ma

New best file: GG-Grits-2-146-8128.ma-8128-mod-12347.ma

Dimensions of G2G2: {3, 126}

Rank of plus basis attempt = 49 and dimension of S_4(K(146))^+ = 49

Rank of minus basis attempt = 0 and dimension of S_4(K(146))^- = 8
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 3 Hecke spreads, the last of which dilates determinants by 144

Need to multiply the minimal max det 111 by the largest det contraction factor
 144 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 15 984

Hecke spreads: 3

Pre-Hecke expansions will have length 260 523

Found file Grits-2-146-15984.ma, Getting it

Viable file found: GG-Grits-2-146-15984.ma-15984-mod-12347.ma

New best file: GG-Grits-2-146-15984.ma-15984-mod-12347.ma

Viable file found: GG-Grits-2-146-28416.ma-28416-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 9

{{2, 2}, {2, 1}} 1

{{2, 2}, {3, 1}} 1

Hecke spread 1 out of 3

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}
```

```
Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 260 523

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 260 523}, output dimensions {3, 260 523}

Length, max det of input vectors to ParaHeckeOp: 260 523, 15 984 (check: 260 523, 15 984)

Desired length, max det of output vectors: 3636, 999

Quotient of the max dets should be 16

Smaller max det is the minimal max det 111
  times the biggest postpended det inverse dilation factor
 9 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 3636, 999

Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 49 and dimension of S_4(K(146))^+ = 49

Rank of minus basis attempt = 3 and dimension of S_4(K(146))^- = 8


Hecke spread 2 out of 3

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 3636

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {3, 3636}, output dimensions {3, 1012}

Length, max det of input vectors to ParaHeckeOp: 1012, 444 (check: 1012, 444)

Desired length, max det of output vectors: 126, 111

Quotient of the max dets should be 4

Smaller max det is the minimal max det 111
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 126, 111
```

```
Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 49 and dimension of S_4(K(146))^+ = 49

Rank of minus basis attempt = 6 and dimension of S_4(K(146))^- = 8


Hecke spread 3 out of 3

Is Hecke spread 3 present?  False

DoOneHecke called {},{{2, 2}, {3, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{3, 1}}

{tp,tpdel} = {3, 1}

Need to compute G2G2T[{{2, 2}, {3, 1}}] having G2G2T[{{2, 2}}] vectors to length 3636

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {3, 3636}, output dimensions {3, 3636}

Length, max det of input vectors to ParaHeckeOp: 3636, 999 (check: 3636, 999)

Desired length, max det of output vectors: 126, 111

Quotient of the max dets should be 9

Smaller max det is the minimal max det 111
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {3, 1}}

About to compute G2G2T[{{2, 2}, {3, 1}}]

Making abstract Hecke formula

Substituting given paramodular forms in abstract formula

Hecke spread 3 length and max det should be as desired: 126, 111

Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 49 and dimension of S_4(K(146))^+ = 49

Rank of minus basis attempt = 7 and dimension of S_4(K(146))^- = 8


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-146-150.ma}

Dimensions of BPcoeffMatProven: {22, 216}

Dimensions of join: {29, 126}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 49 and dimension of S_4(K(146))^+ = 49

Rank of minus basis attempt = 8 and dimension of S_4(K(146))^- = 8


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}
```

```
Found a plus basis of dimension 49

Found minus basis of dimension 8

N = 154

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {3, 0, 0}

lift dimension of S_4(K(154))^+ : 25

nonlift dimension of S_4(K(154))^+ : 30

dimension of S_4(K(154))^- : 3

Ibukiyama-Kitayama dimension of S_4(K(154)) : 58

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(154))^+ = 55

Rank of minus basis attempt = 0 and dimension of S_4(K(154))^- = 3

Initial short vector length: 110

Have vectors of length 112: long enough

Determinant shell containing the vectors of length 110 : 112

Short vector length is 112, and the corresponding determinant is 112

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-154-120.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,154}^{cusp}), which is 25

Rank of plus basis attempt = 25 and dimension of S_4(K(154))^+ = 55

Rank of minus basis attempt = 0 and dimension of S_4(K(154))^- = 3

STEP 2: TRACE DOWN

N = 154  q = 3  Nq = 462

112-th determinant is 112

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 15

Trace down to try to hit S4(K(N)), of dimension 58

Plus lifts, plus nonlifts, minus dimension: {25, 30, 3}

Target plus rank, target minus rank: 55 3

Dimensions of savedTargetMats: {112, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.
```

```
Making G2 for S_2(K(Nq))...

Found file Grits-2-462-1008.ma, Getting it

 ... done making G2.  Dimensions of Grits: {5, 5140}

Determining indices: {1, 2, 5, 37, 41}

Atkin-Lehner truncation: 64

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1, 1}, 3}, {{1, 1, -1, -1}, 1}, {{-1, 1, -1, 1}, 1}}

Need 8 AL signatures; have the signatures
 {{-1, 1, -1, 1}, {-1, 1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-462-1008-to-154-112.ma exists, reading it

Found file Grits-2-462-1008.ma, Getting it

Grit dimensions: {5, 5140}

Viable file found: GG-Grits-2-462-1008.ma-1008-mod-12347.ma

New best file: GG-Grits-2-462-1008.ma-1008-mod-12347.ma

Viable file found: GG-Grits-2-462-1476.ma-1476-mod-12347.ma

Dimensions of G2G2: {15, 5140}

Dimensions of fDown = {15, 112}

Rank of fDown mod pp = 15

Dimensions and rank of plus space fDown = {15, 112}  15

Dimensions and rank of minus space fDown = {15, 112}  0

Rank of plus basis attempt = 40 and dimension of S_4(K(154))^+ = 55

Rank of minus basis attempt = 0 and dimension of S_4(K(154))^- = 3


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-154-112.ma, Getting it

Viable file found: GG-Grits-2-154-12960.ma-12960-mod-12347.ma

New best file: GG-Grits-2-154-12960.ma-12960-mod-12347.ma

Viable file found: GG-Grits-2-154-28672.ma-28672-mod-12347.ma

Viable file found: GG-Grits-2-154-36288.ma-36288-mod-12347.ma

Dimensions of G2G2: {3, 112}

Rank of plus basis attempt = 43 and dimension of S_4(K(154))^+ = 55

Rank of minus basis attempt = 0 and dimension of S_4(K(154))^- = 3


STEP 4: HECKE SPREAD
```

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 2 Hecke spreads, the last of which dilates determinants by 64

Need to multiply the minimal max det 112 by the largest det contraction factor
 64 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 7168

Hecke spreads: 2

Pre-Hecke expansions will have length 91954

Found file Grits-2-154-7168.ma, Getting it

Viable file found: GG-Grits-2-154-12960.ma-12960-mod-12347.ma

New best file: GG-Grits-2-154-12960.ma-12960-mod-12347.ma

Viable file found: GG-Grits-2-154-28672.ma-28672-mod-12347.ma

Viable file found: GG-Grits-2-154-36288.ma-36288-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 4

{{2, 2}, {2, 1}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 228700

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 228700}, output dimensions {3, 91954}

Length, max det of input vectors to ParaHeckeOp: 91954, 7168 (check: 91954, 7168)

Desired length, max det of output vectors: 1122, 448

Quotient of the max dets should be 16

Smaller max det is the minimal max det 112
  times the biggest postpended det inverse dilation factor
 4 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 1122, 448

Rank of T[G2G2]^+/- = (3,2)

Rank of plus basis attempt = 43 and dimension of S_4(K(154))^+ = 55

Rank of minus basis attempt = 2 and dimension of S_4(K(154))^- = 3
```

```
Hecke spread 2 out of 2

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 1122

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {3, 1122}, output dimensions {3, 1122}

Length, max det of input vectors to ParaHeckeOp: 1122, 448 (check: 1122, 448)

Desired length, max det of output vectors: 112, 112

Quotient of the max dets should be 4

Smaller max det is the minimal max det 112
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}
About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 112, 112

Rank of T[G2G2]^+/- = (3,2)

Rank of plus basis attempt = 43 and dimension of S_4(K(154))^+ = 55

Rank of minus basis attempt = 2 and dimension of S_4(K(154))^- = 3
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-154-112.ma}

Dimensions of BPcoeffMatProven: {3, 112}

Dimensions of join: {5, 112}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 43 and dimension of S_4(K(154))^+ = 55

Rank of minus basis attempt = 3 and dimension of S_4(K(154))^- = 3
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {BPinfo-files/BPplusInfo-4-154-400-nomerge.ma}

New dimensions of JSglobalBasisAttempt: {55, 942}

Plus space span attempt may contain non-numerical entries
```

```
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 55 and dimension of S_4(K(154))^+ = 55
Rank of minus basis attempt = 3 and dimension of S_4(K(154))^- = 3

Found a plus basis of dimension 55
Found minus basis of dimension 3

N = 155
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}
lift dimension of S_4(K(155))^+ : 26
nonlift dimension of S_4(K(155))^+ : 22
dimension of S_4(K(155))^- : 5
Ibukiyama-Kitayama dimension of S_4(K(155)) : 53

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(155))^+ = 48
Rank of minus basis attempt = 0 and dimension of S_4(K(155))^- = 5
Initial short vector length: 101
Have vectors of length 121: long enough
Determinant shell containing the vectors of length 101 : 119
Short vector length is 117, and the corresponding determinant is 119
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-155-120.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,155}^{cusp}), which is 26
Rank of plus basis attempt = 26 and dimension of S_4(K(155))^+ = 48
Rank of minus basis attempt = 0 and dimension of S_4(K(155))^- = 5

STEP 2: TRACE DOWN
N = 155  q = 7  Nq = 1085
117-th determinant is 119
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 465
Trace down to try to hit S4(K(N)), of dimension 53
Plus lifts, plus nonlifts, minus dimension: {26, 22, 5}
Target plus rank, target minus rank: 48 5
```

```
Dimensions of savedTargetMats: {117, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1085-5831.ma, Getting it

 ... done making G2.  Dimensions of Grits: {30, 49578}

Determining indices: {1, 2, 3, 4, 9, 10, 15, 16, 23, 24, 31, 32,
  39, 40, 43, 44, 45, 46, 67, 69, 91, 92, 95, 96, 115, 116, 127, 128, 129, 155}

Atkin-Lehner truncation: 190

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 13}, {{1, -1, -1}, 7}, {{-1, 1, -1}, 6}, {{-1, -1, 1}, 4}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1085-5831-to-155-119.ma exists, reading it

Found file Grits-2-1085-5831.ma, Getting it

Grit dimensions: {30, 49578}

Viable file found: GG-Grits-2-1085-5831.ma-5831-mod-12347.ma

New best file: GG-Grits-2-1085-5831.ma-5831-mod-12347.ma

Dimensions of G2G2: {465, 49578}

Dimensions of fDown = {465, 117}

Rank of fDown mod pp = 48

Dimensions and rank of plus space fDown = {465, 117}  48

Dimensions and rank of minus space fDown = {465, 117}  0

Rank of plus basis attempt = 48 and dimension of S_4(K(155))^+ = 48

Rank of minus basis attempt = 0 and dimension of S_4(K(155))^- = 5
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-155-119.ma, Getting it

Viable file found: GG-Grits-2-155-8896.ma-8896-mod-12347.ma

New best file: GG-Grits-2-155-8896.ma-8896-mod-12347.ma

Dimensions of G2G2: {3, 117}

Rank of plus basis attempt = 48 and dimension of S_4(K(155))^+ = 48

Rank of minus basis attempt = 0 and dimension of S_4(K(155))^- = 5
```

STEP 4: HECKE SPREAD

```
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-155-119.ma}
Dimensions of BPcoeffMatProven: {6, 117}
Dimensions of join: {6, 117}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 48 and dimension of S_4(K(155))^+ = 48
Rank of minus basis attempt = 5 and dimension of S_4(K(155))^- = 5

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Found a plus basis of dimension 48
Found minus basis of dimension 5

N = 158
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}
lift dimension of S_4(K(158))^+ : 28
nonlift dimension of S_4(K(158))^+ : 26
dimension of S_4(K(158))^- : 10
Ibukiyama-Kitayama dimension of S_4(K(158)) : 64

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(158))^+ = 54
Rank of minus basis attempt = 0 and dimension of S_4(K(158))^- = 10
Initial short vector length: 130
Have vectors of length 139: long enough
Determinant shell containing the vectors of length 130 : 127
Short vector length is 139, and the corresponding determinant is 127
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-158-130.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,158}^{cusp}), which is 28
```

```
Rank of plus basis attempt = 28 and dimension of S_4(K(158))^+ = 54

Rank of minus basis attempt = 0 and dimension of S_4(K(158))^- = 10
```

STEP 2: TRACE DOWN

```
N = 158  q = 5  Nq = 790

139-th determinant is 127

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 171

Trace down to try to hit S4(K(N)), of dimension 64

Plus lifts, plus nonlifts, minus dimension: {28, 26, 10}

Target plus rank, target minus rank: 54 10

Dimensions of savedTargetMats: {139, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-790-3175.ma, Getting it

 ... done making G2.  Dimensions of Grits: {18, 25782}

Determining indices: {1, 2, 5, 6, 9, 10, 11, 12, 25, 26, 33, 41, 42, 43, 69, 79, 80, 111}

Atkin-Lehner truncation: 114

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 9}, {{1, -1, -1}, 5}, {{-1, 1, -1}, 3}, {{-1, -1, 1}, 1}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-790-3175-to-158-127.ma exists, reading it

Found file Grits-2-790-3175.ma, Getting it

Grit dimensions: {18, 25782}

Viable file found: GG-Grits-2-790-3175.ma-3175-mod-12347.ma

New best file: GG-Grits-2-790-3175.ma-3175-mod-12347.ma

Dimensions of G2G2: {171, 25782}

Dimensions of fDown = {171, 139}

Rank of fDown mod pp = 51

Dimensions and rank of plus space fDown = {171, 139}  51

Dimensions and rank of minus space fDown = {171, 139}  0

Rank of plus basis attempt = 54 and dimension of S_4(K(158))^+ = 54

Rank of minus basis attempt = 0 and dimension of S_4(K(158))^- = 10
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 6

Found file Grits-2-158-127.ma, Getting it

Viable file found: GG-Grits-2-158-32512.ma-32512-mod-12347.ma

New best file: GG-Grits-2-158-32512.ma-32512-mod-12347.ma

Viable file found: GG-Grits-2-158-4896.ma-4896-mod-12347.ma

New best file: GG-Grits-2-158-4896.ma-4896-mod-12347.ma

Dimensions of G2G2: {6, 139}

Rank of plus basis attempt = 54 and dimension of S_4(K(158))^+ = 54

Rank of minus basis attempt = 0 and dimension of S_4(K(158))^- = 10
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 2 Hecke spreads, the last of which dilates determinants by 64

Need to multiply the minimal max det 127 by the largest det contraction factor
 64 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 8128

Hecke spreads: 2

Pre-Hecke expansions will have length 93 225

Found file Grits-2-158-8128.ma, Getting it

Viable file found: GG-Grits-2-158-32512.ma-32512-mod-12347.ma

New best file: GG-Grits-2-158-32512.ma-32512-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 4

{{2, 2}, {2, 1}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 760 807

Called ShortenVecs with G2G2T[{}]

 input dimensions {6, 760 807}, output dimensions {6, 93 225}

Length, max det of input vectors to ParaHeckeOp: 93 225, 8128 (check: 93 225, 8128)

Desired length, max det of output vectors: 1226, 508

Quotient of the max dets should be 16
```

```
Smaller max det is the minimal max det 127
  times the biggest postpended det inverse dilation factor
 4 for the Hecke operator{{2, 2}}
About to compute G2G2T[{{2, 2}}]
Making abstract Hecke formula at a squared bad prime
Using Ralf's formula
Substituting given paramodular forms in abstract formula
Hecke spread 1 length and max det should be as desired: 1226, 508
Rank of T[G2G2]^+/- = (6,4)
Rank of plus basis attempt = 54 and dimension of S_4(K(158))^+ = 54
Rank of minus basis attempt = 4 and dimension of S_4(K(158))^- = 10


Hecke spread 2 out of 2
Is Hecke spread 2 present?  False
DoOneHecke called {},{{2, 2}, {2, 1}}
{tp,tpdel} = {2, 2}
DoOneHecke called {{2, 2}},{{2, 1}}
{tp,tpdel} = {2, 1}
Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 1226
Called ShortenVecs with G2G2T[{{2, 2}}]
 input dimensions {6, 1226}, output dimensions {6, 1226}
Length, max det of input vectors to ParaHeckeOp: 1226, 508 (check: 1226, 508)
Desired length, max det of output vectors: 139, 127
Quotient of the max dets should be 4
Smaller max det is the minimal max det 127
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}
About to compute G2G2T[{{2, 2}, {2, 1}}]
Making abstract Hecke formula at a bad prime
Using Ralf's formula
Substituting given paramodular forms in abstract formula
Hecke spread 2 length and max det should be as desired: 139, 127
Rank of T[G2G2]^+/- = (6,4)
Rank of plus basis attempt = 54 and dimension of S_4(K(158))^+ = 54
Rank of minus basis attempt = 8 and dimension of S_4(K(158))^- = 10


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-158-150.ma}
```

```
Dimensions of BPcoeffMatProven: {22, 177}

Dimensions of join: {30, 139}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 54 and dimension of S_4(K(158))^+ = 54

Rank of minus basis attempt = 10 and dimension of S_4(K(158))^- = 10
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Found a plus basis of dimension 54

Found minus basis of dimension 10
```

N = 159

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}

lift dimension of S_4(K(159))^+ : 26

nonlift dimension of S_4(K(159))^+ : 29

dimension of S_4(K(159))^- : 4

Ibukiyama-Kitayama dimension of S_4(K(159)) : 59
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(159))^+ = 55

Rank of minus basis attempt = 0 and dimension of S_4(K(159))^- = 4

Initial short vector length: 134

Have vectors of length 146: long enough

Determinant shell containing the vectors of length 134 : 135

Short vector length is 146, and the corresponding determinant is 135

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-159-140.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,159}^{cusp}), which is 26

Rank of plus basis attempt = 26 and dimension of S_4(K(159))^+ = 55

Rank of minus basis attempt = 0 and dimension of S_4(K(159))^- = 4
```

STEP 2: TRACE DOWN

```
N = 159  q = 5  Nq = 795
```

```
146-th determinant is 135

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 253

Trace down to try to hit S4(K(N)), of dimension 59

Plus lifts, plus nonlifts, minus dimension: {26, 29, 4}

Target plus rank, target minus rank: 55 4

Dimensions of savedTargetMats: {146, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-795-3375.ma, Getting it

 ... done making G2.  Dimensions of Grits: {22, 25048}

Determining indices:
 {1, 2, 3, 4, 5, 7, 8, 11, 12, 19, 20, 21, 22, 35, 36, 37, 38, 47, 51, 67, 79, 80}

Atkin-Lehner truncation: 90

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 10}, {{1, -1, -1}, 6}, {{-1, 1, -1}, 3}, {{-1, -1, 1}, 3}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-795-3375-to-159-135.ma exists, reading it

Found file Grits-2-795-3375.ma, Getting it

Grit dimensions: {22, 25048}

Viable file found: GG-Grits-2-795-3375.ma-3375-mod-12347.ma

New best file: GG-Grits-2-795-3375.ma-3375-mod-12347.ma

Dimensions of G2G2: {253, 25048}

Dimensions of fDown = {253, 146}

Rank of fDown mod pp = 55

Dimensions and rank of plus space fDown = {253, 146}  55

Dimensions and rank of minus space fDown = {253, 146}  0

Rank of plus basis attempt = 55 and dimension of S_4(K(159))^+ = 55

Rank of minus basis attempt = 0 and dimension of S_4(K(159))^- = 4
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 1

Found file Grits-2-159-135.ma, Getting it

Viable file found: GG-Grits-2-159-46332.ma-46332-mod-12347.ma

New best file: GG-Grits-2-159-46332.ma-46332-mod-12347.ma
```

```
Dimensions of G2G2: {1, 146}

Rank of plus basis attempt = 55 and dimension of S_4(K(159))^+ = 55

Rank of minus basis attempt = 0 and dimension of S_4(K(159))^- = 4
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 135 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 10 935

Hecke spreads: 1

Pre-Hecke expansions will have length 130 252

Found file Grits-2-159-10935.ma, Getting it

Viable file found: GG-Grits-2-159-46332.ma-46332-mod-12347.ma

New best file: GG-Grits-2-159-46332.ma-46332-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 1 160 198

Called ShortenVecs with G2G2T[{}]

 input dimensions {1, 1 160 198}, output dimensions {1, 130 252}

Length, max det of input vectors to ParaHeckeOp: 130 252, 10 935 (check: 130 252, 10 935)

Desired length, max det of output vectors: 146, 135

Quotient of the max dets should be 81

Smaller max det is the minimal max det 135
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 146, 135

Rank of T[G2G2]^+/- = (1,1)
```

```
Rank of plus basis attempt = 55 and dimension of S_4(K(159))^+ = 55
Rank of minus basis attempt = 1 and dimension of S_4(K(159))^- = 4

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-159-135.ma}
Dimensions of BPcoeffMatProven: {5, 146}
Dimensions of join: {6, 146}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 55 and dimension of S_4(K(159))^+ = 55
Rank of minus basis attempt = 4 and dimension of S_4(K(159))^- = 4

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Found a plus basis of dimension 55
Found minus basis of dimension 4

N = 161
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}
lift dimension of S_4(K(161))^+ : 27
nonlift dimension of S_4(K(161))^+ : 28
dimension of S_4(K(161))^- : 2
Ibukiyama-Kitayama dimension of S_4(K(161)) : 57

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(161))^+ = 55
Rank of minus basis attempt = 0 and dimension of S_4(K(161))^- = 2
Initial short vector length: 103
Have vectors of length 110: long enough
Determinant shell containing the vectors of length 103 : 136
Short vector length is 110, and the corresponding determinant is 136
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-161-140.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,161}^{cusp}), which is 27
```

```
Rank of plus basis attempt = 27 and dimension of S_4(K(161))^+ = 55

Rank of minus basis attempt = 0 and dimension of S_4(K(161))^- = 2


STEP 2: TRACE DOWN

N = 161  q = 5  Nq = 805

110-th determinant is 136

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 253

Trace down to try to hit S4(K(N)), of dimension 57

Plus lifts, plus nonlifts, minus dimension: {27, 28, 2}

Target plus rank, target minus rank: 55 2

Dimensions of savedTargetMats: {110, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-805-3400.ma, Getting it

 ... done making G2.  Dimensions of Grits: {22, 21566}

Determining indices:
 {1, 2, 3, 4, 5, 6, 9, 10, 13, 14, 21, 22, 23, 24, 49, 50, 57, 58, 61, 62, 95, 96}

Atkin-Lehner truncation: 110

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 8}, {{1, -1, -1}, 5}, {{-1, 1, -1}, 5}, {{-1, -1, 1}, 4}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-805-3400-to-161-136.ma exists, reading it

Found file Grits-2-805-3400.ma, Getting it

Grit dimensions: {22, 21566}

Viable file found: GG-Grits-2-805-4116.ma-4116-mod-12347.ma

New best file: GG-Grits-2-805-4116.ma-4116-mod-12347.ma

Dimensions of G2G2: {253, 29098}

Dimensions of fDown = {253, 110}

Rank of fDown mod pp = 54

Dimensions and rank of plus space fDown = {253, 110}  54

Dimensions and rank of minus space fDown = {253, 110}  0

Rank of plus basis attempt = 55 and dimension of S_4(K(161))^+ = 55

Rank of minus basis attempt = 0 and dimension of S_4(K(161))^- = 2
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-161-136.ma, Getting it

Viable file found: GG-Grits-2-161-12960.ma-12960-mod-12347.ma

New best file: GG-Grits-2-161-12960.ma-12960-mod-12347.ma

Dimensions of G2G2: {3, 110}

Rank of plus basis attempt = 55 and dimension of S_4(K(161))^+ = 55

Rank of minus basis attempt = 0 and dimension of S_4(K(161))^- = 2

STEP 4: HECKE SPREAD

J2=0, or already have plus space basis and no extra Hecke spreads were specified

Hecke spreads: 0

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-161-136.ma}

Dimensions of BPcoeffMatProven: {2, 110}

Dimensions of join: {2, 110}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 55 and dimension of S_4(K(161))^+ = 55

Rank of minus basis attempt = 2 and dimension of S_4(K(161))^- = 2

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 55

Found minus basis of dimension 2

N = 165

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 0}

lift dimension of S_4(K(165))^+ : 26

nonlift dimension of S_4(K(165))^+ : 29

dimension of S_4(K(165))^- : 4

Ibukiyama-Kitayama dimension of S_4(K(165)) : 59

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(165))^+ = 55

```
Rank of minus basis attempt = 0 and dimension of S_4(K(165))^- = 4

Initial short vector length: 123

Have vectors of length 136: long enough

Determinant shell containing the vectors of length 123 : 140

Short vector length is 136, and the corresponding determinant is 140

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-165-140.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,165}^{cusp}), which is 26

Rank of plus basis attempt = 26 and dimension of S_4(K(165))^+ = 55

Rank of minus basis attempt = 0 and dimension of S_4(K(165))^- = 4
```

STEP 2: TRACE DOWN

```
N = 165  q = 7  Nq = 1155

136-th determinant is 140

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 300

Trace down to try to hit S4(K(N)), of dimension 59

Plus lifts, plus nonlifts, minus dimension: {26, 29, 4}

Target plus rank, target minus rank: 55 4

Dimensions of savedTargetMats: {136, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1155-6860.ma, Getting it

 ... done making G2.  Dimensions of Grits: {24, 85824}

Determining indices:
 {1, 2, 3, 4, 9, 10, 13, 14, 21, 22, 23, 24, 45, 46, 47, 48, 49, 50, 51, 52, 85, 101, 197, 198}

Atkin-Lehner truncation: 204

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims =
 {{{1, 1, 1, 1}, 8}, {{1, 1, -1, -1}, 4}, {{1, -1, 1, -1}, 4}, {{1, -1, -1, 1}, 3},
  {{-1, 1, 1, -1}, 2}, {{-1, 1, -1, 1}, 1}, {{-1, -1, 1, 1}, 1}, {{-1, -1, -1, -1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1155-6860-to-165-140.ma exists, reading it
```

```
Found file Grits-2-1155-6860.ma, Getting it

Grit dimensions: {24, 85 824}

Viable file found: GG-Grits-2-1155-11616.ma-11616-mod-12347.ma

New best file: GG-Grits-2-1155-11616.ma-11616-mod-12347.ma

Dimensions of G2G2: {300, 193 580}

Dimensions of fDown = {300, 136}

Rank of fDown mod pp = 53

Dimensions and rank of plus space fDown = {300, 136}  53

Dimensions and rank of minus space fDown = {300, 136}  0

Rank of plus basis attempt = 55 and dimension of S_4(K(165))^+ = 55

Rank of minus basis attempt = 0 and dimension of S_4(K(165))^- = 4
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-165-140.ma, Getting it

Viable file found: GG-Grits-2-165-13284.ma-13284-mod-12347.ma

New best file: GG-Grits-2-165-13284.ma-13284-mod-12347.ma

Viable file found: GG-Grits-2-165-45360.ma-45360-mod-12347.ma

Dimensions of G2G2: {3, 136}

Rank of plus basis attempt = 55 and dimension of S_4(K(165))^+ = 55

Rank of minus basis attempt = 0 and dimension of S_4(K(165))^- = 4
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 140 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 11 340

Hecke spreads: 1

Pre-Hecke expansions will have length 172 660

Found file Grits-2-165-11340.ma, Getting it

Viable file found: GG-Grits-2-165-13284.ma-13284-mod-12347.ma

New best file: GG-Grits-2-165-13284.ma-13284-mod-12347.ma

Viable file found: GG-Grits-2-165-45360.ma-45360-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another
```

```
{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 220 658

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 220 658}, output dimensions {3, 172 660}

Length, max det of input vectors to ParaHeckeOp: 172 660, 11 340 (check: 172 660, 11 340)

Desired length, max det of output vectors: 136, 140

Quotient of the max dets should be 81

Smaller max det is the minimal max det 140
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 136, 140

Rank of T[G2G2]^+/- = (3,2)

Rank of plus basis attempt = 55 and dimension of S_4(K(165))^+ = 55

Rank of minus basis attempt = 2 and dimension of S_4(K(165))^- = 4


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-165-140.ma}

Dimensions of BPcoeffMatProven: {5, 136}

Dimensions of join: {7, 136}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 55 and dimension of S_4(K(165))^+ = 55

Rank of minus basis attempt = 4 and dimension of S_4(K(165))^- = 4


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Found a plus basis of dimension 55

Found minus basis of dimension 4
```

N = 166

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}

lift dimension of S_4(K(166))^+ : 29

nonlift dimension of S_4(K(166))^+ : 40

dimension of S_4(K(166))^- : 6

Ibukiyama-Kitayama dimension of S_4(K(166)) : 75

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(166))^+ = 69

Rank of minus basis attempt = 0 and dimension of S_4(K(166))^- = 6

Initial short vector length: 153

Have vectors of length 164: long enough

Determinant shell containing the vectors of length 153 : 140

Short vector length is 164, and the corresponding determinant is 140

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-166-140.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,166}^{cusp}), which is 29

Rank of plus basis attempt = 29 and dimension of S_4(K(166))^+ = 69

Rank of minus basis attempt = 0 and dimension of S_4(K(166))^- = 6

STEP 2: TRACE DOWN

N = 166 q = 7 Nq = 1162

164-th determinant is 140

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 595

Trace down to try to hit S4(K(N)), of dimension 75

Plus lifts, plus nonlifts, minus dimension: {29, 40, 6}

Target plus rank, target minus rank: 69 6

Dimensions of savedTargetMats: {164, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1162-6860.ma, Getting it

 ... done making G2. Dimensions of Grits: {34, 78958}

Determining indices: {1, 2, 5, 6, 9, 10, 11, 12, 29, 30, 33, 34, 35, 36, 49, 53, 54,
  65, 66, 85, 86, 105, 106, 107, 108, 125, 126, 133, 145, 146, 185, 225, 281, 301}

```
Atkin-Lehner truncation: 316

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 15}, {{1, -1, -1}, 13}, {{-1, 1, -1}, 3}, {{-1, -1, 1}, 3}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1162-6860-to-166-140.ma exists, reading it

Found file Grits-2-1162-6860.ma, Getting it

Grit dimensions: {34, 78958}

Viable file found: GG-Grits-2-1162-6860.ma-6860-mod-12347.ma

New best file: GG-Grits-2-1162-6860.ma-6860-mod-12347.ma

Dimensions of G2G2: {595, 78958}

Dimensions of fDown = {595, 164}

Rank of fDown mod pp = 67

Dimensions and rank of plus space fDown = {595, 164}  67

Dimensions and rank of minus space fDown = {595, 164}  0

Rank of plus basis attempt = 68 and dimension of S_4(K(166))^+ = 69

Rank of minus basis attempt = 0 and dimension of S_4(K(166))^- = 6
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-166-140.ma, Getting it

Viable file found: GG-Grits-2-166-22896.ma-22896-mod-12347.ma

New best file: GG-Grits-2-166-22896.ma-22896-mod-12347.ma

Viable file found: GG-Grits-2-166-48128.ma-48128-mod-12347.ma

Dimensions of G2G2: {3, 164}

Rank of plus basis attempt = 68 and dimension of S_4(K(166))^+ = 69

Rank of minus basis attempt = 0 and dimension of S_4(K(166))^- = 6
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 2 Hecke spreads, the last of which dilates determinants by 64

Need to multiply the minimal max det 140 by the largest det contraction factor
 64 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 8960

Hecke spreads: 2
```

```
Pre-Hecke expansions will have length 107 903

Found file Grits-2-166-8960.ma, Getting it

Viable file found: GG-Grits-2-166-22896.ma-22896-mod-12347.ma

New best file: GG-Grits-2-166-22896.ma-22896-mod-12347.ma

Viable file found: GG-Grits-2-166-48128.ma-48128-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 4

{{2, 2}, {2, 1}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 448 337

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 448 337}, output dimensions {3, 107 903}

Length, max det of input vectors to ParaHeckeOp: 107 903, 8960 (check: 107 903, 8960)

Desired length, max det of output vectors: 1506, 560

Quotient of the max dets should be 16

Smaller max det is the minimal max det 140
  times the biggest postpended det inverse dilation factor
 4 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 1506, 560

Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 68 and dimension of S_4(K(166))^+ = 69

Rank of minus basis attempt = 3 and dimension of S_4(K(166))^- = 6


Hecke spread 2 out of 2

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}
```

```
Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 1506

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {3, 1506}, output dimensions {3, 1506}

Length, max det of input vectors to ParaHeckeOp: 1506, 560 (check: 1506, 560)

Desired length, max det of output vectors: 164, 140

Quotient of the max dets should be 4

Smaller max det is the minimal max det 140
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 164, 140

Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 68 and dimension of S_4(K(166))^+ = 69

Rank of minus basis attempt = 4 and dimension of S_4(K(166))^- = 6
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-166-200.ma}

Dimensions of BPcoeffMatProven: {10, 267}

Dimensions of join: {14, 164}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 68 and dimension of S_4(K(166))^+ = 69

Rank of minus basis attempt = 6 and dimension of S_4(K(166))^- = 6
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Didn't find a plus basis

Found minus basis of dimension 6
```

N = 170

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 0}

lift dimension of S_4(K(170))^+ : 28

nonlift dimension of S_4(K(170))^+ : 31

dimension of S_4(K(170))^- : 11
```

```
Ibukiyama-Kitayama dimension of S_4(K(170)) : 70
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(170))^+ = 59
Rank of minus basis attempt = 0 and dimension of S_4(K(170))^- = 11
Initial short vector length: 144
Have vectors of length 174: long enough
Determinant shell containing the vectors of length 144 : 135
Short vector length is 158, and the corresponding determinant is 135
Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-170-140.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,170}^{cusp}), which is 28
Rank of plus basis attempt = 28 and dimension of S_4(K(170))^+ = 59
Rank of minus basis attempt = 0 and dimension of S_4(K(170))^- = 11
```

STEP 2: TRACE DOWN

```
N = 170  q = 7  Nq = 1190
158-th determinant is 135
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 300
Trace down to try to hit S4(K(N)), of dimension 70
Plus lifts, plus nonlifts, minus dimension: {28, 31, 11}
Target plus rank, target minus rank: 59 11
Dimensions of savedTargetMats: {158, 3}
Making ordered good sigs for source space S_4(K(Nq))...
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
Found file Grits-2-1190-6615.ma, Getting it
 ... done making G2.  Dimensions of Grits: {24, 87944}
Determining indices: {1, 2, 3, 4, 17, 18, 19, 20,
  41, 42, 49, 50, 51, 52, 73, 74, 75, 76, 137, 157, 158, 165, 193, 225}
Atkin-Lehner truncation: 296
Beginning ALSignaturesAndDims...
 ... done with ALSignaturesAndDims.
ALspacesDims = {{{1, 1, 1, 1}, 8}, {{1, 1, -1, -1}, 4}, {{1, -1, 1, -1}, 5},
  {{1, -1, -1, 1}, 3}, {{-1, 1, 1, -1}, 2}, {{-1, 1, -1, 1}, 1}, {{-1, -1, 1, 1}, 1}}
```

```
Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1190-6615-to-170-135.ma exists, reading it
Found file Grits-2-1190-6615.ma, Getting it
Grit dimensions: {24, 87944}
Viable file found: GG-Grits-2-1190-6615.ma-6615-mod-12347.ma
New best file: GG-Grits-2-1190-6615.ma-6615-mod-12347.ma
Dimensions of G2G2: {300, 87944}
Dimensions of fDown = {300, 158}
Rank of fDown mod pp = 57
Dimensions and rank of plus space fDown = {300, 158}  57
Dimensions and rank of minus space fDown = {300, 158}  0
Rank of plus basis attempt = 59 and dimension of S_4(K(170))^+ = 59
Rank of minus basis attempt = 0 and dimension of S_4(K(170))^- = 11
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 6
Found file Grits-2-170-135.ma, Getting it
Viable file found: GG-Grits-2-170-18271.ma-18271-mod-12347.ma
New best file: GG-Grits-2-170-18271.ma-18271-mod-12347.ma
Viable file found: GG-Grits-2-170-19440.ma-19440-mod-12347.ma
Dimensions of G2G2: {6, 158}
Rank of plus basis attempt = 59 and dimension of S_4(K(170))^+ = 59
Rank of minus basis attempt = 0 and dimension of S_4(K(170))^- = 11
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified
Found custom table of Hecke spreads
Preparing for 3 Hecke spreads, the last of which dilates determinants by 144
Need to multiply the minimal max det 135 by the largest det contraction factor
 144 under Hecke spreads to get the pre-Hecke max det
Need pre-Hecke expansions up to determinant 19440
Hecke spreads: 3
Pre-Hecke expansions will have length 432074
Found file Grits-2-170-19440.ma, Getting it
Viable file found: GG-Grits-2-170-19440.ma-19440-mod-12347.ma
New best file: GG-Grits-2-170-19440.ma-19440-mod-12347.ma
```

```
Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another
{{2, 2}} 9
{{2, 2}, {2, 1}} 1
{{2, 2}, {3, 1}} 1
Hecke spread 1 out of 3
Is Hecke spread 1 present?  False
DoOneHecke called {},{{2, 2}}
{tp,tpdel} = {2, 2}
Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 432 074
Called ShortenVecs with G2G2T[{}]
 input dimensions {6, 432 074}, output dimensions {6, 432 074}
Length, max det of input vectors to ParaHeckeOp: 432 074, 19 440 (check: 432 074, 19 440)
Desired length, max det of output vectors: 5782, 1215
Quotient of the max dets should be 16
Smaller max det is the minimal max det 135
  times the biggest postpended det inverse dilation factor
 9 for the Hecke operator{{2, 2}}
About to compute G2G2T[{{2, 2}}]
Making abstract Hecke formula at a squared bad prime
Using Ralf's formula
Substituting given paramodular forms in abstract formula
Hecke spread 1 length and max det should be as desired: 5782, 1215
Rank of T[G2G2]^+/- = (6,6)
Rank of plus basis attempt = 59 and dimension of S_4(K(170))^+ = 59
Rank of minus basis attempt = 6 and dimension of S_4(K(170))^- = 11


Hecke spread 2 out of 3
Is Hecke spread 2 present?  False
DoOneHecke called {},{{2, 2}, {2, 1}}
{tp,tpdel} = {2, 2}
DoOneHecke called {{2, 2}},{{2, 1}}
{tp,tpdel} = {2, 1}
Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 5782
Called ShortenVecs with G2G2T[{{2, 2}}]
 input dimensions {6, 5782}, output dimensions {6, 1558}
Length, max det of input vectors to ParaHeckeOp: 1558, 540 (check: 1558, 540)
```

```
Desired length, max det of output vectors: 158, 135

Quotient of the max dets should be 4

Smaller max det is the minimal max det 135
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 158, 135

Rank of T[G2G2]^+/- = (6,6)

Rank of plus basis attempt = 59 and dimension of S_4(K(170))^+ = 59

Rank of minus basis attempt = 8 and dimension of S_4(K(170))^- = 11


Hecke spread 3 out of 3

Is Hecke spread 3 present?  False

DoOneHecke called {},{{2, 2}, {3, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{3, 1}}

{tp,tpdel} = {3, 1}

Need to compute G2G2T[{{2, 2}, {3, 1}}] having G2G2T[{{2, 2}}] vectors to length 5782

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {6, 5782}, output dimensions {6, 5782}

Length, max det of input vectors to ParaHeckeOp: 5782, 1215 (check: 5782, 1215)

Desired length, max det of output vectors: 158, 135

Quotient of the max dets should be 9

Smaller max det is the minimal max det 135
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {3, 1}}

About to compute G2G2T[{{2, 2}, {3, 1}}]

Making abstract Hecke formula

Substituting given paramodular forms in abstract formula

Hecke spread 3 length and max det should be as desired: 158, 135

Rank of T[G2G2]^+/- = (6,6)

Rank of plus basis attempt = 59 and dimension of S_4(K(170))^+ = 59

Rank of minus basis attempt = 9 and dimension of S_4(K(170))^- = 11
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-170-200.ma}
Dimensions of BPcoeffMatProven: {36, 302}
Dimensions of join: {45, 158}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 59 and dimension of S_4(K(170))^+ = 59
Rank of minus basis attempt = 11 and dimension of S_4(K(170))^- = 11
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Found a plus basis of dimension 59
Found minus basis of dimension 11
```

N = 174

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 20}
lift dimension of S_4(K(174))^+ : 27
nonlift dimension of S_4(K(174))^+ : 38
dimension of S_4(K(174))^- : 10
Ibukiyama-Kitayama dimension of S_4(K(174)) : 75
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(174))^+ = 65
Rank of minus basis attempt = 0 and dimension of S_4(K(174))^- = 10
Initial short vector length: 161
Have vectors of length 180: long enough
Determinant shell containing the vectors of length 161 : 140
Adding determinant shell 1
Adding determinant shell 2
Adding determinant shell 3
Adding determinant shell 4
Adding determinant shell 5
Adding determinant shell 6
Adding determinant shell 7
Adding determinant shell 8
Adding determinant shell 9
Adding determinant shell 10
```

```
Adding determinant shell 11

Adding determinant shell 12

Adding determinant shell 13

Adding determinant shell 14

Adding determinant shell 15

Adding determinant shell 16

Adding determinant shell 17

Adding determinant shell 18

Adding determinant shell 19

Adding determinant shell 20

Short vector length is 650, and the corresponding determinant is 296

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-174-296.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,174}^{cusp}), which is 27

Rank of plus basis attempt = 27 and dimension of S_4(K(174))^+ = 65

Rank of minus basis attempt = 0 and dimension of S_4(K(174))^- = 10
```

STEP 2: TRACE DOWN

```
N = 174  q = 7  Nq = 1218

650-th determinant is 296

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 378

Trace down to try to hit S4(K(N)), of dimension 75

Plus lifts, plus nonlifts, minus dimension: {27, 38, 10}

Target plus rank, target minus rank: 65 10

Dimensions of savedTargetMats: {650, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1218-14504.ma, Getting it

 ... done making G2.  Dimensions of Grits: {27, 331548}

Determining indices: {1, 2, 3, 4, 9, 10, 13, 14, 29, 30, 31,
  32, 69, 70, 81, 82, 101, 102, 103, 104, 133, 134, 141, 142, 189, 305, 357}

Atkin-Lehner truncation: 428

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.
```

```
ALspacesDims = {{{1, 1, 1, 1}, 10}, {{1, 1, -1, -1}, 7},
  {{1, -1, 1, -1}, 3}, {{1, -1, -1, 1}, 3}, {{-1, 1, 1, -1}, 2}, {{-1, 1, -1, 1}, 2}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1218-14504-to-174-296.ma
  exists, reading it

Found file Grits-2-1218-14504.ma, Getting it

Grit dimensions: {27, 331548}

Viable file found: GG-Grits-2-1218-14504.ma-14504-mod-12347.ma

New best file: GG-Grits-2-1218-14504.ma-14504-mod-12347.ma

Dimensions of G2G2: {378, 331548}

Dimensions of fDown = {378, 650}

Rank of fDown mod pp = 64

Dimensions and rank of plus space fDown = {378, 650}  64

Dimensions and rank of minus space fDown = {378, 650}  0

Rank of plus basis attempt = 64 and dimension of S_4(K(174))^+ = 65

Rank of minus basis attempt = 0 and dimension of S_4(K(174))^- = 10
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 1

Found file Grits-2-174-296.ma, Getting it

Viable file found: GG-Grits-2-174-20160.ma-20160-mod-12347.ma

New best file: GG-Grits-2-174-20160.ma-20160-mod-12347.ma

Viable file found: GG-Grits-2-174-3500.ma-3500-mod-12347.ma

New best file: GG-Grits-2-174-3500.ma-3500-mod-12347.ma

Viable file found: GG-Grits-2-174-4544.ma-4544-mod-12347.ma

Dimensions of G2G2: {1, 650}

Rank of plus basis attempt = 64 and dimension of S_4(K(174))^+ = 65

Rank of minus basis attempt = 0 and dimension of S_4(K(174))^- = 10
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 0 Hecke spreads, the last of which dilates determinants by 1

Need to multiply the minimal max det 296 by the largest det contraction factor
 1 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 296
```

```
Hecke spreads: 0

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-174-200.ma}

Dimensions of BPcoeffMatProven: {13, 288}

Borcherds product vectors are shorter than notebook space vectors

Dimensions of join: {13, 288}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 64 and dimension of S_4(K(174))^+ = 65

Rank of minus basis attempt = 10 and dimension of S_4(K(174))^- = 10

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {BPinfo-files/BPplusInfo-4-174-400-merge.ma}

(from inside plusInfo file) Dimensions of JSglobalBasisAttempt: {64, 650}

(from inside plusInfo file) Rank of JSglobalBasisAttempt: 64

(from inside plusInfo file) Dimensions of BPcoeffMatProven: {60, 1062}

(from inside plusInfo file) Rank of BPcoeffMatProven's numerical columns: 60

(from inside plusInfo file) Dimensions of BPcoeffMatProven after join: {124, 650}

(from inside plusInfo file) Dimensions of numerical BPcoeffMatProven after join:
 {124, 147}

(from inside plusInfo file) Rank of numerical BPcoeffMatProven after join: 65

New dimensions of JSglobalBasisAttempt: {124, 650}

Plus space span attempt may contain non-numerical entries

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 65 and dimension of S_4(K(174))^+ = 65

Rank of minus basis attempt = 10 and dimension of S_4(K(174))^- = 10

Found a plus basis of dimension 65

Found minus basis of dimension 10

N = 177

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 0}

lift dimension of S_4(K(177))^+ : 32

nonlift dimension of S_4(K(177))^+ : 37

dimension of S_4(K(177))^- : 5

Ibukiyama-Kitayama dimension of S_4(K(177)) : 74
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(177))^+ = 69

Rank of minus basis attempt = 0 and dimension of S_4(K(177))^- = 5

Initial short vector length: 130

Have vectors of length 155: long enough

Determinant shell containing the vectors of length 130 : 152

Short vector length is 139, and the corresponding determinant is 152

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-177-160.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,177}^{cusp}), which is 32

Rank of plus basis attempt = 32 and dimension of S_4(K(177))^+ = 69

Rank of minus basis attempt = 0 and dimension of S_4(K(177))^- = 5
```

STEP 2: TRACE DOWN

```
N = 177  q = 7  Nq = 1239

139-th determinant is 152

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 406

Trace down to try to hit S4(K(N)), of dimension 74

Plus lifts, plus nonlifts, minus dimension: {32, 37, 5}

Target plus rank, target minus rank: 69 5

Dimensions of savedTargetMats: {139, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1239-7448.ma, Getting it

 ... done making G2.  Dimensions of Grits: {28, 78 714}

Determining indices: {1, 2, 5, 6, 7, 8, 25, 26, 33, 34, 39,
  40, 41, 42, 51, 52, 63, 64, 79, 80, 81, 82, 99, 100, 107, 108, 109, 131}

Atkin-Lehner truncation: 134

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 11}, {{1, -1, -1}, 8}, {{-1, 1, -1}, 5}, {{-1, -1, 1}, 4}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1239-7448-to-177-152.ma exists, reading it
```

```
Found file Grits-2-1239-7448.ma, Getting it

Grit dimensions: {28, 78 714}

Viable file found: GG-Grits-2-1239-7448.ma-7448-mod-12347.ma

New best file: GG-Grits-2-1239-7448.ma-7448-mod-12347.ma

Dimensions of G2G2: {406, 78 714}

Dimensions of fDown = {406, 139}

Rank of fDown mod pp = 67

Dimensions and rank of plus space fDown = {406, 139}  67

Dimensions and rank of minus space fDown = {406, 139}  0

Rank of plus basis attempt = 69 and dimension of S_4(K(177))^+ = 69

Rank of minus basis attempt = 0 and dimension of S_4(K(177))^- = 5
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 10

Found file Grits-2-177-152.ma, Getting it

Viable file found: GG-Grits-2-177-15471.ma-15471-mod-12347.ma

New best file: GG-Grits-2-177-15471.ma-15471-mod-12347.ma

Dimensions of G2G2: {10, 139}

Rank of plus basis attempt = 69 and dimension of S_4(K(177))^+ = 69

Rank of minus basis attempt = 0 and dimension of S_4(K(177))^- = 5
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 152 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 12 312

Hecke spreads: 1

Pre-Hecke expansions will have length 155 825

Found file Grits-2-177-12312.ma, Getting it

Viable file found: GG-Grits-2-177-15471.ma-15471-mod-12347.ma

New best file: GG-Grits-2-177-15471.ma-15471-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1
```

```
Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 220 064

Called ShortenVecs with G2G2T[{}]

 input dimensions {10, 220 064}, output dimensions {10, 155 825}

Length, max det of input vectors to ParaHeckeOp: 155 825, 12 312 (check: 155 825, 12 312)

Desired length, max det of output vectors: 139, 152

Quotient of the max dets should be 81

Smaller max det is the minimal max det 152
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}
About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 139, 152

Rank of T[G2G2]^+/- = (10,2)

Rank of plus basis attempt = 69 and dimension of S_4(K(177))^+ = 69

Rank of minus basis attempt = 2 and dimension of S_4(K(177))^- = 5
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-177-200.ma}

Dimensions of BPcoeffMatProven: {8, 261}

Dimensions of join: {10, 139}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 69 and dimension of S_4(K(177))^+ = 69

Rank of minus basis attempt = 5 and dimension of S_4(K(177))^- = 5
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}


Found a plus basis of dimension 69

Found minus basis of dimension 5
```

N = 178

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}
```

```
lift dimension of S_4(K(178))^+ : 32

nonlift dimension of S_4(K(178))^+ : 44

dimension of S_4(K(178))^- : 11

Ibukiyama-Kitayama dimension of S_4(K(178)) : 87
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(178))^+ = 76

Rank of minus basis attempt = 0 and dimension of S_4(K(178))^- = 11

Initial short vector length: 153

Have vectors of length 164: long enough

Determinant shell containing the vectors of length 153 : 144

Short vector length is 164, and the corresponding determinant is 144

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-178-150.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,178}^{cusp}), which is 32

Rank of plus basis attempt = 32 and dimension of S_4(K(178))^+ = 76

Rank of minus basis attempt = 0 and dimension of S_4(K(178))^- = 11
```

STEP 2: TRACE DOWN

```
N = 178  q = 5  Nq = 890

164-th determinant is 144

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 351

Trace down to try to hit S4(K(N)), of dimension 87

Plus lifts, plus nonlifts, minus dimension: {32, 44, 11}

Target plus rank, target minus rank: 76 11

Dimensions of savedTargetMats: {164, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-890-3600.ma, Getting it

 ... done making G2.  Dimensions of Grits: {26, 31296}

Determining indices: {1, 2, 3, 4, 9, 11, 12, 17, 18,
  19, 20, 33, 35, 36, 47, 48, 55, 56, 69, 70, 71, 97, 98, 117, 135, 141}

Atkin-Lehner truncation: 172

Beginning ALSignaturesAndDims...
```

```
 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 13}, {{1, -1, -1}, 9}, {{-1, 1, -1}, 3}, {{-1, -1, 1}, 1}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-890-3600-to-178-144.ma exists, reading it

Found file Grits-2-890-3600.ma, Getting it

Grit dimensions: {26, 31296}

Viable file found: GG-Grits-2-890-3600.ma-3600-mod-12347.ma

New best file: GG-Grits-2-890-3600.ma-3600-mod-12347.ma

Dimensions of G2G2: {351, 31296}

Dimensions of fDown = {351, 164}

Rank of fDown mod pp = 75

Dimensions and rank of plus space fDown = {351, 164}  75

Dimensions and rank of minus space fDown = {351, 164}  0

Rank of plus basis attempt = 76 and dimension of S_4(K(178))^+ = 76

Rank of minus basis attempt = 0 and dimension of S_4(K(178))^- = 11


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 6

Found file Grits-2-178-144.ma, Getting it

Viable file found: GG-Grits-2-178-27072.ma-27072-mod-12347.ma

New best file: GG-Grits-2-178-27072.ma-27072-mod-12347.ma

Viable file found: GG-Grits-2-178-36864.ma-36864-mod-12347.ma

Dimensions of G2G2: {6, 164}

Rank of plus basis attempt = 76 and dimension of S_4(K(178))^+ = 76

Rank of minus basis attempt = 0 and dimension of S_4(K(178))^- = 11


STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 2 Hecke spreads, the last of which dilates determinants by 64

Need to multiply the minimal max det 144 by the largest det contraction factor
 64 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 9216

Hecke spreads: 2

Pre-Hecke expansions will have length 112531

Found file Grits-2-178-9216.ma, Getting it
```

```
Viable file found: GG-Grits-2-178-27072.ma-27072-mod-12347.ma

New best file: GG-Grits-2-178-27072.ma-27072-mod-12347.ma

Viable file found: GG-Grits-2-178-36864.ma-36864-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 4

{{2, 2}, {2, 1}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 576 846

Called ShortenVecs with G2G2T[{}]

 input dimensions {6, 576 846}, output dimensions {6, 112 531}

Length, max det of input vectors to ParaHeckeOp: 112 531, 9216 (check: 112 531, 9216)

Desired length, max det of output vectors: 1545, 576

Quotient of the max dets should be 16

Smaller max det is the minimal max det 144
  times the biggest postpended det inverse dilation factor
 4 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 1545, 576

Rank of T[G2G2]^+/- = (6,6)

Rank of plus basis attempt = 76 and dimension of S_4(K(178))^+ = 76

Rank of minus basis attempt = 6 and dimension of S_4(K(178))^- = 11


Hecke spread 2 out of 2

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 1545

Called ShortenVecs with G2G2T[{{2, 2}}]
```

```
 input dimensions {6, 1545}, output dimensions {6, 1545}

Length, max det of input vectors to ParaHeckeOp: 1545, 576 (check: 1545, 576)

Desired length, max det of output vectors: 164, 144

Quotient of the max dets should be 4

Smaller max det is the minimal max det 144
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 164, 144

Rank of T[G2G2]^+/- = (6,6)

Rank of plus basis attempt = 76 and dimension of S_4(K(178))^+ = 76

Rank of minus basis attempt = 8 and dimension of S_4(K(178))^- = 11
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-178-144.ma}

Dimensions of BPcoeffMatProven: {26, 164}

Dimensions of join: {34, 164}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 76 and dimension of S_4(K(178))^+ = 76

Rank of minus basis attempt = 11 and dimension of S_4(K(178))^- = 11
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Found a plus basis of dimension 76

Found minus basis of dimension 11

N = 182

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}

lift dimension of S_4(K(182))^+ : 29

nonlift dimension of S_4(K(182))^+ : 32

dimension of S_4(K(182))^- : 13

Ibukiyama-Kitayama dimension of S_4(K(182)) : 74
```

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(182))^+ = 61

Rank of minus basis attempt = 0 and dimension of S_4(K(182))^- = 13

Initial short vector length: 157

Have vectors of length 220: long enough

Determinant shell containing the vectors of length 157 : 152

Short vector length is 160, and the corresponding determinant is 152

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-182-160.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,182}^{cusp}), which is 29

Rank of plus basis attempt = 29 and dimension of S_4(K(182))^+ = 61

Rank of minus basis attempt = 0 and dimension of S_4(K(182))^- = 13

STEP 2: TRACE DOWN

N = 182 q = 5 Nq = 910

160-th determinant is 152

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 120

Trace down to try to hit S4(K(N)), of dimension 74

Plus lifts, plus nonlifts, minus dimension: {29, 32, 13}

Target plus rank, target minus rank: 61 13

Dimensions of savedTargetMats: {160, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-910-3800.ma, Getting it

 ... done making G2. Dimensions of Grits: {15, 37712}

Determining indices: {1, 2, 5, 6, 7, 8, 21, 22, 29, 30, 41, 53, 54, 55, 57}

Atkin-Lehner truncation: 132

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1, 1}, 5}, {{1, 1, -1, -1}, 3}, {{1, -1, 1, -1}, 2},
  {{1, -1, -1, 1}, 1}, {{-1, 1, 1, -1}, 2}, {{-1, 1, -1, 1}, 1}, {{-1, -1, 1, 1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-910-3800-to-182-152.ma exists, reading it

```
Found file Grits-2-910-3800.ma, Getting it

Grit dimensions: {15, 37712}

Viable file found: GG-Grits-2-910-4900.ma-4900-mod-12347.ma

New best file: GG-Grits-2-910-4900.ma-4900-mod-12347.ma

Dimensions of G2G2: {120, 56020}

Dimensions of fDown = {120, 160}

Rank of fDown mod pp = 59

Dimensions and rank of plus space fDown = {120, 160}  59

Dimensions and rank of minus space fDown = {120, 160}  0

Rank of plus basis attempt = 61 and dimension of S_4(K(182))^+ = 61

Rank of minus basis attempt = 0 and dimension of S_4(K(182))^- = 13
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-182-152.ma, Getting it

Viable file found: GG-Grits-2-182-36000.ma-36000-mod-12347.ma

New best file: GG-Grits-2-182-36000.ma-36000-mod-12347.ma

Dimensions of G2G2: {3, 160}

Rank of plus basis attempt = 61 and dimension of S_4(K(182))^+ = 61

Rank of minus basis attempt = 0 and dimension of S_4(K(182))^- = 13
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 3 Hecke spreads, the last of which dilates determinants by 144

Need to multiply the minimal max det 152 by the largest det contraction factor
 144 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 21888

Hecke spreads: 3

Pre-Hecke expansions will have length 502128

Found file Grits-2-182-21888.ma, Getting it

Viable file found: GG-Grits-2-182-36000.ma-36000-mod-12347.ma

New best file: GG-Grits-2-182-36000.ma-36000-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 9

{{2, 2}, {2, 1}} 1
```

```
{{2, 2}, {3, 1}} 1

Hecke spread 1 out of 3

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 1 064 844

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 1 064 844}, output dimensions {3, 502 128}

Length, max det of input vectors to ParaHeckeOp: 502 128, 21 888 (check: 502 128, 21 888)

Desired length, max det of output vectors: 6860, 1368

Quotient of the max dets should be 16

Smaller max det is the minimal max det 152
  times the biggest postpended det inverse dilation factor
 9 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 6860, 1368

Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 61 and dimension of S_4(K(182))^+ = 61

Rank of minus basis attempt = 3 and dimension of S_4(K(182))^- = 13


Hecke spread 2 out of 3

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 6860

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {3, 6860}, output dimensions {3, 1782}

Length, max det of input vectors to ParaHeckeOp: 1782, 608 (check: 1782, 608)

Desired length, max det of output vectors: 160, 152

Quotient of the max dets should be 4

Smaller max det is the minimal max det 152
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}
```

```
About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 160, 152

Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 61 and dimension of S_4(K(182))^+ = 61

Rank of minus basis attempt = 6 and dimension of S_4(K(182))^- = 13


Hecke spread 3 out of 3

Is Hecke spread 3 present?  False

DoOneHecke called {},{{2, 2}, {3, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{3, 1}}

{tp,tpdel} = {3, 1}

Need to compute G2G2T[{{2, 2}, {3, 1}}] having G2G2T[{{2, 2}}] vectors to length 6860

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {3, 6860}, output dimensions {3, 6860}

Length, max det of input vectors to ParaHeckeOp: 6860, 1368 (check: 6860, 1368)

Desired length, max det of output vectors: 160, 152

Quotient of the max dets should be 9

Smaller max det is the minimal max det 152
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {3, 1}}

About to compute G2G2T[{{2, 2}, {3, 1}}]

Making abstract Hecke formula

Substituting given paramodular forms in abstract formula

Hecke spread 3 length and max det should be as desired: 160, 152

Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 61 and dimension of S_4(K(182))^+ = 61

Rank of minus basis attempt = 7 and dimension of S_4(K(182))^- = 13


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-182-152.ma}

Dimensions of BPcoeffMatProven: {22, 160}

Dimensions of join: {29, 160}

Minus space span attempt may contain non-numerical entries
```

```
Rank of plus basis attempt = 61 and dimension of S_4(K(182))^+ = 61
Rank of minus basis attempt = 13 and dimension of S_4(K(182))^- = 13

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Found a plus basis of dimension 61
Found minus basis of dimension 13

N = 183
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}
lift dimension of S_4(K(183))^+ : 32
nonlift dimension of S_4(K(183))^+ : 38
dimension of S_4(K(183))^- : 10
Ibukiyama-Kitayama dimension of S_4(K(183)) : 80

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(183))^+ = 70
Rank of minus basis attempt = 0 and dimension of S_4(K(183))^- = 10
Initial short vector length: 158
Have vectors of length 160: long enough
Determinant shell containing the vectors of length 158 : 147
Short vector length is 160, and the corresponding determinant is 147
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-183-150.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,183}^{cusp}), which is 32
Rank of plus basis attempt = 32 and dimension of S_4(K(183))^+ = 70
Rank of minus basis attempt = 0 and dimension of S_4(K(183))^- = 10

STEP 2: TRACE DOWN
N = 183  q = 5  Nq = 915
160-th determinant is 147
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 253
Trace down to try to hit S4(K(N)), of dimension 80
```

```
Plus lifts, plus nonlifts, minus dimension: {32, 38, 10}

Target plus rank, target minus rank: 70 10

Dimensions of savedTargetMats: {160, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-915-3675.ma, Getting it

 ... done making G2.  Dimensions of Grits: {22, 28442}

Determining indices:
 {1, 3, 4, 7, 8, 15, 16, 23, 24, 25, 26, 39, 43, 47, 48, 59, 60, 75, 76, 77, 135, 173}
Atkin-Lehner truncation: 188

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 11}, {{1, -1, -1}, 5}, {{-1, 1, -1}, 5}, {{-1, -1, 1}, 1}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-915-3675-to-183-147.ma exists, reading it
Found file Grits-2-915-3675.ma, Getting it

Grit dimensions: {22, 28442}

Viable file found: GG-Grits-2-915-3675.ma-3675-mod-12347.ma

New best file: GG-Grits-2-915-3675.ma-3675-mod-12347.ma

Dimensions of G2G2: {253, 28442}

Dimensions of fDown = {253, 160}

Rank of fDown mod pp = 68

Dimensions and rank of plus space fDown = {253, 160}  68

Dimensions and rank of minus space fDown = {253, 160}  0

Rank of plus basis attempt = 70 and dimension of S_4(K(183))^+ = 70

Rank of minus basis attempt = 0 and dimension of S_4(K(183))^- = 10
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 6

Found file Grits-2-183-147.ma, Getting it

Viable file found: GG-Grits-2-183-26352.ma-26352-mod-12347.ma

New best file: GG-Grits-2-183-26352.ma-26352-mod-12347.ma

Viable file found: GG-Grits-2-183-47628.ma-47628-mod-12347.ma

Dimensions of G2G2: {6, 160}

Rank of plus basis attempt = 70 and dimension of S_4(K(183))^+ = 70
```

```
Rank of minus basis attempt = 0 and dimension of S_4(K(183))^- = 10
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 147 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 11 907

Hecke spreads: 1

Pre-Hecke expansions will have length 148 510

Found file Grits-2-183-11907.ma, Getting it

Viable file found: GG-Grits-2-183-26352.ma-26352-mod-12347.ma

New best file: GG-Grits-2-183-26352.ma-26352-mod-12347.ma

Viable file found: GG-Grits-2-183-47628.ma-47628-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 494 743

Called ShortenVecs with G2G2T[{}]

 input dimensions {6, 494 743}, output dimensions {6, 148 510}

Length, max det of input vectors to ParaHeckeOp: 148 510, 11 907 (check: 148 510, 11 907)

Desired length, max det of output vectors: 160, 147

Quotient of the max dets should be 81

Smaller max det is the minimal max det 147
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 160, 147

Rank of T[G2G2]^+/- = (6,5)

Rank of plus basis attempt = 70 and dimension of S_4(K(183))^+ = 70
```

```
Rank of minus basis attempt = 5 and dimension of S_4(K(183))^- = 10
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-183-147.ma}
Dimensions of BPcoeffMatProven: {24, 160}
Dimensions of join: {29, 160}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 70 and dimension of S_4(K(183))^+ = 70
Rank of minus basis attempt = 10 and dimension of S_4(K(183))^- = 10
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Found a plus basis of dimension 70
Found minus basis of dimension 10

N = 185
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {3, 0, 0}
lift dimension of S_4(K(185))^+ : 33
nonlift dimension of S_4(K(185))^+ : 39
dimension of S_4(K(185))^- : 6
Ibukiyama-Kitayama dimension of S_4(K(185)) : 78
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(185))^+ = 72
Rank of minus basis attempt = 0 and dimension of S_4(K(185))^- = 6
Initial short vector length: 149
Have vectors of length 178: long enough
Determinant shell containing the vectors of length 149 : 155
Short vector length is 152, and the corresponding determinant is 155
Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-185-160.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,185}^{cusp}), which is 33
Rank of plus basis attempt = 33 and dimension of S_4(K(185))^+ = 72
```

```
Rank of minus basis attempt = 0 and dimension of S_4(K(185))^- = 6


STEP 2: TRACE DOWN
N = 185  q = 3  Nq = 555
152-th determinant is 155
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 78
Trace down to try to hit S4(K(N)), of dimension 78
Plus lifts, plus nonlifts, minus dimension: {33, 39, 6}
Target plus rank, target minus rank: 72 6
Dimensions of savedTargetMats: {152, 3}
Making ordered good sigs for source space S_4(K(Nq))...
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
Found file Grits-2-555-1395.ma, Getting it
 ... done making G2.  Dimensions of Grits: {12, 5996}
Determining indices: {1, 2, 3, 4, 5, 6, 13, 14, 15, 16, 29, 69}
Atkin-Lehner truncation: 84
Beginning ALSignaturesAndDims...
 ... done with ALSignaturesAndDims.
ALspacesDims = {{{1, 1, 1}, 5}, {{1, -1, -1}, 3}, {{-1, 1, -1}, 2}, {{-1, -1, 1}, 2}}
Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-555-1395-to-185-155.ma exists, reading it
Found file Grits-2-555-1395.ma, Getting it
Grit dimensions: {12, 5996}
Viable file found: GG-Grits-2-555-2100.ma-2100-mod-12347.ma
New best file: GG-Grits-2-555-2100.ma-2100-mod-12347.ma
Dimensions of G2G2: {78, 11948}
Dimensions of fDown = {78, 152}
Rank of fDown mod pp = 46
Dimensions and rank of plus space fDown = {78, 152}  46
Dimensions and rank of minus space fDown = {78, 152}  0
Rank of plus basis attempt = 69 and dimension of S_4(K(185))^+ = 72
Rank of minus basis attempt = 0 and dimension of S_4(K(185))^- = 6


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS
Pairs from dim(J_{2,N}^{cusp}: 10
```

```
Found file Grits-2-185-155.ma, Getting it
Viable file found: GG-Grits-2-185-17091.ma-17091-mod-12347.ma
New best file: GG-Grits-2-185-17091.ma-17091-mod-12347.ma
Dimensions of G2G2: {10, 152}
Rank of plus basis attempt = 72 and dimension of S_4(K(185))^+ = 72
Rank of minus basis attempt = 0 and dimension of S_4(K(185))^- = 6
```

STEP 4: HECKE SPREAD

```
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-185-155.ma}
Dimensions of BPcoeffMatProven: {13, 152}
Dimensions of join: {13, 152}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 72 and dimension of S_4(K(185))^+ = 72
Rank of minus basis attempt = 6 and dimension of S_4(K(185))^- = 6
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Found a plus basis of dimension 72
Found minus basis of dimension 6
```

N = 186

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 0}
lift dimension of S_4(K(186))^+ : 30
nonlift dimension of S_4(K(186))^+ : 43
dimension of S_4(K(186))^- : 12
Ibukiyama-Kitayama dimension of S_4(K(186)) : 85
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(186))^+ = 73
Rank of minus basis attempt = 0 and dimension of S_4(K(186))^- = 12
Initial short vector length: 182
```

```
Have vectors of length 202: long enough

Determinant shell containing the vectors of length 182 : 135

Short vector length is 202, and the corresponding determinant is 135

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-186-140.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,186}^{cusp}), which is 30

Rank of plus basis attempt = 30 and dimension of S_4(K(186))^+ = 73

Rank of minus basis attempt = 0 and dimension of S_4(K(186))^- = 12
```

STEP 2: TRACE DOWN

```
N = 186  q = 7  Nq = 1302

202-th determinant is 135

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 435

Trace down to try to hit S4(K(N)), of dimension 85

Plus lifts, plus nonlifts, minus dimension: {30, 43, 12}

Target plus rank, target minus rank: 73 12

Dimensions of savedTargetMats: {202, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1302-6615.ma, Getting it

 ... done making G2.  Dimensions of Grits: {29, 97060}

Determining indices: {1, 2, 5, 6, 9, 10, 21, 22, 23, 24, 37, 41, 42,
  61, 62, 63, 64, 113, 114, 115, 116, 153, 154, 155, 156, 157, 158, 241, 245}

Atkin-Lehner truncation: 272

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1, 1}, 11}, {{1, 1, -1, -1}, 7}, {{1, -1, 1, -1}, 4},
  {{1, -1, -1, 1}, 3}, {{-1, 1, 1, -1}, 2}, {{-1, 1, -1, 1}, 1}, {{-1, -1, 1, 1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1302-6615-to-186-135.ma exists, reading it

Found file Grits-2-1302-6615.ma, Getting it

Grit dimensions: {29, 97060}

Viable file found: GG-Grits-2-1302-6615.ma-6615-mod-12347.ma
```

```
New best file: GG-Grits-2-1302-6615.ma-6615-mod-12347.ma

Dimensions of G2G2: {435, 97060}

Dimensions of fDown = {435, 202}

Rank of fDown mod pp = 72

Dimensions and rank of plus space fDown = {435, 202}  72

Dimensions and rank of minus space fDown = {435, 202}  0

Rank of plus basis attempt = 72 and dimension of S_4(K(186))^+ = 73

Rank of minus basis attempt = 0 and dimension of S_4(K(186))^- = 12
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-186-135.ma, Getting it

Viable file found: GG-Grits-2-186-39600.ma-39600-mod-12347.ma

New best file: GG-Grits-2-186-39600.ma-39600-mod-12347.ma

Dimensions of G2G2: {3, 202}

Rank of plus basis attempt = 72 and dimension of S_4(K(186))^+ = 73

Rank of minus basis attempt = 0 and dimension of S_4(K(186))^- = 12
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 3 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 135 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 10935

Hecke spreads: 3

Pre-Hecke expansions will have length 195358

Found file Grits-2-186-10935.ma, Getting it

Viable file found: GG-Grits-2-186-39600.ma-39600-mod-12347.ma

New best file: GG-Grits-2-186-39600.ma-39600-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 4

{{2, 2}, {2, 1}} 1

{{3, 2}} 1

Hecke spread 1 out of 3

Is Hecke spread 1 present?  False
```

```
DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 1 380 914

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 1 380 914}, output dimensions {3, 136 280}

Length, max det of input vectors to ParaHeckeOp: 136 280, 8640 (check: 136 280, 8640)

Desired length, max det of output vectors: 1752, 540

Quotient of the max dets should be 16

Smaller max det is the minimal max det 135
  times the biggest postpended det inverse dilation factor
 4 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 1752, 540

Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 72 and dimension of S_4(K(186))^+ = 73

Rank of minus basis attempt = 3 and dimension of S_4(K(186))^- = 12


Hecke spread 2 out of 3

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 1752

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {3, 1752}, output dimensions {3, 1752}

Length, max det of input vectors to ParaHeckeOp: 1752, 540 (check: 1752, 540)

Desired length, max det of output vectors: 202, 135

Quotient of the max dets should be 4

Smaller max det is the minimal max det 135
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula
```

```
Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 202, 135

Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 72 and dimension of S_4(K(186))^+ = 73

Rank of minus basis attempt = 6 and dimension of S_4(K(186))^- = 12


Hecke spread 3 out of 3

Is Hecke spread 3 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 1 380 914

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 1 380 914}, output dimensions {3, 195 358}

Length, max det of input vectors to ParaHeckeOp: 195 358, 10 935 (check: 195 358, 10 935)

Desired length, max det of output vectors: 202, 135

Quotient of the max dets should be 81

Smaller max det is the minimal max det 135
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 3 length and max det should be as desired: 202, 135

Rank of T[G2G2]^+/- = (3,1)

Rank of plus basis attempt = 72 and dimension of S_4(K(186))^+ = 73

Rank of minus basis attempt = 7 and dimension of S_4(K(186))^- = 12


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-186-200.ma}

Dimensions of BPcoeffMatProven: {28, 300}

Dimensions of join: {35, 202}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 72 and dimension of S_4(K(186))^+ = 73

Rank of minus basis attempt = 12 and dimension of S_4(K(186))^- = 12


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
```

```
Found files: {}

Didn't find a plus basis
Found minus basis of dimension 12

N = 187
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {3, 0, 0}
lift dimension of S_4(K(187))^+ : 35
nonlift dimension of S_4(K(187))^+ : 39
dimension of S_4(K(187))^- : 6
Ibukiyama-Kitayama dimension of S_4(K(187)) : 80

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(187))^+ = 74
Rank of minus basis attempt = 0 and dimension of S_4(K(187))^- = 6
Initial short vector length: 209
Have vectors of length 218: long enough
Determinant shell containing the vectors of length 209 : 208
Short vector length is 218, and the corresponding determinant is 208
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-187-210.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,187}^{cusp}), which is 35
Rank of plus basis attempt = 35 and dimension of S_4(K(187))^+ = 74
Rank of minus basis attempt = 0 and dimension of S_4(K(187))^- = 6

STEP 2: TRACE DOWN
N = 187  q = 3  Nq = 561
218-th determinant is 208
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 120
Trace down to try to hit S4(K(N)), of dimension 80
Plus lifts, plus nonlifts, minus dimension: {35, 39, 6}
Target plus rank, target minus rank: 74 6
Dimensions of savedTargetMats: {218, 3}
Making ordered good sigs for source space S_4(K(Nq))...
```

```
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
Found file Grits-2-561-1872.ma, Getting it
 ... done making G2.  Dimensions of Grits: {15, 9234}
Determining indices: {1, 2, 3, 4, 5, 6, 7, 8, 17, 18, 25, 27, 35, 36, 79}
Atkin-Lehner truncation: 86
Beginning ALSignaturesAndDims...
 ... done with ALSignaturesAndDims.
ALspacesDims = {{{1, 1, 1}, 6}, {{1, -1, -1}, 5}, {{-1, 1, -1}, 2}, {{-1, -1, 1}, 2}}
Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-561-1872-to-187-208.ma exists, reading it
Found file Grits-2-561-1872.ma, Getting it
Grit dimensions: {15, 9234}
Viable file found: GG-Grits-2-561-1872.ma-1872-mod-12347.ma
New best file: GG-Grits-2-561-1872.ma-1872-mod-12347.ma
Dimensions of G2G2: {120, 9234}
Dimensions of fDown = {120, 218}
Rank of fDown mod pp = 66
Dimensions and rank of plus space fDown = {120, 218}  66
Dimensions and rank of minus space fDown = {120, 218}  0
Rank of plus basis attempt = 74 and dimension of S_4(K(187))^+ = 74
Rank of minus basis attempt = 0 and dimension of S_4(K(187))^- = 6

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS
Pairs from dim(J_{2,N}^{cusp}: 15
Found file Grits-2-187-208.ma, Getting it
Viable file found: GG-Grits-2-187-11711.ma-11711-mod-12347.ma
New best file: GG-Grits-2-187-11711.ma-11711-mod-12347.ma
Dimensions of G2G2: {15, 218}
Rank of plus basis attempt = 74 and dimension of S_4(K(187))^+ = 74
Rank of minus basis attempt = 0 and dimension of S_4(K(187))^- = 6

STEP 4: HECKE SPREAD
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-187-208.ma}

Dimensions of BPcoeffMatProven: {8, 218}

Dimensions of join: {8, 218}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 74 and dimension of S_4(K(187))^+ = 74

Rank of minus basis attempt = 6 and dimension of S_4(K(187))^- = 6

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 74

Found minus basis of dimension 6

N = 190

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}

lift dimension of S_4(K(190))^+ : 31

nonlift dimension of S_4(K(190))^+ : 49

dimension of S_4(K(190))^- : 9

Ibukiyama-Kitayama dimension of S_4(K(190)) : 89

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(190))^+ = 80

Rank of minus basis attempt = 0 and dimension of S_4(K(190))^- = 9

Initial short vector length: 196

Have vectors of length 202: long enough

Determinant shell containing the vectors of length 196 : 160

Short vector length is 202, and the corresponding determinant is 160

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-190-160.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,190}^{cusp}), which is 31

Rank of plus basis attempt = 31 and dimension of S_4(K(190))^+ = 80

Rank of minus basis attempt = 0 and dimension of S_4(K(190))^- = 9

STEP 2: TRACE DOWN

```
N = 190  q = 7  Nq = 1330

202-th determinant is 160

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 496

Trace down to try to hit S4(K(N)), of dimension 89

Plus lifts, plus nonlifts, minus dimension: {31, 49, 9}

Target plus rank, target minus rank: 80 9

Dimensions of savedTargetMats: {202, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1330-7840.ma, Getting it

 ... done making G2.  Dimensions of Grits: {31, 116 028}

Determining indices: {1, 2, 3, 4, 5, 6, 7, 8, 25, 26, 29, 30, 37, 38,
  49, 50, 57, 58, 59, 60, 105, 106, 107, 108, 121, 122, 173, 174, 213, 215, 261}

Atkin-Lehner truncation: 332

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims =
 {{{1, 1, 1, 1}, 11}, {{1, 1, -1, -1}, 5}, {{1, -1, 1, -1}, 6}, {{1, -1, -1, 1}, 4},
  {{-1, 1, 1, -1}, 2}, {{-1, 1, -1, 1}, 1}, {{-1, -1, 1, 1}, 1}, {{-1, -1, -1, -1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1330-7840-to-190-160.ma exists, reading it

Found file Grits-2-1330-7840.ma, Getting it

Grit dimensions: {31, 116 028}

Viable file found: GG-Grits-2-1330-7840.ma-7840-mod-12347.ma

New best file: GG-Grits-2-1330-7840.ma-7840-mod-12347.ma

Dimensions of G2G2: {496, 116 028}

Dimensions of fDown = {496, 202}

Rank of fDown mod pp = 79

Dimensions and rank of plus space fDown = {496, 202}  79

Dimensions and rank of minus space fDown = {496, 202}  0

Rank of plus basis attempt = 79 and dimension of S_4(K(190))^+ = 80

Rank of minus basis attempt = 0 and dimension of S_4(K(190))^- = 9
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-190-160.ma, Getting it

Viable file found: GG-Grits-2-190-39984.ma-39984-mod-12347.ma

New best file: GG-Grits-2-190-39984.ma-39984-mod-12347.ma

Dimensions of G2G2: {3, 202}

Rank of plus basis attempt = 79 and dimension of S_4(K(190))^+ = 80

Rank of minus basis attempt = 0 and dimension of S_4(K(190))^- = 9
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 3 Hecke spreads, the last of which dilates determinants by 144

Need to multiply the minimal max det 160 by the largest det contraction factor
 144 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 23 040

Hecke spreads: 3

Pre-Hecke expansions will have length 556 316

Found file Grits-2-190-23040.ma, Getting it

Viable file found: GG-Grits-2-190-39984.ma-39984-mod-12347.ma

New best file: GG-Grits-2-190-39984.ma-39984-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 9

{{2, 2}, {2, 1}} 1

{{2, 2}, {3, 1}} 1

Hecke spread 1 out of 3

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 1 283 094

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 1 283 094}, output dimensions {3, 556 316}

Length, max det of input vectors to ParaHeckeOp: 556 316, 23 040 (check: 556 316, 23 040)

Desired length, max det of output vectors: 7754, 1440

Quotient of the max dets should be 16

Smaller max det is the minimal max det 160
  times the biggest postpended det inverse dilation factor
 9 for the Hecke operator{{2, 2}}
```

```
About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 7754, 1440

Rank of T[G2G2]^+/- = (3,2)

Rank of plus basis attempt = 79 and dimension of S_4(K(190))^+ = 80

Rank of minus basis attempt = 2 and dimension of S_4(K(190))^- = 9


Hecke spread 2 out of 3

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 7754

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {3, 7754}, output dimensions {3, 2082}

Length, max det of input vectors to ParaHeckeOp: 2082, 640 (check: 2082, 640)

Desired length, max det of output vectors: 202, 160

Quotient of the max dets should be 4

Smaller max det is the minimal max det 160
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 202, 160

Rank of T[G2G2]^+/- = (3,2)

Rank of plus basis attempt = 79 and dimension of S_4(K(190))^+ = 80

Rank of minus basis attempt = 3 and dimension of S_4(K(190))^- = 9


Hecke spread 3 out of 3

Is Hecke spread 3 present?  False

DoOneHecke called {},{{2, 2}, {3, 1}}

{tp,tpdel} = {2, 2}
```

```
DoOneHecke called {{2, 2}},{{3, 1}}

{tp,tpdel} = {3, 1}

Need to compute G2G2T[{{2, 2}, {3, 1}}] having G2G2T[{{2, 2}}] vectors to length 7754

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {3, 7754}, output dimensions {3, 7754}

Length, max det of input vectors to ParaHeckeOp: 7754, 1440 (check: 7754, 1440)

Desired length, max det of output vectors: 202, 160

Quotient of the max dets should be 9

Smaller max det is the minimal max det 160
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {3, 1}}

About to compute G2G2T[{{2, 2}, {3, 1}}]

Making abstract Hecke formula

Substituting given paramodular forms in abstract formula

Hecke spread 3 length and max det should be as desired: 202, 160

Rank of T[G2G2]^+/- = (3,2)

Rank of plus basis attempt = 79 and dimension of S_4(K(190))^+ = 80

Rank of minus basis attempt = 4 and dimension of S_4(K(190))^- = 9


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-190-151.ma}

Dimensions of BPcoeffMatProven: {70, 192}

Borcherds product vectors are shorter than notebook space vectors

Dimensions of join: {74, 192}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 79 and dimension of S_4(K(190))^+ = 80

Rank of minus basis attempt = 9 and dimension of S_4(K(190))^- = 9


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Didn't find a plus basis

Found minus basis of dimension 9


N = 194

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}

lift dimension of S_4(K(194))^+ : 34
```

```
nonlift dimension of S_4(K(194))^+ : 45

dimension of S_4(K(194))^- : 17

Ibukiyama-Kitayama dimension of S_4(K(194)) : 96


STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(194))^+ = 79

Rank of minus basis attempt = 0 and dimension of S_4(K(194))^- = 17

Initial short vector length: 191

Have vectors of length 222: long enough

Determinant shell containing the vectors of length 191 : 151

Short vector length is 202, and the corresponding determinant is 151

Vectors fill out a determinant shell, as they should


STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-194-160.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,194}^{cusp}), which is 34

Rank of plus basis attempt = 34 and dimension of S_4(K(194))^+ = 79

Rank of minus basis attempt = 0 and dimension of S_4(K(194))^- = 17


STEP 2: TRACE DOWN

N = 194  q = 5  Nq = 970

202-th determinant is 151

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 406

Trace down to try to hit S4(K(N)), of dimension 96

Plus lifts, plus nonlifts, minus dimension: {34, 45, 17}

Target plus rank, target minus rank: 79 17

Dimensions of savedTargetMats: {202, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-970-3775.ma, Getting it

 ... done making G2.  Dimensions of Grits: {28, 33048}

Determining indices: {1, 2, 3, 4, 9, 10, 13, 14, 15, 16, 25,
  26, 31, 32, 43, 44, 57, 58, 59, 60, 77, 93, 94, 113, 119, 120, 159, 183}

Atkin-Lehner truncation: 194

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.
```

```
ALspacesDims = {{{1, 1, 1}, 14}, {{1, -1, -1}, 10}, {{-1, 1, -1}, 2}, {{-1, -1, 1}, 2}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-970-3775-to-194-151.ma exists, reading it

Found file Grits-2-970-3775.ma, Getting it

Grit dimensions: {28, 33048}

Viable file found: GG-Grits-2-970-3775.ma-3775-mod-12347.ma

New best file: GG-Grits-2-970-3775.ma-3775-mod-12347.ma

Dimensions of G2G2: {406, 33048}

Dimensions of fDown = {406, 202}

Rank of fDown mod pp = 77

Dimensions and rank of plus space fDown = {406, 202}  77

Dimensions and rank of minus space fDown = {406, 202}  0

Rank of plus basis attempt = 79 and dimension of S_4(K(194))^+ = 79

Rank of minus basis attempt = 0 and dimension of S_4(K(194))^- = 17
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 6

Found file Grits-2-194-151.ma, Getting it

Viable file found: GG-Grits-2-194-10688.ma-10688-mod-12347.ma

New best file: GG-Grits-2-194-10688.ma-10688-mod-12347.ma

Viable file found: GG-Grits-2-194-24048.ma-24048-mod-12347.ma

Viable file found: GG-Grits-2-194-38656.ma-38656-mod-12347.ma

Dimensions of G2G2: {6, 202}

Rank of plus basis attempt = 79 and dimension of S_4(K(194))^+ = 79

Rank of minus basis attempt = 0 and dimension of S_4(K(194))^- = 17
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 3 Hecke spreads, the last of which dilates determinants by 144

Need to multiply the minimal max det 151 by the largest det contraction factor
 144 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 21744

Hecke spreads: 3

Pre-Hecke expansions will have length 414205

Found file Grits-2-194-21744.ma, Getting it
```

```
Viable file found: GG-Grits-2-194-24048.ma-24048-mod-12347.ma

New best file: GG-Grits-2-194-24048.ma-24048-mod-12347.ma

Viable file found: GG-Grits-2-194-38656.ma-38656-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 9

{{2, 2}, {2, 1}} 1

{{2, 2}, {3, 1}} 1

Hecke spread 1 out of 3

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 481 857

Called ShortenVecs with G2G2T[{}]

 input dimensions {6, 481 857}, output dimensions {6, 414 205}

Length, max det of input vectors to ParaHeckeOp: 414 205, 21 744 (check: 414 205, 21 744)

Desired length, max det of output vectors: 5865, 1359

Quotient of the max dets should be 16

Smaller max det is the minimal max det 151
  times the biggest postpended det inverse dilation factor
 9 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 5865, 1359

Rank of T[G2G2]^+/- = (6,6)

Rank of plus basis attempt = 79 and dimension of S_4(K(194))^+ = 79

Rank of minus basis attempt = 6 and dimension of S_4(K(194))^- = 17


Hecke spread 2 out of 3

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 5865
```

```
Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {6, 5865}, output dimensions {6, 1645}

Length, max det of input vectors to ParaHeckeOp: 1645, 604 (check: 1645, 604)

Desired length, max det of output vectors: 202, 151

Quotient of the max dets should be 4

Smaller max det is the minimal max det 151
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 202, 151

Rank of T[G2G2]^+/- = (6,6)

Rank of plus basis attempt = 79 and dimension of S_4(K(194))^+ = 79

Rank of minus basis attempt = 12 and dimension of S_4(K(194))^- = 17


Hecke spread 3 out of 3

Is Hecke spread 3 present?  False

DoOneHecke called {},{{2, 2}, {3, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{3, 1}}

{tp,tpdel} = {3, 1}

Need to compute G2G2T[{{2, 2}, {3, 1}}] having G2G2T[{{2, 2}}] vectors to length 5865

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {6, 5865}, output dimensions {6, 5865}

Length, max det of input vectors to ParaHeckeOp: 5865, 1359 (check: 5865, 1359)

Desired length, max det of output vectors: 202, 151

Quotient of the max dets should be 9

Smaller max det is the minimal max det 151
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {3, 1}}

About to compute G2G2T[{{2, 2}, {3, 1}}]

Making abstract Hecke formula

Substituting given paramodular forms in abstract formula

Hecke spread 3 length and max det should be as desired: 202, 151

Rank of T[G2G2]^+/- = (6,6)

Rank of plus basis attempt = 79 and dimension of S_4(K(194))^+ = 79
```

Rank of minus basis attempt = 13 and dimension of S_4(K(194))^- = 17

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-194-200.ma}

Dimensions of BPcoeffMatProven: {58, 356}

Dimensions of join: {71, 202}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 79 and dimension of S_4(K(194))^+ = 79

Rank of minus basis attempt = 17 and dimension of S_4(K(194))^- = 17

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 79

Found minus basis of dimension 17

N = 195

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 0}

lift dimension of S_4(K(195))^+ : 30

nonlift dimension of S_4(K(195))^+ : 39

dimension of S_4(K(195))^- : 10

Ibukiyama-Kitayama dimension of S_4(K(195)) : 79

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(195))^+ = 69

Rank of minus basis attempt = 0 and dimension of S_4(K(195))^- = 10

Initial short vector length: 174

Have vectors of length 192: long enough

Determinant shell containing the vectors of length 174 : 159

Short vector length is 192, and the corresponding determinant is 159

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-195-160.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,195}^{cusp}), which is 30

Rank of plus basis attempt = 30 and dimension of S_4(K(195))^+ = 69

```
Rank of minus basis attempt = 0 and dimension of S_4(K(195))^- = 10
```

STEP 2: TRACE DOWN

```
N = 195  q = 7  Nq = 1365

192-th determinant is 159

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 465

Trace down to try to hit S4(K(N)), of dimension 79

Plus lifts, plus nonlifts, minus dimension: {30, 39, 10}

Target plus rank, target minus rank: 69 10

Dimensions of savedTargetMats: {192, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1365-7791.ma, Getting it

 ... done making G2.  Dimensions of Grits: {30, 103 432}

Determining indices: {1, 2, 5, 6, 7, 8, 21, 22, 29, 30, 31, 32, 53,
  54, 55, 56, 57, 58, 59, 93, 94, 109, 111, 149, 201, 202, 225, 226, 227, 281}

Atkin-Lehner truncation: 312

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims =
 {{{1, 1, 1, 1}, 9}, {{1, 1, -1, -1}, 4}, {{1, -1, 1, -1}, 4}, {{1, -1, -1, 1}, 2},
  {{-1, 1, 1, -1}, 4}, {{-1, 1, -1, 1}, 3}, {{-1, -1, 1, 1}, 2}, {{-1, -1, -1, -1}, 2}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1365-7791-to-195-159.ma exists, reading it

Found file Grits-2-1365-7791.ma, Getting it

Grit dimensions: {30, 103 432}

Viable file found: GG-Grits-2-1365-7791.ma-7791-mod-12347.ma

New best file: GG-Grits-2-1365-7791.ma-7791-mod-12347.ma

Dimensions of G2G2: {465, 103 432}

Dimensions of fDown = {465, 192}

Rank of fDown mod pp = 69

Dimensions and rank of plus space fDown = {465, 192}  69

Dimensions and rank of minus space fDown = {465, 192}  0

Rank of plus basis attempt = 69 and dimension of S_4(K(195))^+ = 69

Rank of minus basis attempt = 0 and dimension of S_4(K(195))^- = 10
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 1

Found file Grits-2-195-159.ma, Getting it

Viable file found: GG-Grits-2-195-12879.ma-12879-mod-12347.ma

New best file: GG-Grits-2-195-12879.ma-12879-mod-12347.ma

Viable file found: GG-Grits-2-195-159.ma-159-mod-12347.ma

New best file: GG-Grits-2-195-159.ma-159-mod-12347.ma

Dimensions of G2G2: {1, 192}

Rank of plus basis attempt = 69 and dimension of S_4(K(195))^+ = 69

Rank of minus basis attempt = 0 and dimension of S_4(K(195))^- = 10

STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 159 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 12 879

Hecke spreads: 1

Pre-Hecke expansions will have length 206 824

Found file Grits-2-195-12879.ma, Getting it

Viable file found: GG-Grits-2-195-12879.ma-12879-mod-12347.ma

New best file: GG-Grits-2-195-12879.ma-12879-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present? False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 206 824

Called ShortenVecs with G2G2T[{}]

 input dimensions {1, 206 824}, output dimensions {1, 206 824}

Length, max det of input vectors to ParaHeckeOp: 206 824, 12 879 (check: 206 824, 12 879)

Desired length, max det of output vectors: 192, 159

Quotient of the max dets should be 81

```
Smaller max det is the minimal max det 159
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}
About to compute G2G2T[{{3, 2}}]
Making abstract Hecke formula at a squared bad prime
Using Ralf's formula
Substituting given paramodular forms in abstract formula
Hecke spread 1 length and max det should be as desired: 192, 159
Rank of T[G2G2]^+/- = (1,1)
Rank of plus basis attempt = 69 and dimension of S_4(K(195))^+ = 69
Rank of minus basis attempt = 1 and dimension of S_4(K(195))^- = 10

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-195-159.ma}
Dimensions of BPcoeffMatProven: {28, 192}
Dimensions of join: {29, 192}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 69 and dimension of S_4(K(195))^+ = 69
Rank of minus basis attempt = 10 and dimension of S_4(K(195))^- = 10

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Found a plus basis of dimension 69
Found minus basis of dimension 10

N = 201
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}
lift dimension of S_4(K(201))^+ : 37
nonlift dimension of S_4(K(201))^+ : 54
dimension of S_4(K(201))^- : 6
Ibukiyama-Kitayama dimension of S_4(K(201)) : 97

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(201))^+ = 91
Rank of minus basis attempt = 0 and dimension of S_4(K(201))^- = 6
Initial short vector length: 181
```

```
Have vectors of length 182: long enough

Determinant shell containing the vectors of length 181 : 168

Short vector length is 182, and the corresponding determinant is 168

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-201-170.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,201}^{cusp}), which is 37

Rank of plus basis attempt = 37 and dimension of S_4(K(201))^+ = 91

Rank of minus basis attempt = 0 and dimension of S_4(K(201))^- = 6
```

STEP 2: TRACE DOWN

```
N = 201  q = 5  Nq = 1005

182-th determinant is 168

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 351

Trace down to try to hit S4(K(N)), of dimension 97

Plus lifts, plus nonlifts, minus dimension: {37, 54, 6}

Target plus rank, target minus rank: 91 6

Dimensions of savedTargetMats: {182, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1005-4200.ma, Getting it

 ... done making G2.  Dimensions of Grits: {26, 34 444}

Determining indices: {1, 2, 3, 4, 5, 6, 9, 10, 11,
  12, 25, 26, 29, 33, 34, 45, 46, 61, 62, 73, 74, 89, 90, 129, 165, 166}

Atkin-Lehner truncation: 204

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 10}, {{1, -1, -1}, 6}, {{-1, 1, -1}, 6}, {{-1, -1, 1}, 4}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1005-4200-to-201-168.ma exists, reading it

Found file Grits-2-1005-4200.ma, Getting it

Grit dimensions: {26, 34 444}

Viable file found: GG-Grits-2-1005-4200.ma-4200-mod-12347.ma

New best file: GG-Grits-2-1005-4200.ma-4200-mod-12347.ma
```

```
Dimensions of G2G2: {351, 34 444}

Dimensions of fDown = {351, 182}

Rank of fDown mod pp = 87

Dimensions and rank of plus space fDown = {351, 182}  87

Dimensions and rank of minus space fDown = {351, 182}  0

Rank of plus basis attempt = 91 and dimension of S_4(K(201))^+ = 91

Rank of minus basis attempt = 0 and dimension of S_4(K(201))^- = 6
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 15

Found file Grits-2-201-168.ma, Getting it

Viable file found: GG-Grits-2-201-13608.ma-13608-mod-12347.ma

New best file: GG-Grits-2-201-13608.ma-13608-mod-12347.ma

Viable file found: GG-Grits-2-201-5000.ma-5000-mod-12347.ma

New best file: GG-Grits-2-201-5000.ma-5000-mod-12347.ma

Dimensions of G2G2: {15, 182}

Rank of plus basis attempt = 91 and dimension of S_4(K(201))^+ = 91

Rank of minus basis attempt = 0 and dimension of S_4(K(201))^- = 6
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 168 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 13 608

Hecke spreads: 1

Pre-Hecke expansions will have length 180 608

Found file Grits-2-201-13608.ma, Getting it

Viable file found: GG-Grits-2-201-13608.ma-13608-mod-12347.ma

New best file: GG-Grits-2-201-13608.ma-13608-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}
```

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 180 608

Called ShortenVecs with G2G2T[{}]

 input dimensions {15, 180 608}, output dimensions {15, 180 608}

Length, max det of input vectors to ParaHeckeOp: 180 608, 13 608 (check: 180 608, 13 608)

Desired length, max det of output vectors: 182, 168

Quotient of the max dets should be 81

Smaller max det is the minimal max det 168
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 182, 168

Rank of T[G2G2]^+/- = (15,4)

Rank of plus basis attempt = 91 and dimension of S_4(K(201))^+ = 91

Rank of minus basis attempt = 4 and dimension of S_4(K(201))^- = 6

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-201-200.ma}

Dimensions of BPcoeffMatProven: {8, 276}

Dimensions of join: {12, 182}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 91 and dimension of S_4(K(201))^+ = 91

Rank of minus basis attempt = 6 and dimension of S_4(K(201))^- = 6

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 91

Found minus basis of dimension 6

N = 202

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}

lift dimension of S_4(K(202))^+ : 37

nonlift dimension of S_4(K(202))^+ : 57

```
dimension of S_4(K(202))^- : 15

Ibukiyama-Kitayama dimension of S_4(K(202)) : 109


STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(202))^+ = 94

Rank of minus basis attempt = 0 and dimension of S_4(K(202))^- = 15

Initial short vector length: 223

Have vectors of length 252: long enough

Determinant shell containing the vectors of length 223 : 183

Short vector length is 232, and the corresponding determinant is 183

Vectors fill out a determinant shell, as they should


STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-202-190.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,202}^{cusp}), which is 37

Rank of plus basis attempt = 37 and dimension of S_4(K(202))^+ = 94

Rank of minus basis attempt = 0 and dimension of S_4(K(202))^- = 15


STEP 2: TRACE DOWN

N = 202  q = 7  Nq = 1414

232-th determinant is 183

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 903

Trace down to try to hit S4(K(N)), of dimension 109

Plus lifts, plus nonlifts, minus dimension: {37, 57, 15}

Target plus rank, target minus rank: 94 15

Dimensions of savedTargetMats: {232, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1414-8967.ma, Getting it

 ... done making G2.  Dimensions of Grits: {42, 119254}

Determining indices:
 {1, 2, 5, 6, 9, 10, 11, 12, 21, 22, 23, 24, 41, 42, 45, 49, 50, 57, 58, 69, 70, 89, 93, 94,
  95, 96, 117, 118, 137, 138, 161, 162, 169, 170, 171, 172, 209, 210, 221, 222, 261, 269}

Atkin-Lehner truncation: 288

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.
```

```
ALspacesDims = {{{1, 1, 1}, 19}, {{1, -1, -1}, 15}, {{-1, 1, -1}, 4}, {{-1, -1, 1}, 4}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1414-8967-to-202-183.ma exists, reading it

Found file Grits-2-1414-8967.ma, Getting it

Grit dimensions: {42, 119 254}

Viable file found: GG-Grits-2-1414-8967.ma-8967-mod-12347.ma

New best file: GG-Grits-2-1414-8967.ma-8967-mod-12347.ma

Dimensions of G2G2: {903, 119 254}

Dimensions of fDown = {903, 232}

Rank of fDown mod pp = 92

Dimensions and rank of plus space fDown = {903, 232}  92

Dimensions and rank of minus space fDown = {903, 232}  0

Rank of plus basis attempt = 93 and dimension of S_4(K(202))^+ = 94

Rank of minus basis attempt = 0 and dimension of S_4(K(202))^- = 15
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 10

Found file Grits-2-202-183.ma, Getting it

Viable file found: GG-Grits-2-202-10192.ma-10192-mod-12347.ma

New best file: GG-Grits-2-202-10192.ma-10192-mod-12347.ma

Viable file found: GG-Grits-2-202-25168.ma-25168-mod-12347.ma

Dimensions of G2G2: {10, 232}

Rank of plus basis attempt = 93 and dimension of S_4(K(202))^+ = 94

Rank of minus basis attempt = 0 and dimension of S_4(K(202))^- = 15
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 2 Hecke spreads, the last of which dilates determinants by 64

Need to multiply the minimal max det 183 by the largest det contraction factor
 64 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 11 712

Hecke spreads: 2

Pre-Hecke expansions will have length 162 106

Found file Grits-2-202-11712.ma, Getting it

Viable file found: GG-Grits-2-202-25168.ma-25168-mod-12347.ma
```

```
New best file: GG-Grits-2-202-25168.ma-25168-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 4

{{2, 2}, {2, 1}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 516 137

Called ShortenVecs with G2G2T[{}]

 input dimensions {10, 516 137}, output dimensions {10, 162 106}

Length, max det of input vectors to ParaHeckeOp: 162 106, 11 712 (check: 162 106, 11 712)

Desired length, max det of output vectors: 2267, 732

Quotient of the max dets should be 16

Smaller max det is the minimal max det 183
  times the biggest postpended det inverse dilation factor
 4 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 2267, 732

Rank of T[G2G2]^+/- = (10,8)

Rank of plus basis attempt = 93 and dimension of S_4(K(202))^+ = 94

Rank of minus basis attempt = 8 and dimension of S_4(K(202))^- = 15


Hecke spread 2 out of 2

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 2267

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {10, 2267}, output dimensions {10, 2267}

Length, max det of input vectors to ParaHeckeOp: 2267, 732 (check: 2267, 732)
```

```
Desired length, max det of output vectors: 232, 183

Quotient of the max dets should be 4

Smaller max det is the minimal max det 183
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 232, 183

Rank of T[G2G2]^+/- = (10,8)

Rank of plus basis attempt = 93 and dimension of S_4(K(202))^+ = 94

Rank of minus basis attempt = 10 and dimension of S_4(K(202))^- = 15


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-202-199.ma}

Dimensions of BPcoeffMatProven: {19, 257}

Dimensions of join: {29, 232}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 93 and dimension of S_4(K(202))^+ = 94

Rank of minus basis attempt = 15 and dimension of S_4(K(202))^- = 15


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Didn't find a plus basis

Found minus basis of dimension 15


N = 203

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {3, 0, 0}

lift dimension of S_4(K(203))^+ : 36

nonlift dimension of S_4(K(203))^+ : 37

dimension of S_4(K(203))^- : 11

Ibukiyama-Kitayama dimension of S_4(K(203)) : 84


STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(203))^+ = 73
```

```
Rank of minus basis attempt = 0 and dimension of S_4(K(203))^- = 11

Initial short vector length: 203

Have vectors of length 217: long enough

Determinant shell containing the vectors of length 203 : 199

Short vector length is 217, and the corresponding determinant is 199

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-203-200.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,203}^{cusp}), which is 36

Rank of plus basis attempt = 36 and dimension of S_4(K(203))^+ = 73

Rank of minus basis attempt = 0 and dimension of S_4(K(203))^- = 11
```

STEP 2: TRACE DOWN

```
N = 203  q = 3  Nq = 609

217-th determinant is 199

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 105

Trace down to try to hit S4(K(N)), of dimension 84

Plus lifts, plus nonlifts, minus dimension: {36, 37, 11}

Target plus rank, target minus rank: 73 11

Dimensions of savedTargetMats: {217, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-609-1791.ma, Getting it

 ... done making G2.  Dimensions of Grits: {14, 8716}

Determining indices: {1, 2, 3, 4, 9, 10, 13, 14, 17, 18, 19, 29, 37, 38}

Atkin-Lehner truncation: 60

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 6}, {{1, -1, -1}, 3}, {{-1, 1, -1}, 2}, {{-1, -1, 1}, 3}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-609-1791-to-203-199.ma exists, reading it

Found file Grits-2-609-1791.ma, Getting it

Grit dimensions: {14, 8716}

Viable file found: GG-Grits-2-609-1791.ma-1791-mod-12347.ma
```

```
New best file: GG-Grits-2-609-1791.ma-1791-mod-12347.ma
Dimensions of G2G2: {105, 8716}
Dimensions of fDown = {105, 217}
Rank of fDown mod pp = 60
Dimensions and rank of plus space fDown = {105, 217}  60
Dimensions and rank of minus space fDown = {105, 217}  0
Rank of plus basis attempt = 73 and dimension of S_4(K(203))^+ = 73
Rank of minus basis attempt = 0 and dimension of S_4(K(203))^- = 11

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS
Pairs from dim(J_{2,N}^{cusp}: 10
Found file Grits-2-203-199.ma, Getting it
Viable file found: GG-Grits-2-203-5075.ma-5075-mod-12347.ma
New best file: GG-Grits-2-203-5075.ma-5075-mod-12347.ma
Dimensions of G2G2: {10, 217}
Rank of plus basis attempt = 73 and dimension of S_4(K(203))^+ = 73
Rank of minus basis attempt = 0 and dimension of S_4(K(203))^- = 11

STEP 4: HECKE SPREAD
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-203-200.ma}
Dimensions of BPcoeffMatProven: {35, 217}
Dimensions of join: {35, 217}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 73 and dimension of S_4(K(203))^+ = 73
Rank of minus basis attempt = 11 and dimension of S_4(K(203))^- = 11

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Found a plus basis of dimension 73
Found minus basis of dimension 11
```

N = 205

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {3, 1, 0}

lift dimension of S_4(K(205))^+ : 38

nonlift dimension of S_4(K(205))^+ : 59

dimension of S_4(K(205))^- : 6

Ibukiyama-Kitayama dimension of S_4(K(205)) : 103

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(205))^+ = 97

Rank of minus basis attempt = 0 and dimension of S_4(K(205))^- = 6

Initial short vector length: 163

Have vectors of length 184: long enough

Determinant shell containing the vectors of length 163 : 159

Short vector length is 178, and the corresponding determinant is 159

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-205-160.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,205}^{cusp}), which is 38

Rank of plus basis attempt = 38 and dimension of S_4(K(205))^+ = 97

Rank of minus basis attempt = 0 and dimension of S_4(K(205))^- = 6

STEP 2: TRACE DOWN

N = 205 q = 3 Nq = 615

178-th determinant is 159

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 66

Trace down to try to hit S4(K(N)), of dimension 103

Plus lifts, plus nonlifts, minus dimension: {38, 59, 6}

Target plus rank, target minus rank: 97 6

Dimensions of savedTargetMats: {178, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-615-1431.ma, Getting it

 ... done making G2. Dimensions of Grits: {11, 6120}

Determining indices: {1, 2, 5, 6, 13, 14, 21, 22, 25, 26, 37}

```
Atkin-Lehner truncation: 48

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 6}, {{1, -1, -1}, 3}, {{-1, 1, -1}, 2}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-615-1431-to-205-159.ma exists, reading it
Found file Grits-2-615-1431.ma, Getting it

Grit dimensions: {11, 6120}

Viable file found: GG-Grits-2-615-2300.ma-2300-mod-12347.ma

New best file: GG-Grits-2-615-2300.ma-2300-mod-12347.ma

Dimensions of G2G2: {66, 13 632}

Dimensions of fDown = {66, 178}

Rank of fDown mod pp = 48

Dimensions and rank of plus space fDown = {66, 178}  48

Dimensions and rank of minus space fDown = {66, 178}  0

Rank of plus basis attempt = 86 and dimension of S_4(K(205))^+ = 97

Rank of minus basis attempt = 0 and dimension of S_4(K(205))^- = 6


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 15

Found file Grits-2-205-159.ma, Getting it

Viable file found: GG-Grits-2-205-6100.ma-6100-mod-12347.ma

New best file: GG-Grits-2-205-6100.ma-6100-mod-12347.ma

Dimensions of G2G2: {15, 178}

Rank of plus basis attempt = 94 and dimension of S_4(K(205))^+ = 97

Rank of minus basis attempt = 0 and dimension of S_4(K(205))^- = 6


STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Using default table of Hecke spreads

Preparing for 2 Hecke spreads, the last of which dilates determinants by 9

Need to multiply the minimal max det 159 by the largest det contraction factor
 9 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 1431

Hecke spreads: 2

Pre-Hecke expansions will have length 5060
```

```
Found file Grits-2-205-1431.ma, Getting it

Viable file found: GG-Grits-2-205-6100.ma-6100-mod-12347.ma

New best file: GG-Grits-2-205-6100.ma-6100-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 1}} 1

{{3, 1}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 1}}] having G2G2T[{}] vectors to length 47 906

Called ShortenVecs with G2G2T[{}]

 input dimensions {15, 47 906}, output dimensions {15, 1510}

Length, max det of input vectors to ParaHeckeOp: 1510, 636 (check: 1510, 636)

Desired length, max det of output vectors: 178, 159

Quotient of the max dets should be 4

Smaller max det is the minimal max det 159
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 1}}

About to compute G2G2T[{{2, 1}}]

Making abstract Hecke formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 178, 159

Rank of T[G2G2]^+/- = (15,0)

Rank of plus basis attempt = 96 and dimension of S_4(K(205))^+ = 97

Rank of minus basis attempt = 0 and dimension of S_4(K(205))^- = 6


Hecke spread 2 out of 2

Is Hecke spread 2 present?  False

DoOneHecke called {},{{3, 1}}

{tp,tpdel} = {3, 1}

Need to compute G2G2T[{{3, 1}}] having G2G2T[{}] vectors to length 47 906

Called ShortenVecs with G2G2T[{}]

 input dimensions {15, 47 906}, output dimensions {15, 5060}

Length, max det of input vectors to ParaHeckeOp: 5060, 1431 (check: 5060, 1431)

Desired length, max det of output vectors: 178, 159
```

```
Quotient of the max dets should be 9

Smaller max det is the minimal max det 159
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 1}}

About to compute G2G2T[{{3, 1}}]

Making abstract Hecke formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 178, 159

Rank of T[G2G2]^+/- = (15,0)

Rank of plus basis attempt = 97 and dimension of S_4(K(205))^+ = 97

Rank of minus basis attempt = 0 and dimension of S_4(K(205))^- = 6
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-205-200.ma}

Dimensions of BPcoeffMatProven: {10, 232}

Dimensions of join: {10, 178}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 97 and dimension of S_4(K(205))^+ = 97

Rank of minus basis attempt = 6 and dimension of S_4(K(205))^- = 6
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Found a plus basis of dimension 97

Found minus basis of dimension 6
```

N = 206

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}

lift dimension of S_4(K(206))^+ : 35

nonlift dimension of S_4(K(206))^+ : 51

dimension of S_4(K(206))^- : 19

Ibukiyama-Kitayama dimension of S_4(K(206)) : 105
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(206))^+ = 86

Rank of minus basis attempt = 0 and dimension of S_4(K(206))^- = 19

Initial short vector length: 226
```

```
Have vectors of length 255: long enough

Determinant shell containing the vectors of length 226 : 172

Short vector length is 231, and the corresponding determinant is 172

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-206-180.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,206}^{cusp}), which is 35

Rank of plus basis attempt = 35 and dimension of S_4(K(206))^+ = 86

Rank of minus basis attempt = 0 and dimension of S_4(K(206))^- = 19
```

STEP 2: TRACE DOWN

```
N = 206  q = 7  Nq = 1442

231-th determinant is 172

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 990

Trace down to try to hit S4(K(N)), of dimension 105

Plus lifts, plus nonlifts, minus dimension: {35, 51, 19}

Target plus rank, target minus rank: 86 19

Dimensions of savedTargetMats: {231, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1442-8428.ma, Getting it

 ... done making G2.  Dimensions of Grits: {44, 108 266}

Determining indices:
 {1, 2, 5, 6, 9, 10, 13, 14, 15, 16, 25, 26, 29, 30, 31, 32, 49, 50, 61, 62, 81, 85, 86, 87, 88,
  109, 110, 129, 130, 139, 140, 155, 156, 179, 191, 192, 193, 194, 231, 232, 251, 267, 268, 387}

Atkin-Lehner truncation: 402

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 19}, {{1, -1, -1}, 16}, {{-1, 1, -1}, 4}, {{-1, -1, 1}, 5}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1442-8428-to-206-172.ma exists, reading it

Found file Grits-2-1442-8428.ma, Getting it

Grit dimensions: {44, 108 266}

Viable file found: GG-Grits-2-1442-8428.ma-8428-mod-12347.ma
```

```
New best file: GG-Grits-2-1442-8428.ma-8428-mod-12347.ma

Dimensions of G2G2: {990, 108 266}

Dimensions of fDown = {990, 231}

Rank of fDown mod pp = 85

Dimensions and rank of plus space fDown = {990, 231}  85

Dimensions and rank of minus space fDown = {990, 231}  0

Rank of plus basis attempt = 85 and dimension of S_4(K(206))^+ = 86

Rank of minus basis attempt = 0 and dimension of S_4(K(206))^- = 19
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-206-172.ma, Getting it

Viable file found: GG-Grits-2-206-11008.ma-11008-mod-12347.ma

New best file: GG-Grits-2-206-11008.ma-11008-mod-12347.ma

Viable file found: GG-Grits-2-206-172.ma-172-mod-12347.ma

New best file: GG-Grits-2-206-172.ma-172-mod-12347.ma

Viable file found: GG-Grits-2-206-24768.ma-24768-mod-12347.ma

Viable file found: GG-Grits-2-206-688.ma-688-mod-12347.ma

Dimensions of G2G2: {3, 231}

Rank of plus basis attempt = 85 and dimension of S_4(K(206))^+ = 86

Rank of minus basis attempt = 0 and dimension of S_4(K(206))^- = 19
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 3 Hecke spreads, the last of which dilates determinants by 144

Need to multiply the minimal max det 172 by the largest det contraction factor
 144 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 24 768

Hecke spreads: 3

Pre-Hecke expansions will have length 504 634

Found file Grits-2-206-24768.ma, Getting it

Viable file found: GG-Grits-2-206-24768.ma-24768-mod-12347.ma

New best file: GG-Grits-2-206-24768.ma-24768-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 9
```

```
{{2, 2}, {2, 1}} 1

{{2, 2}, {3, 1}} 1

Hecke spread 1 out of 3

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 504 634

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 504 634}, output dimensions {3, 504 634}

Length, max det of input vectors to ParaHeckeOp: 504 634, 24 768 (check: 504 634, 24 768)

Desired length, max det of output vectors: 7340, 1548

Quotient of the max dets should be 16

Smaller max det is the minimal max det 172
  times the biggest postpended det inverse dilation factor
 9 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 7340, 1548

Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 85 and dimension of S_4(K(206))^+ = 86

Rank of minus basis attempt = 3 and dimension of S_4(K(206))^- = 19


Hecke spread 2 out of 3

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 7340

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {3, 7340}, output dimensions {3, 2002}

Length, max det of input vectors to ParaHeckeOp: 2002, 688 (check: 2002, 688)

Desired length, max det of output vectors: 231, 172

Quotient of the max dets should be 4
```

```
Smaller max det is the minimal max det 172
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}
About to compute G2G2T[{{2, 2}, {2, 1}}]
Making abstract Hecke formula at a bad prime
Using Ralf's formula
Substituting given paramodular forms in abstract formula
Hecke spread 2 length and max det should be as desired: 231, 172
Rank of T[G2G2]^+/- = (3,3)
Rank of plus basis attempt = 85 and dimension of S_4(K(206))^+ = 86
Rank of minus basis attempt = 6 and dimension of S_4(K(206))^- = 19


Hecke spread 3 out of 3
Is Hecke spread 3 present?  False
DoOneHecke called {},{{2, 2}, {3, 1}}
{tp,tpdel} = {2, 2}
DoOneHecke called {{2, 2}},{{3, 1}}
{tp,tpdel} = {3, 1}
Need to compute G2G2T[{{2, 2}, {3, 1}}] having G2G2T[{{2, 2}}] vectors to length 7340
Called ShortenVecs with G2G2T[{{2, 2}}]
 input dimensions {3, 7340}, output dimensions {3, 7340}
Length, max det of input vectors to ParaHeckeOp: 7340, 1548 (check: 7340, 1548)
Desired length, max det of output vectors: 231, 172
Quotient of the max dets should be 9
Smaller max det is the minimal max det 172
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {3, 1}}
About to compute G2G2T[{{2, 2}, {3, 1}}]
Making abstract Hecke formula
Substituting given paramodular forms in abstract formula
Hecke spread 3 length and max det should be as desired: 231, 172
Rank of T[G2G2]^+/- = (3,3)
Rank of plus basis attempt = 85 and dimension of S_4(K(206))^+ = 86
Rank of minus basis attempt = 9 and dimension of S_4(K(206))^- = 19


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-206-200.ma}
Dimensions of BPcoeffMatProven: {19, 349}
```

```
Dimensions of join: {28, 231}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 85 and dimension of S_4(K(206))^+ = 86
Rank of minus basis attempt = 19 and dimension of S_4(K(206))^- = 19

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Didn't find a plus basis
Found minus basis of dimension 19

N = 209
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {3, 1, 0}
lift dimension of S_4(K(209))^+ : 36
nonlift dimension of S_4(K(209))^+ : 48
dimension of S_4(K(209))^- : 6
Ibukiyama-Kitayama dimension of S_4(K(209)) : 90

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(209))^+ = 84
Rank of minus basis attempt = 0 and dimension of S_4(K(209))^- = 6
Initial short vector length: 143
Have vectors of length 179: long enough
Determinant shell containing the vectors of length 143 : 164
Short vector length is 157, and the corresponding determinant is 164
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-209-170.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,209}^{cusp}), which is 36
Rank of plus basis attempt = 36 and dimension of S_4(K(209))^+ = 84
Rank of minus basis attempt = 0 and dimension of S_4(K(209))^- = 6

STEP 2: TRACE DOWN
N = 209  q = 3  Nq = 627
157-th determinant is 164
```

```
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 136

Trace down to try to hit S4(K(N)), of dimension 90

Plus lifts, plus nonlifts, minus dimension: {36, 48, 6}

Target plus rank, target minus rank: 84 6

Dimensions of savedTargetMats: {157, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-627-1476.ma, Getting it

 ... done making G2.  Dimensions of Grits: {16, 5936}

Determining indices: {1, 2, 3, 4, 5, 6, 7, 8, 17, 18, 21, 22, 33, 57, 58, 59}

Atkin-Lehner truncation: 68

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 6}, {{1, -1, -1}, 4}, {{-1, 1, -1}, 3}, {{-1, -1, 1}, 3}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-627-1476-to-209-164.ma exists, reading it

Found file Grits-2-627-1476.ma, Getting it

Grit dimensions: {16, 5936}

Viable file found: GG-Grits-2-627-1476.ma-1476-mod-12347.ma

New best file: GG-Grits-2-627-1476.ma-1476-mod-12347.ma

Dimensions of G2G2: {136, 5936}

Dimensions of fDown = {136, 157}

Rank of fDown mod pp = 74

Dimensions and rank of plus space fDown = {136, 157}  74

Dimensions and rank of minus space fDown = {136, 157}  0

Rank of plus basis attempt = 82 and dimension of S_4(K(209))^+ = 84

Rank of minus basis attempt = 0 and dimension of S_4(K(209))^- = 6
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 6

Found file Grits-2-209-164.ma, Getting it

Viable file found: GG-Grits-2-209-13504.ma-13504-mod-12347.ma

New best file: GG-Grits-2-209-13504.ma-13504-mod-12347.ma

Viable file found: GG-Grits-2-209-30384.ma-30384-mod-12347.ma

Dimensions of G2G2: {6, 157}
```

```
Rank of plus basis attempt = 82 and dimension of S_4(K(209))^+ = 84

Rank of minus basis attempt = 0 and dimension of S_4(K(209))^- = 6
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 2 Hecke spreads, the last of which dilates determinants by 16

Need to multiply the minimal max det 164 by the largest det contraction factor
 16 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 2624

Hecke spreads: 2

Pre-Hecke expansions will have length 12 174

Found file Grits-2-209-2624.ma, Getting it

Viable file found: GG-Grits-2-209-13504.ma-13504-mod-12347.ma

New best file: GG-Grits-2-209-13504.ma-13504-mod-12347.ma

Viable file found: GG-Grits-2-209-30384.ma-30384-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 1}} 1

{{2, 2}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 1}}

{tp,tpdel} = {3, 1}

Need to compute G2G2T[{{3, 1}}] having G2G2T[{}] vectors to length 150 971

Called ShortenVecs with G2G2T[{}]

 input dimensions {6, 150 971}, output dimensions {6, 4906}

Length, max det of input vectors to ParaHeckeOp: 4906, 1476 (check: 4906, 1476)

Desired length, max det of output vectors: 157, 164

Quotient of the max dets should be 9

Smaller max det is the minimal max det 164
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 1}}

About to compute G2G2T[{{3, 1}}]

Making abstract Hecke formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 157, 164

Rank of T[G2G2]^+/- = (6,0)
```

```
Rank of plus basis attempt = 83 and dimension of S_4(K(209))^+ = 84

Rank of minus basis attempt = 0 and dimension of S_4(K(209))^- = 6


Hecke spread 2 out of 2

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 150 971

Called ShortenVecs with G2G2T[{}]

 input dimensions {6, 150 971}, output dimensions {6, 12 174}

Length, max det of input vectors to ParaHeckeOp: 12 174, 2624 (check: 12 174, 2624)

Desired length, max det of output vectors: 157, 164

Quotient of the max dets should be 16

Smaller max det is the minimal max det 164
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 157, 164

Rank of T[G2G2]^+/- = (6,0)

Rank of plus basis attempt = 84 and dimension of S_4(K(209))^+ = 84

Rank of minus basis attempt = 0 and dimension of S_4(K(209))^- = 6


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-209-200.ma}

Dimensions of BPcoeffMatProven: {6, 222}

Dimensions of join: {6, 157}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 84 and dimension of S_4(K(209))^+ = 84

Rank of minus basis attempt = 6 and dimension of S_4(K(209))^- = 6


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Found a plus basis of dimension 84

Found minus basis of dimension 6
```

N = 210

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {11, 1, 0}

lift dimension of S_4(K(210))^+ : 29

nonlift dimension of S_4(K(210))^+ : 48

dimension of S_4(K(210))^- : 15

Ibukiyama-Kitayama dimension of S_4(K(210)) : 92

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(210))^+ = 77

Rank of minus basis attempt = 0 and dimension of S_4(K(210))^- = 15

Initial short vector length: 228

Have vectors of length 240: long enough

Determinant shell containing the vectors of length 228 : 164

Short vector length is 240, and the corresponding determinant is 164

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-210-170.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,210}^{cusp}), which is 29

Rank of plus basis attempt = 29 and dimension of S_4(K(210))^+ = 77

Rank of minus basis attempt = 0 and dimension of S_4(K(210))^- = 15

STEP 2: TRACE DOWN

N = 210 q = 11 Nq = 2310

240-th determinant is 164

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 1035

Trace down to try to hit S4(K(N)), of dimension 92

Plus lifts, plus nonlifts, minus dimension: {29, 48, 15}

Target plus rank, target minus rank: 77 15

Dimensions of savedTargetMats: {240, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-2310-19844.ma, Getting it

 ... done making G2. Dimensions of Grits: {45, 655264}

```
Determining indices: {1, 2, 3, 4, 9, 10, 17, 18, 19, 20, 57, 58, 81,
  82, 83, 84, 85, 86, 87, 88, 145, 146, 147, 148, 149, 150, 151, 257, 258, 259,
  260, 313, 314, 337, 338, 339, 340, 385, 386, 401, 402, 441, 577, 578, 721}

Atkin-Lehner truncation: 1120

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1, 1, 1}, 12}, {{1, 1, 1, -1, -1}, 6}, {{1, 1, -1, 1, -1}, 6},
  {{1, 1, -1, -1, 1}, 5}, {{1, -1, 1, 1, -1}, 3}, {{1, -1, 1, -1, 1}, 3},
  {{1, -1, -1, 1, 1}, 3}, {{1, -1, -1, -1, -1}, 1}, {{-1, 1, 1, 1, -1}, 2},
  {{-1, 1, 1, -1, 1}, 2}, {{-1, 1, -1, 1, 1}, 1}, {{-1, -1, 1, 1, 1}, 1}}

Need 16 AL signatures; have the signatures
 {{-1, -1, -1, -1, 1}, {-1, -1, -1, 1, -1}, {-1, -1, 1, -1, -1}, {-1, -1, 1, 1, 1},
  {-1, 1, -1, -1, -1}, {-1, 1, -1, 1, 1}, {-1, 1, 1, -1, 1}, {-1, 1, 1, 1, -1},
  {1, -1, -1, -1, -1}, {1, -1, -1, 1, 1}, {1, -1, 1, -1, 1}, {1, -1, 1, 1, -1},
  {1, 1, -1, -1, 1}, {1, 1, -1, 1, -1}, {1, 1, 1, -1, -1}, {1, 1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-2310-19844-to-210-164.ma
  exists, reading it

Found file Grits-2-2310-19844.ma, Getting it

Grit dimensions: {45, 655264}

Viable file found: GG-Grits-2-2310-19844.ma-19844-mod-12347.ma

New best file: GG-Grits-2-2310-19844.ma-19844-mod-12347.ma

Dimensions of G2G2: {259, 655264}

Dimensions of fDown = {259, 240}

Rank of fDown mod pp = 77

Dimensions and rank of plus space fDown = {259, 240}  77

Dimensions and rank of minus space fDown = {259, 240}  0

Rank of plus basis attempt = 77 and dimension of S_4(K(210))^+ = 77

Rank of minus basis attempt = 0 and dimension of S_4(K(210))^- = 15


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 1

Found file Grits-2-210-164.ma, Getting it

Viable file found: GG-Grits-2-210-164.ma-164-mod-12347.ma

New best file: GG-Grits-2-210-164.ma-164-mod-12347.ma

Viable file found: GG-Grits-2-210-23616.ma-23616-mod-12347.ma

Dimensions of G2G2: {1, 240}

Rank of plus basis attempt = 77 and dimension of S_4(K(210))^+ = 77

Rank of minus basis attempt = 0 and dimension of S_4(K(210))^- = 15


STEP 4: HECKE SPREAD
```

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 4 Hecke spreads, the last of which dilates determinants by 144

Need to multiply the minimal max det 164 by the largest det contraction factor
 144 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 23 616

Hecke spreads: 4

Pre-Hecke expansions will have length 824 644

Found file Grits-2-210-23616.ma, Getting it

Viable file found: GG-Grits-2-210-23616.ma-23616-mod-12347.ma

New best file: GG-Grits-2-210-23616.ma-23616-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 9

{{2, 2}, {2, 1}} 1

{{3, 2}} 1

{{2, 2}, {3, 1}} 1

Hecke spread 1 out of 4

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 824 644

Called ShortenVecs with G2G2T[{}]

 input dimensions {1, 824 644}, output dimensions {1, 824 644}

Length, max det of input vectors to ParaHeckeOp: 824 644, 23 616 (check: 824 644, 23 616)

Desired length, max det of output vectors: 10 552, 1476

Quotient of the max dets should be 16

Smaller max det is the minimal max det 164
  times the biggest postpended det inverse dilation factor
 9 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 10 552, 1476

Rank of T[G2G2]^+/- = (1,1)

Rank of plus basis attempt = 77 and dimension of S_4(K(210))^+ = 77

Rank of minus basis attempt = 1 and dimension of S_4(K(210))^- = 15
```

```
Hecke spread 2 out of 4

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 10 552

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {1, 10 552}, output dimensions {1, 2688}

Length, max det of input vectors to ParaHeckeOp: 2688, 656 (check: 2688, 656)

Desired length, max det of output vectors: 240, 164

Quotient of the max dets should be 4

Smaller max det is the minimal max det 164
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 240, 164

Rank of T[G2G2]^+/- = (1,1)

Rank of plus basis attempt = 77 and dimension of S_4(K(210))^+ = 77

Rank of minus basis attempt = 2 and dimension of S_4(K(210))^- = 15


Hecke spread 3 out of 4

Is Hecke spread 3 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 824 644

Called ShortenVecs with G2G2T[{}]

 input dimensions {1, 824 644}, output dimensions {1, 339 820}

Length, max det of input vectors to ParaHeckeOp: 339 820, 13 284 (check: 339 820, 13 284)

Desired length, max det of output vectors: 240, 164

Quotient of the max dets should be 81

Smaller max det is the minimal max det 164
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}
```

```
About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 3 length and max det should be as desired: 240, 164

Rank of T[G2G2]^+/- = (1,1)

Rank of plus basis attempt = 77 and dimension of S_4(K(210))^+ = 77

Rank of minus basis attempt = 3 and dimension of S_4(K(210))^- = 15


Hecke spread 4 out of 4

Is Hecke spread 4 present?  False

DoOneHecke called {},{{2, 2}, {3, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{3, 1}}

{tp,tpdel} = {3, 1}

Need to compute G2G2T[{{2, 2}, {3, 1}}] having G2G2T[{{2, 2}}] vectors to length 10552

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {1, 10552}, output dimensions {1, 10552}

Length, max det of input vectors to ParaHeckeOp: 10552, 1476 (check: 10552, 1476)

Desired length, max det of output vectors: 240, 164

Quotient of the max dets should be 9

Smaller max det is the minimal max det 164
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {3, 1}}

About to compute G2G2T[{{2, 2}, {3, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 4 length and max det should be as desired: 240, 164

Rank of T[G2G2]^+/- = (1,1)

Rank of plus basis attempt = 77 and dimension of S_4(K(210))^+ = 77

Rank of minus basis attempt = 4 and dimension of S_4(K(210))^- = 15


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-210-200.ma}

Dimensions of BPcoeffMatProven: {15, 252}

Dimensions of join: {19, 240}
```

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 77 and dimension of S_4(K(210))^+ = 77

Rank of minus basis attempt = 15 and dimension of S_4(K(210))^- = 15

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 77

Found minus basis of dimension 15

N = 213

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}

lift dimension of S_4(K(213))^+ : 38

nonlift dimension of S_4(K(213))^+ : 53

dimension of S_4(K(213))^- : 13

Ibukiyama-Kitayama dimension of S_4(K(213)) : 104

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(213))^+ = 91

Rank of minus basis attempt = 0 and dimension of S_4(K(213))^- = 13

Initial short vector length: 182

Have vectors of length 195: long enough

Determinant shell containing the vectors of length 182 : 176

Short vector length is 195, and the corresponding determinant is 176

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-213-180.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,213}^{cusp}), which is 38

Rank of plus basis attempt = 38 and dimension of S_4(K(213))^+ = 91

Rank of minus basis attempt = 0 and dimension of S_4(K(213))^- = 13

STEP 2: TRACE DOWN

N = 213 q = 5 Nq = 1065

195-th determinant is 176

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 528

```
Trace down to try to hit S4(K(N)), of dimension 104

Plus lifts, plus nonlifts, minus dimension: {38, 53, 13}

Target plus rank, target minus rank: 91 13

Dimensions of savedTargetMats: {195, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1065-4400.ma, Getting it

 ... done making G2.  Dimensions of Grits: {32, 37428}

Determining indices: {1, 2, 3, 4, 5, 6, 9, 10, 17, 18, 19, 20, 33, 34,
  37, 38, 39, 40, 53, 54, 55, 65, 79, 80, 87, 88, 99, 100, 123, 139, 163, 164}

Atkin-Lehner truncation: 182

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 14}, {{1, -1, -1}, 9}, {{-1, 1, -1}, 6}, {{-1, -1, 1}, 3}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1065-4400-to-213-176.ma exists, reading it

Found file Grits-2-1065-4400.ma, Getting it

Grit dimensions: {32, 37428}

Viable file found: GG-Grits-2-1065-4400.ma-4400-mod-12347.ma

New best file: GG-Grits-2-1065-4400.ma-4400-mod-12347.ma

Dimensions of G2G2: {528, 37428}

Dimensions of fDown = {528, 195}

Rank of fDown mod pp = 91

Dimensions and rank of plus space fDown = {528, 195}  91

Dimensions and rank of minus space fDown = {528, 195}  0

Rank of plus basis attempt = 91 and dimension of S_4(K(213))^+ = 91

Rank of minus basis attempt = 0 and dimension of S_4(K(213))^- = 13


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 10

Found file Grits-2-213-176.ma, Getting it

Viable file found: GG-Grits-2-213-10388.ma-10388-mod-12347.ma

New best file: GG-Grits-2-213-10388.ma-10388-mod-12347.ma

Viable file found: GG-Grits-2-213-14256.ma-14256-mod-12347.ma

Dimensions of G2G2: {10, 195}
```

```
Rank of plus basis attempt = 91 and dimension of S_4(K(213))^+ = 91

Rank of minus basis attempt = 0 and dimension of S_4(K(213))^- = 13


STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 176 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 14 256

Hecke spreads: 1

Pre-Hecke expansions will have length 194 213

Found file Grits-2-213-14256.ma, Getting it

Viable file found: GG-Grits-2-213-14256.ma-14256-mod-12347.ma

New best file: GG-Grits-2-213-14256.ma-14256-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 194 213

Called ShortenVecs with G2G2T[{}]

 input dimensions {10, 194 213}, output dimensions {10, 194 213}

Length, max det of input vectors to ParaHeckeOp: 194 213, 14 256 (check: 194 213, 14 256)

Desired length, max det of output vectors: 195, 176

Quotient of the max dets should be 81

Smaller max det is the minimal max det 176
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 195, 176

Rank of T[G2G2]^+/- = (10,4)

Rank of plus basis attempt = 91 and dimension of S_4(K(213))^+ = 91
```

Rank of minus basis attempt = 4 and dimension of S_4(K(213))^- = 13

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-213-200.ma}

Dimensions of BPcoeffMatProven: {13, 227}

Dimensions of join: {17, 195}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 91 and dimension of S_4(K(213))^+ = 91

Rank of minus basis attempt = 13 and dimension of S_4(K(213))^- = 13

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 91

Found minus basis of dimension 13

N = 214

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 0}

lift dimension of S_4(K(214))^+ : 40

nonlift dimension of S_4(K(214))^+ : 67

dimension of S_4(K(214))^- : 12

Ibukiyama-Kitayama dimension of S_4(K(214)) : 119

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(214))^+ = 107

Rank of minus basis attempt = 0 and dimension of S_4(K(214))^- = 12

Initial short vector length: 229

Have vectors of length 244: long enough

Determinant shell containing the vectors of length 229 : 175

Short vector length is 238, and the corresponding determinant is 175

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-214-180.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,214}^{cusp}), which is 40

Rank of plus basis attempt = 40 and dimension of S_4(K(214))^+ = 107

```
Rank of minus basis attempt = 0 and dimension of S_4(K(214))^- = 12
```

STEP 2: TRACE DOWN

```
N = 214  q = 7  Nq = 1498

238-th determinant is 175

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 1128

Trace down to try to hit S4(K(N)), of dimension 119

Plus lifts, plus nonlifts, minus dimension: {40, 67, 12}

Target plus rank, target minus rank: 107 12

Dimensions of savedTargetMats: {238, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1498-8575.ma, Getting it

 ... done making G2.  Dimensions of Grits: {47, 111816}

Determining indices: {1, 2, 3, 4, 7, 8, 11, 15, 16, 17, 18, 27, 28, 29,
  30, 43, 44, 53, 54, 61, 62, 81, 85, 86, 105, 106, 107, 108, 125, 126, 137, 145,
  146, 169, 170, 181, 182, 183, 225, 239, 240, 251, 252, 299, 359, 379, 395}

Atkin-Lehner truncation: 434

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 24}, {{1, -1, -1}, 14}, {{-1, 1, -1}, 6}, {{-1, -1, 1}, 3}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1498-8575-to-214-175.ma exists, reading it

Found file Grits-2-1498-8575.ma, Getting it

Grit dimensions: {47, 111816}

Viable file found: GG-Grits-2-1498-8575.ma-8575-mod-12347.ma

New best file: GG-Grits-2-1498-8575.ma-8575-mod-12347.ma

Dimensions of G2G2: {1128, 111816}

Dimensions of fDown = {1128, 238}

Rank of fDown mod pp = 104

Dimensions and rank of plus space fDown = {1128, 238}  104

Dimensions and rank of minus space fDown = {1128, 238}  0

Rank of plus basis attempt = 105 and dimension of S_4(K(214))^+ = 107

Rank of minus basis attempt = 0 and dimension of S_4(K(214))^- = 12
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 15

Found file Grits-2-214-175.ma, Getting it

Viable file found: GG-Grits-2-214-18711.ma-18711-mod-12347.ma

New best file: GG-Grits-2-214-18711.ma-18711-mod-12347.ma

Viable file found: GG-Grits-2-214-23100.ma-23100-mod-12347.ma

Viable file found: GG-Grits-2-214-44800.ma-44800-mod-12347.ma

Viable file found: GG-Grits-2-214-8316.ma-8316-mod-12347.ma

New best file: GG-Grits-2-214-8316.ma-8316-mod-12347.ma

Dimensions of G2G2: {15, 238}

Rank of plus basis attempt = 105 and dimension of S_4(K(214))^+ = 107

Rank of minus basis attempt = 0 and dimension of S_4(K(214))^- = 12

STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 16

Need to multiply the minimal max det 175 by the largest det contraction factor
 16 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 2800

Hecke spreads: 1

Pre-Hecke expansions will have length 18 280

Found file Grits-2-214-2800.ma, Getting it

Viable file found: GG-Grits-2-214-18711.ma-18711-mod-12347.ma

New best file: GG-Grits-2-214-18711.ma-18711-mod-12347.ma

Viable file found: GG-Grits-2-214-23100.ma-23100-mod-12347.ma

Viable file found: GG-Grits-2-214-44800.ma-44800-mod-12347.ma

Viable file found: GG-Grits-2-214-8316.ma-8316-mod-12347.ma

New best file: GG-Grits-2-214-8316.ma-8316-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present? False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 95 758

```
Called ShortenVecs with G2G2T[{}]

 input dimensions {15, 95758}, output dimensions {15, 18280}

Length, max det of input vectors to ParaHeckeOp: 18280, 2800 (check: 18280, 2800)

Desired length, max det of output vectors: 238, 175

Quotient of the max dets should be 16

Smaller max det is the minimal max det 175
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 238, 175

Rank of T[G2G2]^+/- = (15,8)

Rank of plus basis attempt = 105 and dimension of S_4(K(214))^+ = 107

Rank of minus basis attempt = 8 and dimension of S_4(K(214))^- = 12


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-214-300.ma}

Dimensions of BPcoeffMatProven: {12, 558}

Dimensions of join: {20, 238}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 105 and dimension of S_4(K(214))^+ = 107

Rank of minus basis attempt = 12 and dimension of S_4(K(214))^- = 12


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Didn't find a plus basis

Found minus basis of dimension 12


N = 215

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}

lift dimension of S_4(K(215))^+ : 36

nonlift dimension of S_4(K(215))^+ : 47

dimension of S_4(K(215))^- : 14

Ibukiyama-Kitayama dimension of S_4(K(215)) : 97
```

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(215))^+ = 83

Rank of minus basis attempt = 0 and dimension of S_4(K(215))^- = 14

Initial short vector length: 189

Have vectors of length 216: long enough

Determinant shell containing the vectors of length 189 : 175

Short vector length is 200, and the corresponding determinant is 175

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-215-180.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,215}^{cusp}), which is 36

Rank of plus basis attempt = 36 and dimension of S_4(K(215))^+ = 83

Rank of minus basis attempt = 0 and dimension of S_4(K(215))^- = 14

STEP 2: TRACE DOWN

N = 215 q = 7 Nq = 1505

200-th determinant is 175

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 946

Trace down to try to hit S4(K(N)), of dimension 97

Plus lifts, plus nonlifts, minus dimension: {36, 47, 14}

Target plus rank, target minus rank: 83 14

Dimensions of savedTargetMats: {200, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1505-8575.ma, Getting it

 ... done making G2. Dimensions of Grits: {43, 88854}

Determining indices:
 {1, 2, 3, 4, 5, 6, 9, 10, 17, 18, 25, 26, 27, 28, 41, 42, 53, 54, 57, 58, 59, 60, 81, 82, 85,
  86, 105, 106, 107, 108, 125, 126, 127, 128, 141, 142, 143, 144, 181, 182, 183, 225, 227}

Atkin-Lehner truncation: 260

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 15}, {{1, -1, -1}, 12}, {{-1, 1, -1}, 8}, {{-1, -1, 1}, 8}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

```
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1505-8575-to-215-175.ma exists, reading it
Found file Grits-2-1505-8575.ma, Getting it
Grit dimensions: {43, 88 854}
Viable file found: GG-Grits-2-1505-8575.ma-8575-mod-12347.ma
New best file: GG-Grits-2-1505-8575.ma-8575-mod-12347.ma
Dimensions of G2G2: {946, 88 854}
Dimensions of fDown = {946, 200}
Rank of fDown mod pp = 83
Dimensions and rank of plus space fDown = {946, 200}  83
Dimensions and rank of minus space fDown = {946, 200}  0
Rank of plus basis attempt = 83 and dimension of S_4(K(215))^+ = 83
Rank of minus basis attempt = 0 and dimension of S_4(K(215))^- = 14

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS
Pairs from dim(J_{2,N}^{cusp}: 3
Found file Grits-2-215-175.ma, Getting it
Viable file found: GG-Grits-2-215-175.ma-175-mod-12347.ma
New best file: GG-Grits-2-215-175.ma-175-mod-12347.ma
Dimensions of G2G2: {3, 200}
Rank of plus basis attempt = 83 and dimension of S_4(K(215))^+ = 83
Rank of minus basis attempt = 0 and dimension of S_4(K(215))^- = 14

STEP 4: HECKE SPREAD
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-215-200.ma}
Dimensions of BPcoeffMatProven: {14, 280}
Dimensions of join: {14, 200}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 83 and dimension of S_4(K(215))^+ = 83
Rank of minus basis attempt = 14 and dimension of S_4(K(215))^- = 14

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}
```

```
Found a plus basis of dimension 83

Found minus basis of dimension 14


N = 217

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}

lift dimension of S_4(K(217))^+ : 41

nonlift dimension of S_4(K(217))^+ : 66

dimension of S_4(K(217))^- : 6

Ibukiyama-Kitayama dimension of S_4(K(217)) : 113


STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(217))^+ = 107

Rank of minus basis attempt = 0 and dimension of S_4(K(217))^- = 6

Initial short vector length: 170

Have vectors of length 199: long enough

Determinant shell containing the vectors of length 170 : 192

Short vector length is 181, and the corresponding determinant is 192

Vectors fill out a determinant shell, as they should


STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-217-200.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,217}^{cusp}), which is 41

Rank of plus basis attempt = 41 and dimension of S_4(K(217))^+ = 107

Rank of minus basis attempt = 0 and dimension of S_4(K(217))^- = 6


STEP 2: TRACE DOWN

N = 217  q = 5  Nq = 1085

181-th determinant is 192

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 465

Trace down to try to hit S4(K(N)), of dimension 113

Plus lifts, plus nonlifts, minus dimension: {41, 66, 6}

Target plus rank, target minus rank: 107 6

Dimensions of savedTargetMats: {181, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.
```

```
Making G2 for S_2(K(Nq))...

Found file Grits-2-1085-4800.ma, Getting it

 ... done making G2.  Dimensions of Grits: {30, 36 340}

Determining indices: {1, 2, 3, 4, 9, 10, 15, 16, 23, 24, 31, 32,
  39, 40, 43, 44, 45, 46, 67, 69, 91, 92, 95, 96, 115, 116, 127, 128, 129, 155}

Atkin-Lehner truncation: 190

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 13}, {{1, -1, -1}, 7}, {{-1, 1, -1}, 6}, {{-1, -1, 1}, 4}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1085-4800-to-217-192.ma exists, reading it

Found file Grits-2-1085-4800.ma, Getting it

Grit dimensions: {30, 36 340}

Viable file found: GG-Grits-2-1085-5831.ma-5831-mod-12347.ma

New best file: GG-Grits-2-1085-5831.ma-5831-mod-12347.ma

Dimensions of G2G2: {465, 49 578}

Dimensions of fDown = {465, 181}

Rank of fDown mod pp = 103

Dimensions and rank of plus space fDown = {465, 181}  103

Dimensions and rank of minus space fDown = {465, 181}  0

Rank of plus basis attempt = 107 and dimension of S_4(K(217))^+ = 107

Rank of minus basis attempt = 0 and dimension of S_4(K(217))^- = 6


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 21

Found file Grits-2-217-192.ma, Getting it

Viable file found: GG-Grits-2-217-6275.ma-6275-mod-12347.ma

New best file: GG-Grits-2-217-6275.ma-6275-mod-12347.ma

Dimensions of G2G2: {21, 181}

Rank of plus basis attempt = 107 and dimension of S_4(K(217))^+ = 107

Rank of minus basis attempt = 0 and dimension of S_4(K(217))^- = 6


STEP 4: HECKE SPREAD

J2=0, or already have plus space basis and no extra Hecke spreads were specified

Hecke spreads: 0
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-217-300.ma}

Dimensions of BPcoeffMatProven: {6, 426}

Dimensions of join: {6, 181}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 107 and dimension of S_4(K(217))^+ = 107

Rank of minus basis attempt = 6 and dimension of S_4(K(217))^- = 6

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 107

Found minus basis of dimension 6

N = 218

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}

lift dimension of S_4(K(218))^+ : 41

nonlift dimension of S_4(K(218))^+ : 55

dimension of S_4(K(218))^- : 23

Ibukiyama-Kitayama dimension of S_4(K(218)) : 119

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(218))^+ = 96

Rank of minus basis attempt = 0 and dimension of S_4(K(218))^- = 23

Initial short vector length: 226

Have vectors of length 240: long enough

Determinant shell containing the vectors of length 226 : 175

Short vector length is 234, and the corresponding determinant is 175

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-218-180.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,218}^{cusp}), which is 41

Rank of plus basis attempt = 41 and dimension of S_4(K(218))^+ = 96

Rank of minus basis attempt = 0 and dimension of S_4(K(218))^- = 23

STEP 2: TRACE DOWN

N = 218 q = 5 Nq = 1090

234-th determinant is 175

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 528

Trace down to try to hit S4(K(N)), of dimension 119

Plus lifts, plus nonlifts, minus dimension: {41, 55, 23}

Target plus rank, target minus rank: 96 23

Dimensions of savedTargetMats: {234, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1090-4375.ma, Getting it

 ... done making G2. Dimensions of Grits: {32, 41954}

Determining indices: {1, 2, 3, 4, 7, 8, 13, 15, 16, 17, 18, 27, 28, 33, 41,
  42, 55, 56, 57, 83, 93, 94, 101, 107, 108, 119, 120, 131, 132, 155, 156, 179}

Atkin-Lehner truncation: 190

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 17}, {{1, -1, -1}, 10}, {{-1, 1, -1}, 4}, {{-1, -1, 1}, 1}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1090-4375-to-218-175.ma exists, reading it

Found file Grits-2-1090-4375.ma, Getting it

Grit dimensions: {32, 41954}

Viable file found: GG-Grits-2-1090-4375.ma-4375-mod-12347.ma

New best file: GG-Grits-2-1090-4375.ma-4375-mod-12347.ma

Dimensions of G2G2: {528, 41954}

Dimensions of fDown = {528, 234}

Rank of fDown mod pp = 93

Dimensions and rank of plus space fDown = {528, 234} 93

Dimensions and rank of minus space fDown = {528, 234} 0

Rank of plus basis attempt = 96 and dimension of S_4(K(218))^+ = 96

Rank of minus basis attempt = 0 and dimension of S_4(K(218))^- = 23

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 21

Found file Grits-2-218-175.ma, Getting it

```
Viable file found: GG-Grits-2-218-25200.ma-25200-mod-12347.ma

New best file: GG-Grits-2-218-25200.ma-25200-mod-12347.ma

Viable file found: GG-Grits-2-218-3072.ma-3072-mod-12347.ma

New best file: GG-Grits-2-218-3072.ma-3072-mod-12347.ma

Dimensions of G2G2: {21, 234}

Rank of plus basis attempt = 96 and dimension of S_4(K(218))^+ = 96

Rank of minus basis attempt = 0 and dimension of S_4(K(218))^- = 23


STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 2 Hecke spreads, the last of which dilates determinants by 64

Need to multiply the minimal max det 175 by the largest det contraction factor
 64 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 11 200

Hecke spreads: 2

Pre-Hecke expansions will have length 150 828

Found file Grits-2-218-11200.ma, Getting it

Viable file found: GG-Grits-2-218-25200.ma-25200-mod-12347.ma

New best file: GG-Grits-2-218-25200.ma-25200-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 4

{{2, 2}, {2, 1}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 516 985

Called ShortenVecs with G2G2T[{}]

 input dimensions {21, 516 985}, output dimensions {21, 150 828}

Length, max det of input vectors to ParaHeckeOp: 150 828, 11 200 (check: 150 828, 11 200)

Desired length, max det of output vectors: 2055, 700

Quotient of the max dets should be 16

Smaller max det is the minimal max det 175
  times the biggest postpended det inverse dilation factor
 4 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]
```

```
Making abstract Hecke formula at a squared bad prime
Using Ralf's formula
Substituting given paramodular forms in abstract formula
Hecke spread 1 length and max det should be as desired: 2055, 700
Rank of T[G2G2]^+/- = (21,14)
Rank of plus basis attempt = 96 and dimension of S_4(K(218))^+ = 96
Rank of minus basis attempt = 14 and dimension of S_4(K(218))^- = 23

Hecke spread 2 out of 2
Is Hecke spread 2 present?  False
DoOneHecke called {},{{2, 2}, {2, 1}}
{tp,tpdel} = {2, 2}
DoOneHecke called {{2, 2}},{{2, 1}}
{tp,tpdel} = {2, 1}
Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 2055
Called ShortenVecs with G2G2T[{{2, 2}}]
 input dimensions {21, 2055}, output dimensions {21, 2055}
Length, max det of input vectors to ParaHeckeOp: 2055, 700 (check: 2055, 700)
Desired length, max det of output vectors: 234, 175
Quotient of the max dets should be 4
Smaller max det is the minimal max det 175
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}
About to compute G2G2T[{{2, 2}, {2, 1}}]
Making abstract Hecke formula at a bad prime
Using Ralf's formula
Substituting given paramodular forms in abstract formula
Hecke spread 2 length and max det should be as desired: 234, 175
Rank of T[G2G2]^+/- = (21,14)
Rank of plus basis attempt = 96 and dimension of S_4(K(218))^+ = 96
Rank of minus basis attempt = 17 and dimension of S_4(K(218))^- = 23

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-218-300.ma}
Dimensions of BPcoeffMatProven: {20, 566}
Dimensions of join: {37, 234}
Minus space span attempt may contain non-numerical entries
```

```
Rank of plus basis attempt = 96 and dimension of S_4(K(218))^+ = 96
Rank of minus basis attempt = 22 and dimension of S_4(K(218))^- = 23

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Found a plus basis of dimension 96
Didn't find a minus basis

N = 219
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}
lift dimension of S_4(K(219))^+ : 40
nonlift dimension of S_4(K(219))^+ : 58
dimension of S_4(K(219))^- : 13
Ibukiyama-Kitayama dimension of S_4(K(219)) : 111

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(219))^+ = 98
Rank of minus basis attempt = 0 and dimension of S_4(K(219))^- = 13
Initial short vector length: 219
Have vectors of length 224: long enough
Determinant shell containing the vectors of length 219 : 171
Short vector length is 224, and the corresponding determinant is 171
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-219-180.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,219}^{cusp}), which is 40
Rank of plus basis attempt = 40 and dimension of S_4(K(219))^+ = 98
Rank of minus basis attempt = 0 and dimension of S_4(K(219))^- = 13

STEP 2: TRACE DOWN
N = 219  q = 5  Nq = 1095
224-th determinant is 171
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 276
Trace down to try to hit S4(K(N)), of dimension 111
```

```
Plus lifts, plus nonlifts, minus dimension: {40, 58, 13}

Target plus rank, target minus rank: 98 13

Dimensions of savedTargetMats: {224, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1095-4275.ma, Getting it

 ... done making G2.  Dimensions of Grits: {23, 35616}

Determining indices:
 {1, 2, 5, 6, 9, 10, 17, 18, 19, 20, 45, 49, 50, 61, 62, 77, 78, 79, 80, 117, 118, 133, 157}
Atkin-Lehner truncation: 164

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 11}, {{1, -1, -1}, 6}, {{-1, 1, -1}, 4}, {{-1, -1, 1}, 2}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1095-4275-to-219-171.ma exists, reading it
Found file Grits-2-1095-4275.ma, Getting it

Grit dimensions: {23, 35616}

Viable file found: GG-Grits-2-1095-4275.ma-4275-mod-12347.ma

New best file: GG-Grits-2-1095-4275.ma-4275-mod-12347.ma

Dimensions of G2G2: {276, 35616}

Dimensions of fDown = {276, 224}

Rank of fDown mod pp = 94

Dimensions and rank of plus space fDown = {276, 224}  94

Dimensions and rank of minus space fDown = {276, 224}  0

Rank of plus basis attempt = 98 and dimension of S_4(K(219))^+ = 98

Rank of minus basis attempt = 0 and dimension of S_4(K(219))^- = 13
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 15

Found file Grits-2-219-171.ma, Getting it

Viable file found: GG-Grits-2-219-13851.ma-13851-mod-12347.ma

New best file: GG-Grits-2-219-13851.ma-13851-mod-12347.ma

Viable file found: GG-Grits-2-219-5375.ma-5375-mod-12347.ma

New best file: GG-Grits-2-219-5375.ma-5375-mod-12347.ma

Dimensions of G2G2: {15, 224}
```

```
Rank of plus basis attempt = 98 and dimension of S_4(K(219))^+ = 98

Rank of minus basis attempt = 0 and dimension of S_4(K(219))^- = 13
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 171 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 13 851

Hecke spreads: 1

Pre-Hecke expansions will have length 185 736

Found file Grits-2-219-13851.ma, Getting it

Viable file found: GG-Grits-2-219-13851.ma-13851-mod-12347.ma

New best file: GG-Grits-2-219-13851.ma-13851-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 185 736

Called ShortenVecs with G2G2T[{}]

 input dimensions {15, 185 736}, output dimensions {15, 185 736}

Length, max det of input vectors to ParaHeckeOp: 185 736, 13 851 (check: 185 736, 13 851)

Desired length, max det of output vectors: 224, 171

Quotient of the max dets should be 81

Smaller max det is the minimal max det 171
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 224, 171

Rank of T[G2G2]^+/- = (15,7)

Rank of plus basis attempt = 98 and dimension of S_4(K(219))^+ = 98
```

Rank of minus basis attempt = 7 and dimension of S_4(K(219))^- = 13

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-219-400-rank7.ma}

Dimensions of BPcoeffMatProven: {7, 773}

Dimensions of join: {14, 224}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 98 and dimension of S_4(K(219))^+ = 98

Rank of minus basis attempt = 13 and dimension of S_4(K(219))^- = 13

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 98

Found minus basis of dimension 13

N = 221

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 2}

lift dimension of S_4(K(221))^+ : 40

nonlift dimension of S_4(K(221))^+ : 55

dimension of S_4(K(221))^- : 9

Ibukiyama-Kitayama dimension of S_4(K(221)) : 104

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(221))^+ = 95

Rank of minus basis attempt = 0 and dimension of S_4(K(221))^- = 9

Initial short vector length: 196

Have vectors of length 228: long enough

Determinant shell containing the vectors of length 196 : 191

Adding determinant shell 1

Adding determinant shell 2

Short vector length is 228, and the corresponding determinant is 196

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-221-196.ma, Getting it

```
Rank of plus basis attempt should match dim(J_{4,221}^{cusp}), which is 40

Rank of plus basis attempt = 40 and dimension of S_4(K(221))^+ = 95

Rank of minus basis attempt = 0 and dimension of S_4(K(221))^- = 9


STEP 2: TRACE DOWN

N = 221  q = 5  Nq = 1105

228-th determinant is 196

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 666

Trace down to try to hit S4(K(N)), of dimension 104

Plus lifts, plus nonlifts, minus dimension: {40, 55, 9}

Target plus rank, target minus rank: 95 9

Dimensions of savedTargetMats: {228, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1105-4900.ma, Getting it

 ... done making G2.  Dimensions of Grits: {36, 36476}

Determining indices: {1, 2, 3, 4, 5, 6, 7, 8, 13, 14, 17, 18, 19, 20, 29, 30, 33,
  34, 41, 42, 43, 44, 57, 58, 69, 70, 81, 82, 89, 105, 106, 165, 166, 167, 217, 221}

Atkin-Lehner truncation: 240

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 13}, {{1, -1, -1}, 8}, {{-1, 1, -1}, 7}, {{-1, -1, 1}, 8}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1105-4900-to-221-196.ma exists, reading it

Found file Grits-2-1105-4900.ma, Getting it

Grit dimensions: {36, 36476}

Viable file found: GG-Grits-2-1105-4900.ma-4900-mod-12347.ma

New best file: GG-Grits-2-1105-4900.ma-4900-mod-12347.ma

Dimensions of G2G2: {666, 36476}

Dimensions of fDown = {666, 228}

Rank of fDown mod pp = 95

Dimensions and rank of plus space fDown = {666, 228}  95

Dimensions and rank of minus space fDown = {666, 228}  0

Rank of plus basis attempt = 95 and dimension of S_4(K(221))^+ = 95

Rank of minus basis attempt = 0 and dimension of S_4(K(221))^- = 9
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 15

Found file Grits-2-221-196.ma, Getting it

Viable file found: GG-Grits-2-221-6275.ma-6275-mod-12347.ma

New best file: GG-Grits-2-221-6275.ma-6275-mod-12347.ma

Viable file found: GG-Grits-2-221-6575.ma-6575-mod-12347.ma

Dimensions of G2G2: {15, 228}

Rank of plus basis attempt = 95 and dimension of S_4(K(221))^+ = 95

Rank of minus basis attempt = 0 and dimension of S_4(K(221))^- = 9

STEP 4: HECKE SPREAD

J2=0, or already have plus space basis and no extra Hecke spreads were specified

Hecke spreads: 0

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-221-500.ma}

Dimensions of BPcoeffMatProven: {6, 870}

Dimensions of join: {6, 228}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 95 and dimension of S_4(K(221))^+ = 95

Rank of minus basis attempt = 6 and dimension of S_4(K(221))^- = 9

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 95

Didn't find a minus basis

N = 222

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 20}

lift dimension of S_4(K(222))^+ : 35

nonlift dimension of S_4(K(222))^+ : 61

dimension of S_4(K(222))^- : 24

Ibukiyama-Kitayama dimension of S_4(K(222)) : 120

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(222))^+ = 96
Rank of minus basis attempt = 0 and dimension of S_4(K(222))^- = 24
Initial short vector length: 277
Have vectors of length 310: long enough
Determinant shell containing the vectors of length 277 : 188
Adding determinant shell 1
Adding determinant shell 2
Adding determinant shell 3
Adding determinant shell 4
Adding determinant shell 5
Adding determinant shell 6
Adding determinant shell 7
Adding determinant shell 8
Adding determinant shell 9
Adding determinant shell 10
Adding determinant shell 11
Adding determinant shell 12
Adding determinant shell 13
Adding determinant shell 14
Adding determinant shell 15
Adding determinant shell 16
Adding determinant shell 17
Adding determinant shell 18
Adding determinant shell 19
Adding determinant shell 20
Short vector length is 824, and the corresponding determinant is 344
Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-222-344.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,222}^{cusp}), which is 35
Rank of plus basis attempt = 35 and dimension of S_4(K(222))^+ = 96
Rank of minus basis attempt = 0 and dimension of S_4(K(222))^- = 24
```

STEP 2: TRACE DOWN

```
N = 222  q = 7  Nq = 1554

824-th determinant is 344

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 780

Trace down to try to hit S4(K(N)), of dimension 120

Plus lifts, plus nonlifts, minus dimension: {35, 61, 24}

Target plus rank, target minus rank: 96 24

Dimensions of savedTargetMats: {824, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1554-16856.ma, Getting it

 ... done making G2.  Dimensions of Grits: {39, 416268}

Determining indices: {1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 45, 46, 57, 58, 73, 77, 78, 79, 80, 101,
  102, 129, 130, 145, 146, 153, 154, 155, 156, 177, 178, 179, 217, 218, 219, 249, 250, 357, 661}

Atkin-Lehner truncation: 688

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims =
 {{{1, 1, 1, 1}, 14}, {{1, 1, -1, -1}, 10}, {{1, -1, 1, -1}, 4}, {{1, -1, -1, 1}, 4},
  {{-1, 1, 1, -1}, 3}, {{-1, 1, -1, 1}, 2}, {{-1, -1, 1, 1}, 1}, {{-1, -1, -1, -1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1554-16856-to-222-344.ma
  exists, reading it

Found file Grits-2-1554-16856.ma, Getting it

Grit dimensions: {39, 416268}

Viable file found: GG-Grits-2-1554-16856.ma-16856-mod-12347.ma

New best file: GG-Grits-2-1554-16856.ma-16856-mod-12347.ma

Dimensions of G2G2: {780, 416268}

Dimensions of fDown = {780, 824}

Rank of fDown mod pp = 95

Dimensions and rank of plus space fDown = {780, 824}  95

Dimensions and rank of minus space fDown = {780, 824}  0

Rank of plus basis attempt = 95 and dimension of S_4(K(222))^+ = 96

Rank of minus basis attempt = 0 and dimension of S_4(K(222))^- = 24
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 1
```

```
Found file Grits-2-222-344.ma, Getting it

Viable file found: GG-Grits-2-222-752.ma-752-mod-12347.ma

New best file: GG-Grits-2-222-752.ma-752-mod-12347.ma

Dimensions of G2G2: {1, 824}

Rank of plus basis attempt = 95 and dimension of S_4(K(222))^+ = 96

Rank of minus basis attempt = 0 and dimension of S_4(K(222))^- = 24
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 0 Hecke spreads, the last of which dilates determinants by 1

Need to multiply the minimal max det 344 by the largest det contraction factor
 1 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 344

Hecke spreads: 0
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-222-500-5-0.ma}

Dimensions of BPcoeffMatProven: {24, 1520}

Dimensions of join: {24, 824}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 95 and dimension of S_4(K(222))^+ = 96

Rank of minus basis attempt = 24 and dimension of S_4(K(222))^- = 24
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {BPinfo-files/BPplusInfo-4-222-700-merge.ma}

(from inside plusInfo file) Dimensions of JSglobalBasisAttempt: {95, 824}

(from inside plusInfo file) Rank of JSglobalBasisAttempt: 95

(from inside plusInfo file) Dimensions of BPcoeffMatProven: {42, 2534}

(from inside plusInfo file) Rank of BPcoeffMatProven's numerical columns: 42

(from inside plusInfo file) Dimensions of BPcoeffMatProven after join: {137, 824}

(from inside plusInfo file) Dimensions of numerical BPcoeffMatProven after join:
 {137, 173}

(from inside plusInfo file) Rank of numerical BPcoeffMatProven after join: 96

New dimensions of JSglobalBasisAttempt: {137, 824}

Plus space span attempt may contain non-numerical entries

Minus space span attempt may contain non-numerical entries
```

```
Rank of plus basis attempt = 96 and dimension of S_4(K(222))^+ = 96
Rank of minus basis attempt = 24 and dimension of S_4(K(222))^- = 24

Found a plus basis of dimension 96
Found minus basis of dimension 24

N = 226
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 0}
lift dimension of S_4(K(226))^+ : 43
nonlift dimension of S_4(K(226))^+ : 75
dimension of S_4(K(226))^- : 16
Ibukiyama-Kitayama dimension of S_4(K(226)) : 134

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(226))^+ = 118
Rank of minus basis attempt = 0 and dimension of S_4(K(226))^- = 16
Initial short vector length: 244
Have vectors of length 260: long enough
Determinant shell containing the vectors of length 244 : 176
Short vector length is 260, and the corresponding determinant is 176
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-226-180.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,226}^{cusp}), which is 43
Rank of plus basis attempt = 43 and dimension of S_4(K(226))^+ = 118
Rank of minus basis attempt = 0 and dimension of S_4(K(226))^- = 16

STEP 2: TRACE DOWN
N = 226  q = 7  Nq = 1582
260-th determinant is 176
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 946
Trace down to try to hit S4(K(N)), of dimension 134
Plus lifts, plus nonlifts, minus dimension: {43, 75, 16}
Target plus rank, target minus rank: 118 16
Dimensions of savedTargetMats: {260, 3}
```

```
Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1582-8624.ma, Getting it

 ... done making G2.  Dimensions of Grits: {43, 111932}

Determining indices:
 {1, 2, 3, 7, 8, 9, 10, 19, 20, 23, 27, 28, 37, 38, 39, 40, 61, 62, 73, 74, 75, 76, 105, 113, 114,
  137, 138, 139, 140, 177, 178, 193, 194, 207, 215, 216, 235, 236, 291, 339, 359, 375, 435}

Atkin-Lehner truncation: 462

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 19}, {{1, -1, -1}, 11}, {{-1, 1, -1}, 9}, {{-1, -1, 1}, 4}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1582-8624-to-226-176.ma exists, reading it

Found file Grits-2-1582-8624.ma, Getting it

Grit dimensions: {43, 111932}

Viable file found: GG-Grits-2-1582-10192.ma-10192-mod-12347.ma

New best file: GG-Grits-2-1582-10192.ma-10192-mod-12347.ma

Viable file found: GG-Grits-2-1582-10388.ma-10388-mod-12347.ma

Viable file found: GG-Grits-2-1582-11760.ma-11760-mod-12347.ma

Viable file found: GG-Grits-2-1582-12152.ma-12152-mod-12347.ma

Viable file found: GG-Grits-2-1582-13524.ma-13524-mod-12347.ma

Viable file found: GG-Grits-2-1582-14847.ma-14847-mod-12347.ma

Viable file found: GG-Grits-2-1582-16268.ma-16268-mod-12347.ma

Viable file found: GG-Grits-2-1582-8624.ma-8624-mod-12347.ma

New best file: GG-Grits-2-1582-8624.ma-8624-mod-12347.ma

Viable file found: GG-Grits-2-1582-9604.ma-9604-mod-12347.ma

Viable file found: GG-Grits-2-1582-9800.ma-9800-mod-12347.ma

Viable file found: GG-Grits-2-1582-9996.ma-9996-mod-12347.ma

Dimensions of G2G2: {946, 111932}

Dimensions of fDown = {946, 260}

Rank of fDown mod pp = 115

Dimensions and rank of plus space fDown = {946, 260}  115

Dimensions and rank of minus space fDown = {946, 260}  0

Rank of plus basis attempt = 117 and dimension of S_4(K(226))^+ = 118

Rank of minus basis attempt = 0 and dimension of S_4(K(226))^- = 16
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 21

Found file Grits-2-226-176.ma, Getting it

Viable file found: GG-Grits-2-226-176.ma-176-mod-12347.ma

New best file: GG-Grits-2-226-176.ma-176-mod-12347.ma

Viable file found: GG-Grits-2-226-45056.ma-45056-mod-12347.ma

Viable file found: GG-Grits-2-226-51200.ma-51200-mod-12347.ma

Viable file found: GG-Grits-2-226-704.ma-704-mod-12347.ma

Dimensions of G2G2: {21, 260}

Rank of plus basis attempt = 117 and dimension of S_4(K(226))^+ = 118

Rank of minus basis attempt = 0 and dimension of S_4(K(226))^- = 16

STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 16

Need to multiply the minimal max det 176 by the largest det contraction factor
 16 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 2816

Hecke spreads: 1

Pre-Hecke expansions will have length 18 665

Found file Grits-2-226-2816.ma, Getting it

Viable file found: GG-Grits-2-226-45056.ma-45056-mod-12347.ma

New best file: GG-Grits-2-226-45056.ma-45056-mod-12347.ma

Viable file found: GG-Grits-2-226-51200.ma-51200-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present? False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 1 241 815

Called ShortenVecs with G2G2T[{}]

 input dimensions {21, 1 241 815}, output dimensions {21, 18 665}

Length, max det of input vectors to ParaHeckeOp: 18 665, 2816 (check: 18 665, 2816)

```
Desired length, max det of output vectors: 260, 176

Quotient of the max dets should be 16

Smaller max det is the minimal max det 176
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 260, 176

Rank of T[G2G2]^+/- = (21,12)

Rank of plus basis attempt = 117 and dimension of S_4(K(226))^+ = 118

Rank of minus basis attempt = 12 and dimension of S_4(K(226))^- = 16


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {}


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Didn't find a plus basis

Didn't find a minus basis


N = 230

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}

lift dimension of S_4(K(230))^+ : 37

nonlift dimension of S_4(K(230))^+ : 60

dimension of S_4(K(230))^- : 25

Ibukiyama-Kitayama dimension of S_4(K(230)) : 122


STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(230))^+ = 97

Rank of minus basis attempt = 0 and dimension of S_4(K(230))^- = 25

Initial short vector length: 280

Have vectors of length 354: long enough

Determinant shell containing the vectors of length 280 : 191

Short vector length is 318, and the corresponding determinant is 191
```

```
Vectors fill out a determinant shell, as they should


STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-230-200.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,230}^{cusp}), which is 37

Rank of plus basis attempt = 37 and dimension of S_4(K(230))^+ = 97

Rank of minus basis attempt = 0 and dimension of S_4(K(230))^- = 25


STEP 2: TRACE DOWN

N = 230  q = 7  Nq = 1610

318-th determinant is 191

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 861

Trace down to try to hit S4(K(N)), of dimension 122

Plus lifts, plus nonlifts, minus dimension: {37, 60, 25}

Target plus rank, target minus rank: 97 25

Dimensions of savedTargetMats: {318, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1610-9359.ma, Getting it

 ... done making G2.  Dimensions of Grits: {41, 150 228}

Determining indices:
 {1, 2, 5, 6, 9, 10, 17, 18, 19, 20, 41, 42, 61, 62, 69, 70, 71, 72, 73, 74, 75, 76, 133, 134,
  135, 136, 149, 150, 151, 152, 153, 154, 229, 230, 249, 273, 274, 285, 286, 287, 453}

Atkin-Lehner truncation: 468

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims =
 {{{1, 1, 1, 1}, 13}, {{1, 1, -1, -1}, 9}, {{1, -1, 1, -1}, 7}, {{1, -1, -1, 1}, 6},
  {{-1, 1, 1, -1}, 2}, {{-1, 1, -1, 1}, 2}, {{-1, -1, 1, 1}, 1}, {{-1, -1, -1, -1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1610-9359-to-230-191.ma exists, reading it

Found file Grits-2-1610-9359.ma, Getting it

Grit dimensions: {41, 150 228}

Viable file found: GG-Grits-2-1610-9359.ma-9359-mod-12347.ma

New best file: GG-Grits-2-1610-9359.ma-9359-mod-12347.ma

Dimensions of G2G2: {861, 150 228}
```

```
Dimensions of fDown = {861, 318}

Rank of fDown mod pp = 94

Dimensions and rank of plus space fDown = {861, 318}  94

Dimensions and rank of minus space fDown = {861, 318}  0

Rank of plus basis attempt = 96 and dimension of S_4(K(230))^+ = 97

Rank of minus basis attempt = 0 and dimension of S_4(K(230))^- = 25
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-230-191.ma, Getting it

Viable file found: GG-Grits-2-230-191.ma-191-mod-12347.ma

New best file: GG-Grits-2-230-191.ma-191-mod-12347.ma

Viable file found: GG-Grits-2-230-27504.ma-27504-mod-12347.ma

Viable file found: GG-Grits-2-230-764.ma-764-mod-12347.ma

Dimensions of G2G2: {3, 318}

Rank of plus basis attempt = 96 and dimension of S_4(K(230))^+ = 97

Rank of minus basis attempt = 0 and dimension of S_4(K(230))^- = 25
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 3 Hecke spreads, the last of which dilates determinants by 144

Need to multiply the minimal max det 191 by the largest det contraction factor
 144 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 27504

Hecke spreads: 3

Pre-Hecke expansions will have length 720846

Found file Grits-2-230-27504.ma, Getting it

Viable file found: GG-Grits-2-230-27504.ma-27504-mod-12347.ma

New best file: GG-Grits-2-230-27504.ma-27504-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 9

{{2, 2}, {2, 1}} 1

{{2, 2}, {3, 1}} 1

Hecke spread 1 out of 3

Is Hecke spread 1 present?  False
```

```
DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 720 846

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 720 846}, output dimensions {3, 720 846}

Length, max det of input vectors to ParaHeckeOp: 720 846, 27 504 (check: 720 846, 27 504)

Desired length, max det of output vectors: 9958, 1719

Quotient of the max dets should be 16

Smaller max det is the minimal max det 191
  times the biggest postpended det inverse dilation factor
 9 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 9958, 1719

Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 96 and dimension of S_4(K(230))^+ = 97

Rank of minus basis attempt = 3 and dimension of S_4(K(230))^- = 25


Hecke spread 2 out of 3

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 9958

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {3, 9958}, output dimensions {3, 2780}

Length, max det of input vectors to ParaHeckeOp: 2780, 764 (check: 2780, 764)

Desired length, max det of output vectors: 318, 191

Quotient of the max dets should be 4

Smaller max det is the minimal max det 191
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula
```

```
Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 318, 191

Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 96 and dimension of S_4(K(230))^+ = 97

Rank of minus basis attempt = 6 and dimension of S_4(K(230))^- = 25


Hecke spread 3 out of 3

Is Hecke spread 3 present?  False

DoOneHecke called {},{{2, 2}, {3, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{3, 1}}

{tp,tpdel} = {3, 1}

Need to compute G2G2T[{{2, 2}, {3, 1}}] having G2G2T[{{2, 2}}] vectors to length 9958

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {3, 9958}, output dimensions {3, 9958}

Length, max det of input vectors to ParaHeckeOp: 9958, 1719 (check: 9958, 1719)

Desired length, max det of output vectors: 318, 191

Quotient of the max dets should be 9

Smaller max det is the minimal max det 191
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {3, 1}}

About to compute G2G2T[{{2, 2}, {3, 1}}]

Making abstract Hecke formula

Substituting given paramodular forms in abstract formula

Hecke spread 3 length and max det should be as desired: 318, 191

Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 96 and dimension of S_4(K(230))^+ = 97

Rank of minus basis attempt = 7 and dimension of S_4(K(230))^- = 25


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-230-410.ma}

Dimensions of BPcoeffMatProven: {22, 958}

Dimensions of join: {29, 318}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 96 and dimension of S_4(K(230))^+ = 97

Rank of minus basis attempt = 25 and dimension of S_4(K(230))^- = 25
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Didn't find a plus basis

Found minus basis of dimension 25

N = 231

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}

lift dimension of S_4(K(231))^+ : 36

nonlift dimension of S_4(K(231))^+ : 59

dimension of S_4(K(231))^- : 11

Ibukiyama-Kitayama dimension of S_4(K(231)) : 106

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(231))^+ = 95

Rank of minus basis attempt = 0 and dimension of S_4(K(231))^- = 11

Initial short vector length: 225

Have vectors of length 232: long enough

Determinant shell containing the vectors of length 225 : 195

Short vector length is 232, and the corresponding determinant is 195

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-231-200.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,231}^{cusp}), which is 36

Rank of plus basis attempt = 36 and dimension of S_4(K(231))^+ = 95

Rank of minus basis attempt = 0 and dimension of S_4(K(231))^- = 11

STEP 2: TRACE DOWN

N = 231 q = 5 Nq = 1155

232-th determinant is 195

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 300

Trace down to try to hit S4(K(N)), of dimension 106

Plus lifts, plus nonlifts, minus dimension: {36, 59, 11}

Target plus rank, target minus rank: 95 11

Dimensions of savedTargetMats: {232, 3}

```
Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1155-4875.ma, Getting it

 ... done making G2.  Dimensions of Grits: {24, 50 048}

Determining indices:
 {1, 2, 3, 4, 9, 10, 13, 14, 21, 22, 23, 24, 45, 46, 47, 48, 49, 50, 51, 52, 85, 101, 197, 198}

Atkin-Lehner truncation: 204

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims =
 {{{1, 1, 1, 1}, 8}, {{1, 1, -1, -1}, 4}, {{1, -1, 1, -1}, 4}, {{1, -1, -1, 1}, 3},
  {{-1, 1, 1, -1}, 2}, {{-1, 1, -1, 1}, 1}, {{-1, -1, 1, 1}, 1}, {{-1, -1, -1, -1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1155-4875-to-231-195.ma exists, reading it

Found file Grits-2-1155-4875.ma, Getting it

Grit dimensions: {24, 50 048}

Viable file found: GG-Grits-2-1155-11616.ma-11616-mod-12347.ma

New best file: GG-Grits-2-1155-11616.ma-11616-mod-12347.ma

Dimensions of G2G2: {300, 193 580}

Dimensions of fDown = {300, 232}

Rank of fDown mod pp = 95

Dimensions and rank of plus space fDown = {300, 232}  95

Dimensions and rank of minus space fDown = {300, 232}  0

Rank of plus basis attempt = 95 and dimension of S_4(K(231))^+ = 95

Rank of minus basis attempt = 0 and dimension of S_4(K(231))^- = 11


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 1

Found file Grits-2-231-195.ma, Getting it

Viable file found: GG-Grits-2-231-195.ma-195-mod-12347.ma

New best file: GG-Grits-2-231-195.ma-195-mod-12347.ma

Dimensions of G2G2: {1, 232}

Rank of plus basis attempt = 95 and dimension of S_4(K(231))^+ = 95

Rank of minus basis attempt = 0 and dimension of S_4(K(231))^- = 11
```

STEP 4: HECKE SPREAD

J2=0, or already have plus space basis and no extra Hecke spreads were specified

Hecke spreads: 0

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-231-500.ma}

Dimensions of BPcoeffMatProven: {11, 1064}

Dimensions of join: {11, 232}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 95 and dimension of S_4(K(231))^+ = 95

Rank of minus basis attempt = 11 and dimension of S_4(K(231))^- = 11

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 95

Found minus basis of dimension 11

N = 235

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}

lift dimension of S_4(K(235))^+ : 44

nonlift dimension of S_4(K(235))^+ : 70

dimension of S_4(K(235))^- : 11

Ibukiyama-Kitayama dimension of S_4(K(235)) : 125

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(235))^+ = 114

Rank of minus basis attempt = 0 and dimension of S_4(K(235))^- = 11

Initial short vector length: 220

Have vectors of length 222: long enough

Determinant shell containing the vectors of length 220 : 180

Short vector length is 222, and the corresponding determinant is 180

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-235-180.ma, Getting it

```
Rank of plus basis attempt should match dim(J_{4,235}^{cusp}), which is 44

Rank of plus basis attempt = 44 and dimension of S_4(K(235))^+ = 114

Rank of minus basis attempt = 0 and dimension of S_4(K(235))^- = 11


STEP 2: TRACE DOWN

N = 235  q = 7  Nq = 1645

222-th determinant is 180

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 1485

Trace down to try to hit S4(K(N)), of dimension 125

Plus lifts, plus nonlifts, minus dimension: {44, 70, 11}

Target plus rank, target minus rank: 114 11

Dimensions of savedTargetMats: {222, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1645-8820.ma, Getting it

 ... done making G2.  Dimensions of Grits: {54, 92 912}

Determining indices: {1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 21, 23, 24, 27, 28, 29, 30, 43,
  44, 55, 56, 59, 60, 61, 62, 83, 84, 85, 86, 107, 108, 109, 110, 119, 127, 128, 139,
  141, 171, 172, 173, 174, 191, 192, 203, 204, 205, 206, 239, 240, 319, 320, 321, 411}

Atkin-Lehner truncation: 414

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 19}, {{1, -1, -1}, 13}, {{-1, 1, -1}, 12}, {{-1, -1, 1}, 10}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1645-8820-to-235-180.ma exists, reading it

Found file Grits-2-1645-8820.ma, Getting it

Grit dimensions: {54, 92 912}

Viable file found: GG-Grits-2-1645-8820.ma-8820-mod-12347.ma

New best file: GG-Grits-2-1645-8820.ma-8820-mod-12347.ma

Dimensions of G2G2: {1485, 92 912}

Dimensions of fDown = {1485, 222}

Rank of fDown mod pp = 112

Dimensions and rank of plus space fDown = {1485, 222}  112

Dimensions and rank of minus space fDown = {1485, 222}  0

Rank of plus basis attempt = 114 and dimension of S_4(K(235))^+ = 114
```

```
Rank of minus basis attempt = 0 and dimension of S_4(K(235))^- = 11
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 21
Found file Grits-2-235-180.ma, Getting it
Viable file found: GG-Grits-2-235-17664.ma-17664-mod-12347.ma
New best file: GG-Grits-2-235-17664.ma-17664-mod-12347.ma
Viable file found: GG-Grits-2-235-6900.ma-6900-mod-12347.ma
New best file: GG-Grits-2-235-6900.ma-6900-mod-12347.ma
Dimensions of G2G2: {21, 222}
Rank of plus basis attempt = 114 and dimension of S_4(K(235))^+ = 114
Rank of minus basis attempt = 0 and dimension of S_4(K(235))^- = 11
```

STEP 4: HECKE SPREAD

```
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-235-400.ma}
Dimensions of BPcoeffMatProven: {7, 728}
Dimensions of join: {7, 222}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 114 and dimension of S_4(K(235))^+ = 114
Rank of minus basis attempt = 7 and dimension of S_4(K(235))^- = 11
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Found a plus basis of dimension 114
Didn't find a minus basis
```

N = 237

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}
lift dimension of S_4(K(237))^+ : 43
nonlift dimension of S_4(K(237))^+ : 69
dimension of S_4(K(237))^- : 19
```

```
Ibukiyama-Kitayama dimension of S_4(K(237)) : 131

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(237))^+ = 112
Rank of minus basis attempt = 0 and dimension of S_4(K(237))^- = 19
Initial short vector length: 347
Have vectors of length 368: long enough
Determinant shell containing the vectors of length 347 : 236
Short vector length is 360, and the corresponding determinant is 236
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-237-240.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,237}^{cusp}), which is 43
Rank of plus basis attempt = 43 and dimension of S_4(K(237))^+ = 112
Rank of minus basis attempt = 0 and dimension of S_4(K(237))^- = 19

STEP 2: TRACE DOWN
N = 237  q = 5  Nq = 1185
360-th determinant is 236
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 561
Trace down to try to hit S4(K(N)), of dimension 131
Plus lifts, plus nonlifts, minus dimension: {43, 69, 19}
Target plus rank, target minus rank: 112 19
Dimensions of savedTargetMats: {360, 3}
Making ordered good sigs for source space S_4(K(Nq))...
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
Found file Grits-2-1185-5900.ma, Getting it
 ... done making G2.  Dimensions of Grits: {33, 58472}
Determining indices: {1, 3, 4, 7, 8, 11, 12, 19, 20, 21, 22, 35, 36, 37, 38,
  47, 51, 52, 53, 54, 79, 83, 84, 95, 97, 119, 123, 137, 138, 153, 154, 169, 170}
Atkin-Lehner truncation: 200
Beginning ALSignaturesAndDims...
 ... done with ALSignaturesAndDims.
ALspacesDims = {{{1, 1, 1}, 15}, {{1, -1, -1}, 7}, {{-1, 1, -1}, 7}, {{-1, -1, 1}, 4}}
Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}
```

```
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1185-5900-to-237-236.ma exists, reading it
Found file Grits-2-1185-5900.ma, Getting it
Grit dimensions: {33, 58 472}
Viable file found: GG-Grits-2-1185-5900.ma-5900-mod-12347.ma
New best file: GG-Grits-2-1185-5900.ma-5900-mod-12347.ma
Dimensions of G2G2: {561, 58 472}
Dimensions of fDown = {561, 360}
Rank of fDown mod pp = 110
Dimensions and rank of plus space fDown = {561, 360}  110
Dimensions and rank of minus space fDown = {561, 360}  0
Rank of plus basis attempt = 112 and dimension of S_4(K(237))^+ = 112
Rank of minus basis attempt = 0 and dimension of S_4(K(237))^- = 19

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS
Pairs from dim(J_{2,N}^{cusp}: 15
Found file Grits-2-237-236.ma, Getting it
Viable file found: GG-Grits-2-237-19116.ma-19116-mod-12347.ma
New best file: GG-Grits-2-237-19116.ma-19116-mod-12347.ma
Viable file found: GG-Grits-2-237-236.ma-236-mod-12347.ma
New best file: GG-Grits-2-237-236.ma-236-mod-12347.ma
Dimensions of G2G2: {15, 360}
Rank of plus basis attempt = 112 and dimension of S_4(K(237))^+ = 112
Rank of minus basis attempt = 0 and dimension of S_4(K(237))^- = 19

STEP 4: HECKE SPREAD
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified
Found custom table of Hecke spreads
Preparing for 1 Hecke spreads, the last of which dilates determinants by 81
Need to multiply the minimal max det 236 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det
Need pre-Hecke expansions up to determinant 19 116
Hecke spreads: 1
Pre-Hecke expansions will have length 302 478
Found file Grits-2-237-19116.ma, Getting it
Viable file found: GG-Grits-2-237-19116.ma-19116-mod-12347.ma
New best file: GG-Grits-2-237-19116.ma-19116-mod-12347.ma
```

```
Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 302 478

Called ShortenVecs with G2G2T[{}]

 input dimensions {15, 302 478}, output dimensions {15, 302 478}

Length, max det of input vectors to ParaHeckeOp: 302 478, 19 116 (check: 302 478, 19 116)

Desired length, max det of output vectors: 360, 236

Quotient of the max dets should be 81

Smaller max det is the minimal max det 236
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 360, 236

Rank of T[G2G2]^+/- = (15,8)

Rank of plus basis attempt = 112 and dimension of S_4(K(237))^+ = 112

Rank of minus basis attempt = 8 and dimension of S_4(K(237))^- = 19


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-237-500.ma}

Dimensions of BPcoeffMatProven: {8, 1024}

Dimensions of join: {16, 360}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 112 and dimension of S_4(K(237))^+ = 112

Rank of minus basis attempt = 16 and dimension of S_4(K(237))^- = 19


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Found a plus basis of dimension 112
```

```
Didn't find a minus basis

N = 238
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {3, 1, 0}
lift dimension of S_4(K(238))^+ : 39
nonlift dimension of S_4(K(238))^+ : 75
dimension of S_4(K(238))^- : 20
Ibukiyama-Kitayama dimension of S_4(K(238)) : 134

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(238))^+ = 114
Rank of minus basis attempt = 0 and dimension of S_4(K(238))^- = 20
Initial short vector length: 275
Have vectors of length 298: long enough
Determinant shell containing the vectors of length 275 : 220
Short vector length is 298, and the corresponding determinant is 220
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-238-220.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,238}^{cusp}), which is 39
Rank of plus basis attempt = 39 and dimension of S_4(K(238))^+ = 114
Rank of minus basis attempt = 0 and dimension of S_4(K(238))^- = 20

STEP 2: TRACE DOWN
N = 238  q = 3  Nq = 714
298-th determinant is 220
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 78
Trace down to try to hit S4(K(N)), of dimension 134
Plus lifts, plus nonlifts, minus dimension: {39, 75, 20}
Target plus rank, target minus rank: 114 20
Dimensions of savedTargetMats: {298, 3}
Making ordered good sigs for source space S_4(K(Nq))...
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
Found file Grits-2-714-1980.ma, Getting it
```

```
 ... done making G2.  Dimensions of Grits: {12, 14 732}

Determining indices: {1, 2, 3, 4, 5, 7, 41, 49, 53, 54, 153, 161}

Atkin-Lehner truncation: 184

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1, 1}, 5}, {{1, 1, -1, -1}, 2},
  {{1, -1, 1, -1}, 2}, {{1, -1, -1, 1}, 1}, {{-1, 1, 1, -1}, 1}, {{-1, 1, -1, 1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-714-1980-to-238-220.ma exists, reading it

Found file Grits-2-714-1980.ma, Getting it

Grit dimensions: {12, 14 732}

Viable file found: GG-Grits-2-714-4116.ma-4116-mod-12347.ma

New best file: GG-Grits-2-714-4116.ma-4116-mod-12347.ma

Viable file found: GG-Grits-2-714-4263.ma-4263-mod-12347.ma

Dimensions of G2G2: {78, 48 472}

Dimensions of fDown = {78, 298}

Rank of fDown mod pp = 51

Dimensions and rank of plus space fDown = {78, 298}  51

Dimensions and rank of minus space fDown = {78, 298}  0

Rank of plus basis attempt = 84 and dimension of S_4(K(238))^+ = 114

Rank of minus basis attempt = 0 and dimension of S_4(K(238))^- = 20


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-238-220.ma, Getting it

Viable file found: GG-Grits-2-238-14080.ma-14080-mod-12347.ma

New best file: GG-Grits-2-238-14080.ma-14080-mod-12347.ma

Viable file found: GG-Grits-2-238-220.ma-220-mod-12347.ma

New best file: GG-Grits-2-238-220.ma-220-mod-12347.ma

Viable file found: GG-Grits-2-238-31680.ma-31680-mod-12347.ma

Viable file found: GG-Grits-2-238-63580.ma-63580-mod-12347.ma

Dimensions of G2G2: {3, 298}

Rank of plus basis attempt = 85 and dimension of S_4(K(238))^+ = 114

Rank of minus basis attempt = 0 and dimension of S_4(K(238))^- = 20


STEP 4: HECKE SPREAD
```

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 3 Hecke spreads, the last of which dilates determinants by 144

Need to multiply the minimal max det 220 by the largest det contraction factor
 144 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 31 680

Hecke spreads: 3

Pre-Hecke expansions will have length 861 602

Found file Grits-2-238-31680.ma, Getting it

Viable file found: GG-Grits-2-238-31680.ma-31680-mod-12347.ma

New best file: GG-Grits-2-238-31680.ma-31680-mod-12347.ma

Viable file found: GG-Grits-2-238-63580.ma-63580-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 9

{{2, 2}, {2, 1}} 1

{{2, 2}, {3, 1}} 1

Hecke spread 1 out of 3

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 861 602

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 861 602}, output dimensions {3, 861 602}

Length, max det of input vectors to ParaHeckeOp: 861 602, 31 680 (check: 861 602, 31 680)

Desired length, max det of output vectors: 12 222, 1980

Quotient of the max dets should be 16

Smaller max det is the minimal max det 220
  times the biggest postpended det inverse dilation factor
 9 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 12 222, 1980

Rank of T[G2G2]^+/- = (3,2)

Rank of plus basis attempt = 86 and dimension of S_4(K(238))^+ = 114

Rank of minus basis attempt = 2 and dimension of S_4(K(238))^- = 20
```

```
Hecke spread 2 out of 3

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 12 222

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {3, 12 222}, output dimensions {3, 3300}

Length, max det of input vectors to ParaHeckeOp: 3300, 880 (check: 3300, 880)

Desired length, max det of output vectors: 298, 220

Quotient of the max dets should be 4

Smaller max det is the minimal max det 220
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}
About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 298, 220

Rank of T[G2G2]^+/- = (3,2)

Rank of plus basis attempt = 88 and dimension of S_4(K(238))^+ = 114

Rank of minus basis attempt = 4 and dimension of S_4(K(238))^- = 20


Hecke spread 3 out of 3

Is Hecke spread 3 present?  False

DoOneHecke called {},{{2, 2}, {3, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{3, 1}}

{tp,tpdel} = {3, 1}

Need to compute G2G2T[{{2, 2}, {3, 1}}] having G2G2T[{{2, 2}}] vectors to length 12 222

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {3, 12 222}, output dimensions {3, 12 222}

Length, max det of input vectors to ParaHeckeOp: 12 222, 1980 (check: 12 222, 1980)

Desired length, max det of output vectors: 298, 220

Quotient of the max dets should be 9
```

```
Smaller max det is the minimal max det 220
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {3, 1}}
About to compute G2G2T[{{2, 2}, {3, 1}}]
Making abstract Hecke formula
Substituting given paramodular forms in abstract formula
Hecke spread 3 length and max det should be as desired: 298, 220
Rank of T[G2G2]^+/- = (3,2)
Rank of plus basis attempt = 89 and dimension of S_4(K(238))^+ = 114
Rank of minus basis attempt = 5 and dimension of S_4(K(238))^- = 20

SPECIAL STEP FOR N=238 ONLY: MERGE IN THE POSITIVE BASIS ATTEMPT FOR q=5
Rank of plus basis attempt = 113 and dimension of S_4(K(238))^+ = 114
Rank of minus basis attempt = 5 and dimension of S_4(K(238))^- = 20

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-238-501-400-merged.ma}
Dimensions of BPcoeffMatProven: {38, 894}
Dimensions of join: {43, 298}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 113 and dimension of S_4(K(238))^+ = 114
Rank of minus basis attempt = 20 and dimension of S_4(K(238))^- = 20

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Didn't find a plus basis
Found minus basis of dimension 20

N = 246
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 0}
lift dimension of S_4(K(246))^+ : 42
nonlift dimension of S_4(K(246))^+ : 82
dimension of S_4(K(246))^- : 25
Ibukiyama-Kitayama dimension of S_4(K(246)) : 149

STEP 0: SET UP
```

```
Rank of plus basis attempt = 0 and dimension of S_4(K(246))^+ = 124

Rank of minus basis attempt = 0 and dimension of S_4(K(246))^- = 25

Initial short vector length: 346

Have vectors of length 358: long enough

Determinant shell containing the vectors of length 346 : 207

Short vector length is 358, and the corresponding determinant is 207

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-246-210.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,246}^{cusp}), which is 42

Rank of plus basis attempt = 42 and dimension of S_4(K(246))^+ = 124

Rank of minus basis attempt = 0 and dimension of S_4(K(246))^- = 25
```

STEP 2: TRACE DOWN

```
N = 246  q = 7  Nq = 1722

358-th determinant is 207

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 946

Trace down to try to hit S4(K(N)), of dimension 149

Plus lifts, plus nonlifts, minus dimension: {42, 82, 25}

Target plus rank, target minus rank: 124 25

Dimensions of savedTargetMats: {358, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1722-10143.ma, Getting it

 ... done making G2.  Dimensions of Grits: {43, 188608}

Determining indices: {1, 2, 3, 4, 9, 10, 11, 12, 49, 53, 54, 55,
  56, 77, 78, 79, 80, 117, 118, 125, 126, 127, 128, 129, 130, 205, 206, 207,
  208, 245, 261, 265, 266, 289, 290, 291, 292, 401, 402, 693, 694, 709, 861}

Atkin-Lehner truncation: 932

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1, 1}, 15}, {{1, 1, -1, -1}, 9}, {{1, -1, 1, -1}, 7},
  {{1, -1, -1, 1}, 4}, {{-1, 1, 1, -1}, 4}, {{-1, 1, -1, 1}, 3}, {{-1, -1, 1, 1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}
```

```
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1722-10143-to-246-207.ma
  exists, reading it
Found file Grits-2-1722-10143.ma, Getting it
Grit dimensions: {43, 188 608}
Viable file found: GG-Grits-2-1722-10143.ma-10143-mod-12347.ma
New best file: GG-Grits-2-1722-10143.ma-10143-mod-12347.ma
Dimensions of G2G2: {946, 188 608}
Dimensions of fDown = {946, 358}
Rank of fDown mod pp = 119
Dimensions and rank of plus space fDown = {946, 358}  119
Dimensions and rank of minus space fDown = {946, 358}  0
Rank of plus basis attempt = 121 and dimension of S_4(K(246))^+ = 124
Rank of minus basis attempt = 0 and dimension of S_4(K(246))^- = 25
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 10
Found file Grits-2-246-207.ma, Getting it
Viable file found: GG-Grits-2-246-13248.ma-13248-mod-12347.ma
New best file: GG-Grits-2-246-13248.ma-13248-mod-12347.ma
Viable file found: GG-Grits-2-246-207.ma-207-mod-12347.ma
New best file: GG-Grits-2-246-207.ma-207-mod-12347.ma
Viable file found: GG-Grits-2-246-29808.ma-29808-mod-12347.ma
Dimensions of G2G2: {10, 358}
Rank of plus basis attempt = 121 and dimension of S_4(K(246))^+ = 124
Rank of minus basis attempt = 0 and dimension of S_4(K(246))^- = 25
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified
Found custom table of Hecke spreads
Preparing for 4 Hecke spreads, the last of which dilates determinants by 144
Need to multiply the minimal max det 207 by the largest det contraction factor
 144 under Hecke spreads to get the pre-Hecke max det
Need pre-Hecke expansions up to determinant 29 808
Hecke spreads: 4
Pre-Hecke expansions will have length 892 322
Found file Grits-2-246-29808.ma, Getting it
```

```
Viable file found: GG-Grits-2-246-29808.ma-29808-mod-12347.ma

New best file: GG-Grits-2-246-29808.ma-29808-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 9

{{2, 2}, {2, 1}} 1

{{3, 2}} 1

{{2, 2}, {3, 1}} 1

Hecke spread 1 out of 4

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 892 322

Called ShortenVecs with G2G2T[{}]

 input dimensions {10, 892 322}, output dimensions {10, 892 322}

Length, max det of input vectors to ParaHeckeOp: 892 322, 29 808 (check: 892 322, 29 808)

Desired length, max det of output vectors: 12 514, 1863

Quotient of the max dets should be 16

Smaller max det is the minimal max det 207
  times the biggest postpended det inverse dilation factor
 9 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 12 514, 1863

Rank of T[G2G2]^+/- = (10,7)

Rank of plus basis attempt = 121 and dimension of S_4(K(246))^+ = 124

Rank of minus basis attempt = 7 and dimension of S_4(K(246))^- = 25


Hecke spread 2 out of 4

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 12 514
```

```
Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {10, 12514}, output dimensions {10, 3492}

Length, max det of input vectors to ParaHeckeOp: 3492, 828 (check: 3492, 828)

Desired length, max det of output vectors: 358, 207

Quotient of the max dets should be 4

Smaller max det is the minimal max det 207
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 358, 207

Rank of T[G2G2]^+/- = (10,7)

Rank of plus basis attempt = 121 and dimension of S_4(K(246))^+ = 124

Rank of minus basis attempt = 11 and dimension of S_4(K(246))^- = 25


Hecke spread 3 out of 4

Is Hecke spread 3 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 892322

Called ShortenVecs with G2G2T[{}]

 input dimensions {10, 892322}, output dimensions {10, 372922}

Length, max det of input vectors to ParaHeckeOp: 372922, 16767 (check: 372922, 16767)

Desired length, max det of output vectors: 358, 207

Quotient of the max dets should be 81

Smaller max det is the minimal max det 207
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 3 length and max det should be as desired: 358, 207

Rank of T[G2G2]^+/- = (10,6)

Rank of plus basis attempt = 121 and dimension of S_4(K(246))^+ = 124

Rank of minus basis attempt = 17 and dimension of S_4(K(246))^- = 25
```

```
Hecke spread 4 out of 4

Is Hecke spread 4 present?  False

DoOneHecke called {},{{2, 2}, {3, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{3, 1}}

{tp,tpdel} = {3, 1}

Need to compute G2G2T[{{2, 2}, {3, 1}}] having G2G2T[{{2, 2}}] vectors to length 12514

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {10, 12514}, output dimensions {10, 12514}

Length, max det of input vectors to ParaHeckeOp: 12514, 1863 (check: 12514, 1863)

Desired length, max det of output vectors: 358, 207

Quotient of the max dets should be 9

Smaller max det is the minimal max det 207
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {3, 1}}
About to compute G2G2T[{{2, 2}, {3, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 4 length and max det should be as desired: 358, 207

Rank of T[G2G2]^+/- = (10,7)

Rank of plus basis attempt = 121 and dimension of S_4(K(246))^+ = 124

Rank of minus basis attempt = 19 and dimension of S_4(K(246))^- = 25


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-246-Extended-rank-9-mod-12347-A.ma}

Dimensions of BPcoeffMatProven: {9, 1000}

Dimensions of join: {28, 358}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 121 and dimension of S_4(K(246))^+ = 124

Rank of minus basis attempt = 25 and dimension of S_4(K(246))^- = 25


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Didn't find a plus basis
```

```
Found minus basis of dimension 25

N = 247
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 1}
lift dimension of S_4(K(247))^+ : 47
nonlift dimension of S_4(K(247))^+ : 77
dimension of S_4(K(247))^- : 13
Ibukiyama-Kitayama dimension of S_4(K(247)) : 137

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(247))^+ = 124
Rank of minus basis attempt = 0 and dimension of S_4(K(247))^- = 13
Initial short vector length: 244
Have vectors of length 264: long enough
Determinant shell containing the vectors of length 244 : 224
Adding determinant shell 1
Short vector length is 288, and the corresponding determinant is 231
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-247-231.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,247}^{cusp}), which is 47
Rank of plus basis attempt = 47 and dimension of S_4(K(247))^+ = 124
Rank of minus basis attempt = 0 and dimension of S_4(K(247))^- = 13

STEP 2: TRACE DOWN
N = 247  q = 5  Nq = 1235
288-th determinant is 231
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 528
Trace down to try to hit S4(K(N)), of dimension 137
Plus lifts, plus nonlifts, minus dimension: {47, 77, 13}
Target plus rank, target minus rank: 124 13
Dimensions of savedTargetMats: {288, 3}
Making ordered good sigs for source space S_4(K(Nq))...
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
```

```
Found file Grits-2-1235-5775.ma, Getting it

 ... done making G2.  Dimensions of Grits: {32, 46840}

Determining indices: {1, 2, 5, 6, 7, 8, 13, 14, 15, 16, 29, 30, 37, 38,
  39, 40, 57, 58, 61, 69, 70, 71, 72, 93, 94, 101, 102, 113, 165, 166, 167, 177}

Atkin-Lehner truncation: 224

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 12}, {{1, -1, -1}, 10}, {{-1, 1, -1}, 5}, {{-1, -1, 1}, 5}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1235-5775-to-247-231.ma exists, reading it

Found file Grits-2-1235-5775.ma, Getting it

Grit dimensions: {32, 46840}

Viable file found: GG-Grits-2-1235-5775.ma-5775-mod-12347.ma

New best file: GG-Grits-2-1235-5775.ma-5775-mod-12347.ma

Dimensions of G2G2: {528, 46840}

Dimensions of fDown = {528, 288}

Rank of fDown mod pp = 120

Dimensions and rank of plus space fDown = {528, 288}  120

Dimensions and rank of minus space fDown = {528, 288}  0

Rank of plus basis attempt = 124 and dimension of S_4(K(247))^+ = 124

Rank of minus basis attempt = 0 and dimension of S_4(K(247))^- = 13


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 28

Found file Grits-2-247-231.ma, Getting it

Viable file found: GG-Grits-2-247-4720.ma-4720-mod-12347.ma

New best file: GG-Grits-2-247-4720.ma-4720-mod-12347.ma

Dimensions of G2G2: {28, 288}

Rank of plus basis attempt = 124 and dimension of S_4(K(247))^+ = 124

Rank of minus basis attempt = 0 and dimension of S_4(K(247))^- = 13


STEP 4: HECKE SPREAD

J2=0, or already have plus space basis and no extra Hecke spreads were specified

Hecke spreads: 0


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
```

```
Found files: {BPinfo-files/BPminusInfo-4-247-500.ma}
Dimensions of BPcoeffMatProven: {7, 892}
Dimensions of join: {7, 288}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 124 and dimension of S_4(K(247))^+ = 124
Rank of minus basis attempt = 7 and dimension of S_4(K(247))^- = 13
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Found a plus basis of dimension 124
Didn't find a minus basis
```

N = 249

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 0}
lift dimension of S_4(K(249))^+ : 45
nonlift dimension of S_4(K(249))^+ : 82
dimension of S_4(K(249))^- : 12
Ibukiyama-Kitayama dimension of S_4(K(249)) : 139
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(249))^+ = 127
Rank of minus basis attempt = 0 and dimension of S_4(K(249))^- = 12
Initial short vector length: 253
Have vectors of length 281: long enough
Determinant shell containing the vectors of length 253 : 212
Short vector length is 263, and the corresponding determinant is 212
Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-249-220.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,249}^{cusp}), which is 45
Rank of plus basis attempt = 45 and dimension of S_4(K(249))^+ = 127
Rank of minus basis attempt = 0 and dimension of S_4(K(249))^- = 12
```

STEP 2: TRACE DOWN

```
N = 249  q = 7  Nq = 1743

263-th determinant is 212

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 1225

Trace down to try to hit S4(K(N)), of dimension 139

Plus lifts, plus nonlifts, minus dimension: {45, 82, 12}

Target plus rank, target minus rank: 127 12

Dimensions of savedTargetMats: {263, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1743-10388.ma, Getting it

 ... done making G2.  Dimensions of Grits: {49, 131942}

Determining indices: {1, 2, 3, 4, 9, 10, 13, 14, 17, 18, 19, 20, 37, 38, 45,
  46, 47, 48, 69, 70, 75, 76, 87, 88, 103, 104, 105, 106, 143, 144, 163, 167, 168,
  183, 184, 207, 208, 209, 210, 247, 248, 275, 276, 299, 300, 301, 302, 331, 332}

Atkin-Lehner truncation: 358

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 19}, {{1, -1, -1}, 13}, {{-1, 1, -1}, 10}, {{-1, -1, 1}, 7}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1743-10388-to-249-212.ma
  exists, reading it

Found file Grits-2-1743-10388.ma, Getting it

Grit dimensions: {49, 131942}

Viable file found: GG-Grits-2-1743-10388.ma-10388-mod-12347.ma

New best file: GG-Grits-2-1743-10388.ma-10388-mod-12347.ma

Dimensions of G2G2: {1225, 131942}

Dimensions of fDown = {1225, 263}

Rank of fDown mod pp = 124

Dimensions and rank of plus space fDown = {1225, 263}  124

Dimensions and rank of minus space fDown = {1225, 263}  0

Rank of plus basis attempt = 126 and dimension of S_4(K(249))^+ = 127

Rank of minus basis attempt = 0 and dimension of S_4(K(249))^- = 12
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 15

Found file Grits-2-249-212.ma, Getting it
```

```
Viable file found: GG-Grits-2-249-12495.ma-12495-mod-12347.ma

New best file: GG-Grits-2-249-12495.ma-12495-mod-12347.ma

Viable file found: GG-Grits-2-249-30855.ma-30855-mod-12347.ma

Viable file found: GG-Grits-2-249-34364.ma-34364-mod-12347.ma

Dimensions of G2G2: {15, 263}

Rank of plus basis attempt = 126 and dimension of S_4(K(249))^+ = 127

Rank of minus basis attempt = 0 and dimension of S_4(K(249))^- = 12


STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 212 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 17 172

Hecke spreads: 1

Pre-Hecke expansions will have length 256 351

Found file Grits-2-249-17172.ma, Getting it

Viable file found: GG-Grits-2-249-30855.ma-30855-mod-12347.ma

New best file: GG-Grits-2-249-30855.ma-30855-mod-12347.ma

Viable file found: GG-Grits-2-249-34364.ma-34364-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 623 190

Called ShortenVecs with G2G2T[{}]

 input dimensions {15, 623 190}, output dimensions {15, 256 351}

Length, max det of input vectors to ParaHeckeOp: 256 351, 17 172 (check: 256 351, 17 172)

Desired length, max det of output vectors: 263, 212

Quotient of the max dets should be 81

Smaller max det is the minimal max det 212
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]
```

```
Making abstract Hecke formula at a squared bad prime
Using Ralf's formula
Substituting given paramodular forms in abstract formula
Hecke spread 1 length and max det should be as desired: 263, 212
Rank of T[G2G2]^+/- = (15,4)
Rank of plus basis attempt = 126 and dimension of S_4(K(249))^+ = 127
Rank of minus basis attempt = 4 and dimension of S_4(K(249))^- = 12

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-249-400.ma}
Dimensions of BPcoeffMatProven: {6, 765}
Dimensions of join: {10, 263}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 126 and dimension of S_4(K(249))^+ = 127
Rank of minus basis attempt = 10 and dimension of S_4(K(249))^- = 12

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Didn't find a plus basis
Didn't find a minus basis

N = 253
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}
lift dimension of S_4(K(253))^+ : 49
nonlift dimension of S_4(K(253))^+ : 82
dimension of S_4(K(253))^- : 10
Ibukiyama-Kitayama dimension of S_4(K(253)) : 141

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(253))^+ = 131
Rank of minus basis attempt = 0 and dimension of S_4(K(253))^- = 10
Initial short vector length: 227
Have vectors of length 230: long enough
Determinant shell containing the vectors of length 227 : 228
Short vector length is 230, and the corresponding determinant is 228
```

```
Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-253-230.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,253}^{cusp}), which is 49

Rank of plus basis attempt = 49 and dimension of S_4(K(253))^+ = 131

Rank of minus basis attempt = 0 and dimension of S_4(K(253))^- = 10
```

STEP 2: TRACE DOWN

```
N = 253  q = 5  Nq = 1265

230-th determinant is 228

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 741

Trace down to try to hit S4(K(N)), of dimension 141

Plus lifts, plus nonlifts, minus dimension: {49, 82, 10}

Target plus rank, target minus rank: 131 10

Dimensions of savedTargetMats: {230, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1265-5700.ma, Getting it

 ... done making G2.  Dimensions of Grits: {38, 45454}

Determining indices: {1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 19, 20, 21, 22, 27, 28, 29, 30, 43,
  44, 45, 46, 63, 64, 79, 80, 85, 86, 93, 94, 105, 106, 107, 108, 125, 126, 149, 169}

Atkin-Lehner truncation: 176

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 15}, {{1, -1, -1}, 10}, {{-1, 1, -1}, 8}, {{-1, -1, 1}, 5}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1265-5700-to-253-228.ma exists, reading it

Found file Grits-2-1265-5700.ma, Getting it

Grit dimensions: {38, 45454}

Viable file found: GG-Grits-2-1265-5700.ma-5700-mod-12347.ma

New best file: GG-Grits-2-1265-5700.ma-5700-mod-12347.ma

Dimensions of G2G2: {741, 45454}

Dimensions of fDown = {741, 230}

Rank of fDown mod pp = 129
```

```
Dimensions and rank of plus space fDown = {741, 230}  129
Dimensions and rank of minus space fDown = {741, 230}  0
Rank of plus basis attempt = 131 and dimension of S_4(K(253))^+ = 131
Rank of minus basis attempt = 0 and dimension of S_4(K(253))^- = 10

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS
Pairs from dim(J_{2,N}^{cusp}: 36
Found file Grits-2-253-228.ma, Getting it
Viable file found: GG-Grits-2-253-4864.ma-4864-mod-12347.ma
New best file: GG-Grits-2-253-4864.ma-4864-mod-12347.ma
Dimensions of G2G2: {36, 230}
Rank of plus basis attempt = 131 and dimension of S_4(K(253))^+ = 131
Rank of minus basis attempt = 0 and dimension of S_4(K(253))^- = 10

STEP 4: HECKE SPREAD
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-253-500.ma}
Dimensions of BPcoeffMatProven: {3, 865}
Dimensions of join: {3, 230}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 131 and dimension of S_4(K(253))^+ = 131
Rank of minus basis attempt = 3 and dimension of S_4(K(253))^- = 10

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Found a plus basis of dimension 131
Didn't find a minus basis

N = 254
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}
lift dimension of S_4(K(254))^+ : 46
nonlift dimension of S_4(K(254))^+ : 81
```

```
dimension of S_4(K(254))^- : 29

Ibukiyama-Kitayama dimension of S_4(K(254)) : 156


STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(254))^+ = 127

Rank of minus basis attempt = 0 and dimension of S_4(K(254))^- = 29

Initial short vector length: 307

Have vectors of length 323: long enough

Determinant shell containing the vectors of length 307 : 207

Short vector length is 323, and the corresponding determinant is 207

Vectors fill out a determinant shell, as they should


STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-254-210.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,254}^{cusp}), which is 46

Rank of plus basis attempt = 46 and dimension of S_4(K(254))^+ = 127

Rank of minus basis attempt = 0 and dimension of S_4(K(254))^- = 29


STEP 2: TRACE DOWN

N = 254  q = 7  Nq = 1778

323-th determinant is 207

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 1540

Trace down to try to hit S4(K(N)), of dimension 156

Plus lifts, plus nonlifts, minus dimension: {46, 81, 29}

Target plus rank, target minus rank: 127 29

Dimensions of savedTargetMats: {323, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1778-10143.ma, Getting it

 ... done making G2.  Dimensions of Grits: {55, 144 106}

Determining indices: {1, 2, 3, 4, 7, 8, 11, 12, 15, 19, 20, 23, 24, 35, 36, 37, 38, 51, 55, 56,
  65, 66, 85, 86, 105, 106, 121, 122, 123, 124, 153, 161, 162, 181, 182, 189, 190, 209,
  210, 253, 254, 267, 268, 279, 280, 307, 308, 355, 356, 357, 358, 383, 439, 503, 531}

Atkin-Lehner truncation: 538

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.
```

```
ALspacesDims = {{{1, 1, 1}, 28}, {{1, -1, -1}, 16}, {{-1, 1, -1}, 8}, {{-1, -1, 1}, 3}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1778-10143-to-254-207.ma
  exists, reading it

Found file Grits-2-1778-10143.ma, Getting it

Grit dimensions: {55, 144106}

Viable file found: GG-Grits-2-1778-10143.ma-10143-mod-12347.ma

New best file: GG-Grits-2-1778-10143.ma-10143-mod-12347.ma

Dimensions of G2G2: {1540, 144106}

Dimensions of fDown = {1540, 323}

Rank of fDown mod pp = 125

Dimensions and rank of plus space fDown = {1540, 323}  125

Dimensions and rank of minus space fDown = {1540, 323}  0

Rank of plus basis attempt = 125 and dimension of S_4(K(254))^+ = 127

Rank of minus basis attempt = 0 and dimension of S_4(K(254))^- = 29


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 15

Found file Grits-2-254-207.ma, Getting it

Viable file found: GG-Grits-2-254-10976.ma-10976-mod-12347.ma

New best file: GG-Grits-2-254-10976.ma-10976-mod-12347.ma

Viable file found: GG-Grits-2-254-14336.ma-14336-mod-12347.ma

Viable file found: GG-Grits-2-254-16128.ma-16128-mod-12347.ma

Viable file found: GG-Grits-2-254-18144.ma-18144-mod-12347.ma

Viable file found: GG-Grits-2-254-27104.ma-27104-mod-12347.ma

Viable file found: GG-Grits-2-254-30492.ma-30492-mod-12347.ma

Viable file found: GG-Grits-2-254-34727.ma-34727-mod-12347.ma

Dimensions of G2G2: {15, 323}

Rank of plus basis attempt = 125 and dimension of S_4(K(254))^+ = 127

Rank of minus basis attempt = 0 and dimension of S_4(K(254))^- = 29


STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 2 Hecke spreads, the last of which dilates determinants by 64
```

```
Need to multiply the minimal max det 207 by the largest det contraction factor
 64 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 13 248

Hecke spreads: 2

Pre-Hecke expansions will have length 195 061

Found file Grits-2-254-13248.ma, Getting it

Viable file found: GG-Grits-2-254-14336.ma-14336-mod-12347.ma

New best file: GG-Grits-2-254-14336.ma-14336-mod-12347.ma

Viable file found: GG-Grits-2-254-16128.ma-16128-mod-12347.ma

Viable file found: GG-Grits-2-254-18144.ma-18144-mod-12347.ma

Viable file found: GG-Grits-2-254-27104.ma-27104-mod-12347.ma

Viable file found: GG-Grits-2-254-30492.ma-30492-mod-12347.ma

Viable file found: GG-Grits-2-254-34727.ma-34727-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 4

{{2, 2}, {2, 1}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 219 745

Called ShortenVecs with G2G2T[{}]

 input dimensions {15, 219 745}, output dimensions {15, 195 061}

Length, max det of input vectors to ParaHeckeOp: 195 061, 13 248 (check: 195 061, 13 248)

Desired length, max det of output vectors: 2664, 828

Quotient of the max dets should be 16

Smaller max det is the minimal max det 207
  times the biggest postpended det inverse dilation factor
 4 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 2664, 828

Rank of T[G2G2]^+/- = (15,11)

Rank of plus basis attempt = 125 and dimension of S_4(K(254))^+ = 127

Rank of minus basis attempt = 11 and dimension of S_4(K(254))^- = 29
```

```
Hecke spread 2 out of 2

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 2664

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {15, 2664}, output dimensions {15, 2664}

Length, max det of input vectors to ParaHeckeOp: 2664, 828 (check: 2664, 828)

Desired length, max det of output vectors: 323, 207

Quotient of the max dets should be 4

Smaller max det is the minimal max det 207
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}
About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 323, 207

Rank of T[G2G2]^+/- = (15,11)

Rank of plus basis attempt = 125 and dimension of S_4(K(254))^+ = 127

Rank of minus basis attempt = 20 and dimension of S_4(K(254))^- = 29


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-254-207-rank14.ma}

Dimensions of BPcoeffMatProven: {14, 323}

Dimensions of join: {34, 323}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 125 and dimension of S_4(K(254))^+ = 127

Rank of minus basis attempt = 27 and dimension of S_4(K(254))^- = 29


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Didn't find a plus basis
```

```
Didn't find a minus basis
```

N = 255

```
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 0}
lift dimension of S_4(K(255))^+ : 41
nonlift dimension of S_4(K(255))^+ : 73
dimension of S_4(K(255))^- : 22
Ibukiyama-Kitayama dimension of S_4(K(255)) : 136
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(255))^+ = 114
Rank of minus basis attempt = 0 and dimension of S_4(K(255))^- = 22
Initial short vector length: 291
Have vectors of length 298: long enough
Determinant shell containing the vectors of length 291 : 219
Short vector length is 298, and the corresponding determinant is 219
Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-255-220.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,255}^{cusp}), which is 41
Rank of plus basis attempt = 41 and dimension of S_4(K(255))^+ = 114
Rank of minus basis attempt = 0 and dimension of S_4(K(255))^- = 22
```

STEP 2: TRACE DOWN

```
N = 255  q = 7  Nq = 1785
298-th determinant is 219
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 1035
Trace down to try to hit S4(K(N)), of dimension 136
Plus lifts, plus nonlifts, minus dimension: {41, 73, 22}
Target plus rank, target minus rank: 114 22
Dimensions of savedTargetMats: {298, 3}
Making ordered good sigs for source space S_4(K(Nq))...
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
Found file Grits-2-1785-10731.ma, Getting it
```

```
 ... done making G2.  Dimensions of Grits: {45, 164 612}

Determining indices: {1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 29, 30, 37,
  38, 39, 40, 41, 42, 43, 85, 86, 87, 88, 117, 137, 138, 153, 154, 155, 156,
  185, 186, 193, 194, 195, 196, 198, 199, 289, 290, 291, 292, 345, 457, 459}

Atkin-Lehner truncation: 480

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims =
 {{{1, 1, 1, 1}, 12}, {{1, 1, -1, -1}, 8}, {{1, -1, 1, -1}, 6}, {{1, -1, -1, 1}, 7},
  {{-1, 1, 1, -1}, 5}, {{-1, 1, -1, 1}, 3}, {{-1, -1, 1, 1}, 3}, {{-1, -1, -1, -1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1785-10731-to-255-219.ma
  exists, reading it

Found file Grits-2-1785-10731.ma, Getting it

Grit dimensions: {45, 164 612}

Viable file found: GG-Grits-2-1785-10731.ma-10731-mod-12347.ma

New best file: GG-Grits-2-1785-10731.ma-10731-mod-12347.ma

Dimensions of G2G2: {1035, 164 612}

Dimensions of fDown = {1035, 298}

Rank of fDown mod pp = 113

Dimensions and rank of plus space fDown = {1035, 298}  113

Dimensions and rank of minus space fDown = {1035, 298}  0

Rank of plus basis attempt = 114 and dimension of S_4(K(255))^+ = 114

Rank of minus basis attempt = 0 and dimension of S_4(K(255))^- = 22


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 3

Found file Grits-2-255-219.ma, Getting it

Viable file found: GG-Grits-2-255-17739.ma-17739-mod-12347.ma

New best file: GG-Grits-2-255-17739.ma-17739-mod-12347.ma

Viable file found: GG-Grits-2-255-219.ma-219-mod-12347.ma

New best file: GG-Grits-2-255-219.ma-219-mod-12347.ma

Viable file found: GG-Grits-2-255-70956.ma-70956-mod-12347.ma

Dimensions of G2G2: {3, 298}

Rank of plus basis attempt = 114 and dimension of S_4(K(255))^+ = 114

Rank of minus basis attempt = 0 and dimension of S_4(K(255))^- = 22
```

STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 219 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 17 739

Hecke spreads: 1

Pre-Hecke expansions will have length 331 536

Found file Grits-2-255-17739.ma, Getting it

Viable file found: GG-Grits-2-255-17739.ma-17739-mod-12347.ma

New best file: GG-Grits-2-255-17739.ma-17739-mod-12347.ma

Viable file found: GG-Grits-2-255-70956.ma-70956-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present? False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 331 536

Called ShortenVecs with G2G2T[{}]

 input dimensions {3, 331 536}, output dimensions {3, 331 536}

Length, max det of input vectors to ParaHeckeOp: 331 536, 17 739 (check: 331 536, 17 739)

Desired length, max det of output vectors: 298, 219

Quotient of the max dets should be 81

Smaller max det is the minimal max det 219
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 298, 219

Rank of T[G2G2]^+/- = (3,3)

Rank of plus basis attempt = 114 and dimension of S_4(K(255))^+ = 114

Rank of minus basis attempt = 3 and dimension of S_4(K(255))^- = 22

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-255-500-5-0.ma}

Dimensions of BPcoeffMatProven: {19, 1162}

Dimensions of join: {22, 298}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 114 and dimension of S_4(K(255))^+ = 114

Rank of minus basis attempt = 21 and dimension of S_4(K(255))^- = 22

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 114

Didn't find a minus basis

N = 258

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}

lift dimension of S_4(K(258))^+ : 44

nonlift dimension of S_4(K(258))^+ : 84

dimension of S_4(K(258))^- : 35

Ibukiyama-Kitayama dimension of S_4(K(258)) : 163

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(258))^+ = 128

Rank of minus basis attempt = 0 and dimension of S_4(K(258))^- = 35

Initial short vector length: 349

Have vectors of length 358: long enough

Determinant shell containing the vectors of length 349 : 204

Short vector length is 358, and the corresponding determinant is 204

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-258-210.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,258}^{cusp}), which is 44

Rank of plus basis attempt = 44 and dimension of S_4(K(258))^+ = 128

Rank of minus basis attempt = 0 and dimension of S_4(K(258))^- = 35

STEP 2: TRACE DOWN

```
N = 258  q = 7  Nq = 1806

358-th determinant is 204

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 946

Trace down to try to hit S4(K(N)), of dimension 163

Plus lifts, plus nonlifts, minus dimension: {44, 84, 35}

Target plus rank, target minus rank: 128 35

Dimensions of savedTargetMats: {358, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1806-9996.ma, Getting it

 ... done making G2.  Dimensions of Grits: {43, 184848}

Determining indices:
 {1, 2, 5, 6, 7, 8, 13, 14, 25, 26, 41, 42, 43, 44, 81, 82, 83, 84, 105, 106, 133, 134, 135, 136,
  173, 174, 181, 182, 183, 184, 221, 225, 226, 249, 250, 277, 278, 333, 334, 365, 497, 617, 665}

Atkin-Lehner truncation: 808

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims =
 {{{1, 1, 1, 1}, 16}, {{1, 1, -1, -1}, 10}, {{1, -1, 1, -1}, 6}, {{1, -1, -1, 1}, 4},
  {{-1, 1, 1, -1}, 3}, {{-1, 1, -1, 1}, 2}, {{-1, -1, 1, 1}, 1}, {{-1, -1, -1, -1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1806-9996-to-258-204.ma exists, reading it

Found file Grits-2-1806-9996.ma, Getting it

Grit dimensions: {43, 184848}

Viable file found: GG-Grits-2-1806-9996.ma-9996-mod-12347.ma

New best file: GG-Grits-2-1806-9996.ma-9996-mod-12347.ma

Dimensions of G2G2: {946, 184848}

Dimensions of fDown = {946, 358}

Rank of fDown mod pp = 125

Dimensions and rank of plus space fDown = {946, 358}  125

Dimensions and rank of minus space fDown = {946, 358}  0

Rank of plus basis attempt = 127 and dimension of S_4(K(258))^+ = 128

Rank of minus basis attempt = 0 and dimension of S_4(K(258))^- = 35
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 10

Found file Grits-2-258-204.ma, Getting it

Viable file found: GG-Grits-2-258-36335.ma-36335-mod-12347.ma

New best file: GG-Grits-2-258-36335.ma-36335-mod-12347.ma

Dimensions of G2G2: {10, 358}

Rank of plus basis attempt = 127 and dimension of S_4(K(258))^+ = 128

Rank of minus basis attempt = 0 and dimension of S_4(K(258))^- = 35


STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 4 Hecke spreads, the last of which dilates determinants by 144

Need to multiply the minimal max det 204 by the largest det contraction factor
 144 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 29 376

Hecke spreads: 4

Pre-Hecke expansions will have length 870 734

Found file Grits-2-258-29376.ma, Getting it

Viable file found: GG-Grits-2-258-36335.ma-36335-mod-12347.ma

New best file: GG-Grits-2-258-36335.ma-36335-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 9

{{2, 2}, {2, 1}} 1

{{3, 2}} 1

{{2, 2}, {3, 1}} 1

Hecke spread 1 out of 4

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 1 201 826

Called ShortenVecs with G2G2T[{}]

 input dimensions {10, 1 201 826}, output dimensions {10, 870 734}

Length, max det of input vectors to ParaHeckeOp: 870 734, 29 376 (check: 870 734, 29 376)

Desired length, max det of output vectors: 12 032, 1836

Quotient of the max dets should be 16
```

```
Smaller max det is the minimal max det 204
  times the biggest postpended det inverse dilation factor
 9 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 12 032, 1836

Rank of T[G2G2]^+/- = (10,10)

Rank of plus basis attempt = 127 and dimension of S_4(K(258))^+ = 128

Rank of minus basis attempt = 10 and dimension of S_4(K(258))^- = 35


Hecke spread 2 out of 4

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 12 032

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {10, 12 032}, output dimensions {10, 3318}

Length, max det of input vectors to ParaHeckeOp: 3318, 816 (check: 3318, 816)

Desired length, max det of output vectors: 358, 204

Quotient of the max dets should be 4

Smaller max det is the minimal max det 204
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 358, 204

Rank of T[G2G2]^+/- = (10,10)

Rank of plus basis attempt = 127 and dimension of S_4(K(258))^+ = 128

Rank of minus basis attempt = 20 and dimension of S_4(K(258))^- = 35


Hecke spread 3 out of 4

Is Hecke spread 3 present?  False
```

```
DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 1 201 826

Called ShortenVecs with G2G2T[{}]

 input dimensions {10, 1 201 826}, output dimensions {10, 364 336}

Length, max det of input vectors to ParaHeckeOp: 364 336, 16 524 (check: 364 336, 16 524)

Desired length, max det of output vectors: 358, 204

Quotient of the max dets should be 81

Smaller max det is the minimal max det 204
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 3 length and max det should be as desired: 358, 204

Rank of T[G2G2]^+/- = (10,3)

Rank of plus basis attempt = 127 and dimension of S_4(K(258))^+ = 128

Rank of minus basis attempt = 23 and dimension of S_4(K(258))^- = 35


Hecke spread 4 out of 4

Is Hecke spread 4 present?  False

DoOneHecke called {},{{2, 2}, {3, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{3, 1}}

{tp,tpdel} = {3, 1}

Need to compute G2G2T[{{2, 2}, {3, 1}}] having G2G2T[{{2, 2}}] vectors to length 12 032

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {10, 12 032}, output dimensions {10, 12 032}

Length, max det of input vectors to ParaHeckeOp: 12 032, 1836 (check: 12 032, 1836)

Desired length, max det of output vectors: 358, 204

Quotient of the max dets should be 9

Smaller max det is the minimal max det 204
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {3, 1}}

About to compute G2G2T[{{2, 2}, {3, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula
```

```
Substituting given paramodular forms in abstract formula
Hecke spread 4 length and max det should be as desired: 358, 204
Rank of T[G2G2]^+/- = (10,10)
Rank of plus basis attempt = 127 and dimension of S_4(K(258))^+ = 128
Rank of minus basis attempt = 24 and dimension of S_4(K(258))^- = 35

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-258-500.ma}
Dimensions of BPcoeffMatProven: {11, 1520}
Dimensions of join: {35, 358}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 127 and dimension of S_4(K(258))^+ = 128
Rank of minus basis attempt = 34 and dimension of S_4(K(258))^- = 35

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Didn't find a plus basis
Didn't find a minus basis

N = 259
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}
lift dimension of S_4(K(259))^+ : 49
nonlift dimension of S_4(K(259))^+ : 88
dimension of S_4(K(259))^- : 13
Ibukiyama-Kitayama dimension of S_4(K(259)) : 150

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(259))^+ = 137
Rank of minus basis attempt = 0 and dimension of S_4(K(259))^- = 13
Initial short vector length: 300
Have vectors of length 323: long enough
Determinant shell containing the vectors of length 300 : 243
Short vector length is 307, and the corresponding determinant is 243
Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-259-250.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,259}^{cusp}), which is 49

Rank of plus basis attempt = 49 and dimension of S_4(K(259))^+ = 137

Rank of minus basis attempt = 0 and dimension of S_4(K(259))^- = 13
```

STEP 2: TRACE DOWN

```
N = 259  q = 5  Nq = 1295

307-th determinant is 243

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 406

Trace down to try to hit S4(K(N)), of dimension 150

Plus lifts, plus nonlifts, minus dimension: {49, 88, 13}

Target plus rank, target minus rank: 137 13

Dimensions of savedTargetMats: {307, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1295-6075.ma, Getting it

 ... done making G2.  Dimensions of Grits: {28, 52 104}

Determining indices: {1, 2, 5, 6, 13, 14, 21, 22, 23, 24,
  45, 46, 61, 62, 65, 66, 67, 68, 81, 82, 83, 84, 93, 94, 95, 96, 133, 145}

Atkin-Lehner truncation: 176

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 11}, {{1, -1, -1}, 7}, {{-1, 1, -1}, 5}, {{-1, -1, 1}, 5}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1295-6075-to-259-243.ma exists, reading it

Found file Grits-2-1295-6075.ma, Getting it

Grit dimensions: {28, 52 104}

Viable file found: GG-Grits-2-1295-6075.ma-6075-mod-12347.ma

New best file: GG-Grits-2-1295-6075.ma-6075-mod-12347.ma

Dimensions of G2G2: {406, 52 104}

Dimensions of fDown = {406, 307}

Rank of fDown mod pp = 132

Dimensions and rank of plus space fDown = {406, 307}  132

Dimensions and rank of minus space fDown = {406, 307}  0
```

Rank of plus basis attempt = 137 and dimension of S_4(K(259))^+ = 137

Rank of minus basis attempt = 0 and dimension of S_4(K(259))^- = 13

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 28

Found file Grits-2-259-243.ma, Getting it

Viable file found: GG-Grits-2-259-24219.ma-24219-mod-12347.ma

New best file: GG-Grits-2-259-24219.ma-24219-mod-12347.ma

Dimensions of G2G2: {28, 307}

Rank of plus basis attempt = 137 and dimension of S_4(K(259))^+ = 137

Rank of minus basis attempt = 0 and dimension of S_4(K(259))^- = 13

STEP 4: HECKE SPREAD

J2=0, or already have plus space basis and no extra Hecke spreads were specified

Hecke spreads: 0

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-259-500.ma}

Dimensions of BPcoeffMatProven: {9, 917}

Dimensions of join: {9, 307}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 137 and dimension of S_4(K(259))^+ = 137

Rank of minus basis attempt = 9 and dimension of S_4(K(259))^- = 13

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Found a plus basis of dimension 137

Didn't find a minus basis

N = 262

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}

lift dimension of S_4(K(262))^+ : 48

nonlift dimension of S_4(K(262))^+ : 96

dimension of S_4(K(262))^- : 29

Ibukiyama-Kitayama dimension of S_4(K(262)) : 173

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(262))^+ = 144

Rank of minus basis attempt = 0 and dimension of S_4(K(262))^- = 29

Initial short vector length: 327

Have vectors of length 359: long enough

Determinant shell containing the vectors of length 327 : 223

Short vector length is 335, and the corresponding determinant is 223

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-262-230.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,262}^{cusp}), which is 48

Rank of plus basis attempt = 48 and dimension of S_4(K(262))^+ = 144

Rank of minus basis attempt = 0 and dimension of S_4(K(262))^- = 29

STEP 2: TRACE DOWN

N = 262 q = 7 Nq = 1834

335-th determinant is 223

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 1653

Trace down to try to hit S4(K(N)), of dimension 173

Plus lifts, plus nonlifts, minus dimension: {48, 96, 29}

Target plus rank, target minus rank: 144 29

Dimensions of savedTargetMats: {335, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1834-10927.ma, Getting it

 ... done making G2. Dimensions of Grits: {57, 161550}

Determining indices: {1, 2, 5, 6, 7, 8, 17, 18, 21, 22, 23, 24, 41, 45, 46, 53, 54, 65, 66, 67,
  68, 89, 90, 109, 110, 111, 112, 129, 130, 141, 142, 143, 173, 193, 194, 217, 218, 237, 241,
  242, 281, 282, 283, 284, 317, 318, 333, 334, 361, 381, 429, 430, 445, 446, 561, 609, 653}

Atkin-Lehner truncation: 724

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 25}, {{1, -1, -1}, 21}, {{-1, 1, -1}, 6}, {{-1, -1, 1}, 5}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

```
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1834-10927-to-262-223.ma
  exists, reading it
Found file Grits-2-1834-10927.ma, Getting it
Grit dimensions: {57, 161550}
Viable file found: GG-Grits-2-1834-10927.ma-10927-mod-12347.ma
New best file: GG-Grits-2-1834-10927.ma-10927-mod-12347.ma
Dimensions of G2G2: {1653, 161550}
Dimensions of fDown = {1653, 335}
Rank of fDown mod pp = 140
Dimensions and rank of plus space fDown = {1653, 335}  140
Dimensions and rank of minus space fDown = {1653, 335}  0
Rank of plus basis attempt = 141 and dimension of S_4(K(262))^+ = 144
Rank of minus basis attempt = 0 and dimension of S_4(K(262))^- = 29
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 15
Found file Grits-2-262-223.ma, Getting it
Viable file found: GG-Grits-2-262-223.ma-223-mod-12347.ma
New best file: GG-Grits-2-262-223.ma-223-mod-12347.ma
Viable file found: GG-Grits-2-262-32112.ma-32112-mod-12347.ma
Viable file found: GG-Grits-2-262-892.ma-892-mod-12347.ma
Dimensions of G2G2: {15, 335}
Rank of plus basis attempt = 141 and dimension of S_4(K(262))^+ = 144
Rank of minus basis attempt = 0 and dimension of S_4(K(262))^- = 29
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified
Found custom table of Hecke spreads
Preparing for 2 Hecke spreads, the last of which dilates determinants by 64
Need to multiply the minimal max det 223 by the largest det contraction factor
 64 under Hecke spreads to get the pre-Hecke max det
Need pre-Hecke expansions up to determinant 14272
Hecke spreads: 2
Pre-Hecke expansions will have length 217406
Found file Grits-2-262-14272.ma, Getting it
Viable file found: GG-Grits-2-262-32112.ma-32112-mod-12347.ma
```

```
New best file: GG-Grits-2-262-32112.ma-32112-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 4

{{2, 2}, {2, 1}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 742 623

Called ShortenVecs with G2G2T[{}]

 input dimensions {15, 742 623}, output dimensions {15, 217 406}

Length, max det of input vectors to ParaHeckeOp: 217 406, 14 272 (check: 217 406, 14 272)

Desired length, max det of output vectors: 3040, 892

Quotient of the max dets should be 16

Smaller max det is the minimal max det 223
  times the biggest postpended det inverse dilation factor
 4 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 3040, 892

Rank of T[G2G2]^+/- = (15,10)

Rank of plus basis attempt = 141 and dimension of S_4(K(262))^+ = 144

Rank of minus basis attempt = 10 and dimension of S_4(K(262))^- = 29


Hecke spread 2 out of 2

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 3040

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {15, 3040}, output dimensions {15, 3040}

Length, max det of input vectors to ParaHeckeOp: 3040, 892 (check: 3040, 892)
```

```
Desired length, max det of output vectors: 335, 223

Quotient of the max dets should be 4

Smaller max det is the minimal max det 223
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 335, 223

Rank of T[G2G2]^+/- = (15,10)

Rank of plus basis attempt = 141 and dimension of S_4(K(262))^+ = 144

Rank of minus basis attempt = 16 and dimension of S_4(K(262))^- = 29


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-262-505-5-0.ma}

Dimensions of BPcoeffMatProven: {21, 1367}

Dimensions of join: {37, 335}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 141 and dimension of S_4(K(262))^+ = 144

Rank of minus basis attempt = 28 and dimension of S_4(K(262))^- = 29


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Didn't find a plus basis

Didn't find a minus basis


N = 265

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}

lift dimension of S_4(K(265))^+ : 51

nonlift dimension of S_4(K(265))^+ : 100

dimension of S_4(K(265))^- : 11

Ibukiyama-Kitayama dimension of S_4(K(265)) : 162


STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(265))^+ = 151
```

```
Rank of minus basis attempt = 0 and dimension of S_4(K(265))^- = 11

Initial short vector length: 354

Have vectors of length 358: long enough

Determinant shell containing the vectors of length 354 : 264

Short vector length is 358, and the corresponding determinant is 264

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-265-270.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,265}^{cusp}), which is 51

Rank of plus basis attempt = 51 and dimension of S_4(K(265))^+ = 151

Rank of minus basis attempt = 0 and dimension of S_4(K(265))^- = 11
```

STEP 2: TRACE DOWN

```
N = 265  q = 7  Nq = 1855

358-th determinant is 264

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 1378

Trace down to try to hit S4(K(N)), of dimension 162

Plus lifts, plus nonlifts, minus dimension: {51, 100, 11}

Target plus rank, target minus rank: 151 11

Dimensions of savedTargetMats: {358, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1855-12936.ma, Getting it

 ... done making G2.  Dimensions of Grits: {52, 166692}

Determining indices: {1, 2, 3, 4, 9, 10, 13, 14, 15, 16, 25, 26, 29, 30, 31, 32, 53, 54,
  57, 58, 77, 78, 79, 80, 97, 98, 117, 118, 129, 141, 142, 149, 150, 165, 166, 167,
  168, 201, 202, 203, 204, 233, 234, 235, 236, 281, 305, 306, 313, 314, 315, 425}

Atkin-Lehner truncation: 500

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 19}, {{1, -1, -1}, 12}, {{-1, 1, -1}, 12}, {{-1, -1, 1}, 9}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1855-12936-to-265-264.ma
  exists, reading it

Found file Grits-2-1855-12936.ma, Getting it
```

```
Grit dimensions: {52, 166 692}
Viable file found: GG-Grits-2-1855-12936.ma-12936-mod-12347.ma
New best file: GG-Grits-2-1855-12936.ma-12936-mod-12347.ma
Dimensions of G2G2: {1378, 166 692}
Dimensions of fDown = {1378, 358}
Rank of fDown mod pp = 147
Dimensions and rank of plus space fDown = {1378, 358}  147
Dimensions and rank of minus space fDown = {1378, 358}  0
Rank of plus basis attempt = 151 and dimension of S_4(K(265))^+ = 151
Rank of minus basis attempt = 0 and dimension of S_4(K(265))^- = 11
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 36
Found file Grits-2-265-264.ma, Getting it
Viable file found: GG-Grits-2-265-11484.ma-11484-mod-12347.ma
New best file: GG-Grits-2-265-11484.ma-11484-mod-12347.ma
Viable file found: GG-Grits-2-265-5104.ma-5104-mod-12347.ma
New best file: GG-Grits-2-265-5104.ma-5104-mod-12347.ma
Dimensions of G2G2: {36, 358}
Rank of plus basis attempt = 151 and dimension of S_4(K(265))^+ = 151
Rank of minus basis attempt = 0 and dimension of S_4(K(265))^- = 11
```

STEP 4: HECKE SPREAD

```
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-265-500.ma}
Dimensions of BPcoeffMatProven: {5, 972}
Dimensions of join: {5, 358}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 151 and dimension of S_4(K(265))^+ = 151
Rank of minus basis attempt = 5 and dimension of S_4(K(265))^- = 11
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}
```

```
Found a plus basis of dimension 151

Didn't find a minus basis

N = 266

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}

lift dimension of S_4(K(266))^+ : 45

nonlift dimension of S_4(K(266))^+ : 81

dimension of S_4(K(266))^- : 32

Ibukiyama-Kitayama dimension of S_4(K(266)) : 158

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(266))^+ = 126

Rank of minus basis attempt = 0 and dimension of S_4(K(266))^- = 32

Initial short vector length: 302

Have vectors of length 348: long enough

Determinant shell containing the vectors of length 302 : 223

Short vector length is 328, and the corresponding determinant is 223

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-266-230.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,266}^{cusp}), which is 45

Rank of plus basis attempt = 45 and dimension of S_4(K(266))^+ = 126

Rank of minus basis attempt = 0 and dimension of S_4(K(266))^- = 32

STEP 2: TRACE DOWN

N = 266  q = 5  Nq = 1330

328-th determinant is 223

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 496

Trace down to try to hit S4(K(N)), of dimension 158

Plus lifts, plus nonlifts, minus dimension: {45, 81, 32}

Target plus rank, target minus rank: 126 32

Dimensions of savedTargetMats: {328, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.
```

```
Making G2 for S_2(K(Nq))...

Found file Grits-2-1330-5575.ma, Getting it

 ... done making G2.  Dimensions of Grits: {31, 67652}

Determining indices: {1, 2, 3, 4, 5, 6, 7, 8, 25, 26, 29, 30, 37, 38,
  49, 50, 57, 58, 59, 60, 105, 106, 107, 108, 121, 122, 173, 174, 213, 215, 261}

Atkin-Lehner truncation: 332

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims =
 {{{1, 1, 1, 1}, 11}, {{1, 1, -1, -1}, 5}, {{1, -1, 1, -1}, 6}, {{1, -1, -1, 1}, 4},
  {{-1, 1, 1, -1}, 2}, {{-1, 1, -1, 1}, 1}, {{-1, -1, 1, 1}, 1}, {{-1, -1, -1, -1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1330-5575-to-266-223.ma exists, reading it

Found file Grits-2-1330-5575.ma, Getting it

Grit dimensions: {31, 67652}

Viable file found: GG-Grits-2-1330-7840.ma-7840-mod-12347.ma

New best file: GG-Grits-2-1330-7840.ma-7840-mod-12347.ma

Dimensions of G2G2: {496, 116028}

Dimensions of fDown = {496, 328}

Rank of fDown mod pp = 123

Dimensions and rank of plus space fDown = {496, 328}  123

Dimensions and rank of minus space fDown = {496, 328}  0

Rank of plus basis attempt = 125 and dimension of S_4(K(266))^+ = 126

Rank of minus basis attempt = 0 and dimension of S_4(K(266))^- = 32


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 10

Found file Grits-2-266-223.ma, Getting it

Viable file found: GG-Grits-2-266-14272.ma-14272-mod-12347.ma

New best file: GG-Grits-2-266-14272.ma-14272-mod-12347.ma

Viable file found: GG-Grits-2-266-223.ma-223-mod-12347.ma

New best file: GG-Grits-2-266-223.ma-223-mod-12347.ma

Viable file found: GG-Grits-2-266-32112.ma-32112-mod-12347.ma

Viable file found: GG-Grits-2-266-892.ma-892-mod-12347.ma

Dimensions of G2G2: {10, 328}

Rank of plus basis attempt = 125 and dimension of S_4(K(266))^+ = 126
```

```
Rank of minus basis attempt = 0 and dimension of S_4(K(266))^- = 32
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 3 Hecke spreads, the last of which dilates determinants by 144

Need to multiply the minimal max det 223 by the largest det contraction factor
 144 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 32 112

Hecke spreads: 3

Pre-Hecke expansions will have length 876 440

Found file Grits-2-266-32112.ma, Getting it

Viable file found: GG-Grits-2-266-32112.ma-32112-mod-12347.ma

New best file: GG-Grits-2-266-32112.ma-32112-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 9

{{2, 2}, {2, 1}} 1

{{2, 2}, {3, 1}} 1

Hecke spread 1 out of 3

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 876 440

Called ShortenVecs with G2G2T[{}]

 input dimensions {10, 876 440}, output dimensions {10, 876 440}

Length, max det of input vectors to ParaHeckeOp: 876 440, 32 112 (check: 876 440, 32 112)

Desired length, max det of output vectors: 12 300, 2007

Quotient of the max dets should be 16

Smaller max det is the minimal max det 223
  times the biggest postpended det inverse dilation factor
 9 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 12 300, 2007

Rank of T[G2G2]^+/- = (10,10)
```

```
Rank of plus basis attempt = 125 and dimension of S_4(K(266))^+ = 126

Rank of minus basis attempt = 10 and dimension of S_4(K(266))^- = 32


Hecke spread 2 out of 3

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 12 300

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {10, 12 300}, output dimensions {10, 3270}

Length, max det of input vectors to ParaHeckeOp: 3270, 892 (check: 3270, 892)

Desired length, max det of output vectors: 328, 223

Quotient of the max dets should be 4

Smaller max det is the minimal max det 223
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 328, 223

Rank of T[G2G2]^+/- = (10,10)

Rank of plus basis attempt = 125 and dimension of S_4(K(266))^+ = 126

Rank of minus basis attempt = 19 and dimension of S_4(K(266))^- = 32


Hecke spread 3 out of 3

Is Hecke spread 3 present?  False

DoOneHecke called {},{{2, 2}, {3, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{3, 1}}

{tp,tpdel} = {3, 1}

Need to compute G2G2T[{{2, 2}, {3, 1}}] having G2G2T[{{2, 2}}] vectors to length 12 300

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {10, 12 300}, output dimensions {10, 12 300}

Length, max det of input vectors to ParaHeckeOp: 12 300, 2007 (check: 12 300, 2007)
```

```
Desired length, max det of output vectors: 328, 223

Quotient of the max dets should be 9

Smaller max det is the minimal max det 223
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {3, 1}}

About to compute G2G2T[{{2, 2}, {3, 1}}]

Making abstract Hecke formula

Substituting given paramodular forms in abstract formula

Hecke spread 3 length and max det should be as desired: 328, 223

Rank of T[G2G2]^+/- = (10,10)

Rank of plus basis attempt = 125 and dimension of S_4(K(266))^+ = 126

Rank of minus basis attempt = 23 and dimension of S_4(K(266))^- = 32


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-266-Extended-rank-15-mod-12347-A.ma}

Dimensions of BPcoeffMatProven: {15, 1000}

Dimensions of join: {38, 328}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 125 and dimension of S_4(K(266))^+ = 126

Rank of minus basis attempt = 29 and dimension of S_4(K(266))^- = 32


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Didn't find a plus basis

Didn't find a minus basis


N = 267

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 7}

lift dimension of S_4(K(267))^+ : 50

nonlift dimension of S_4(K(267))^+ : 80

dimension of S_4(K(267))^- : 25

Ibukiyama-Kitayama dimension of S_4(K(267)) : 155


STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(267))^+ = 130

Rank of minus basis attempt = 0 and dimension of S_4(K(267))^- = 25
```

```
Initial short vector length: 285

Have vectors of length 290: long enough

Determinant shell containing the vectors of length 285 : 212

Adding determinant shell 1

Adding determinant shell 2

Adding determinant shell 3

Adding determinant shell 4

Adding determinant shell 5

Adding determinant shell 6

Adding determinant shell 7

Short vector length is 382, and the corresponding determinant is 272

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-267-272.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,267}^{cusp}), which is 50

Rank of plus basis attempt = 50 and dimension of S_4(K(267))^+ = 130

Rank of minus basis attempt = 0 and dimension of S_4(K(267))^- = 25
```

STEP 2: TRACE DOWN

```
N = 267  q = 5  Nq = 1335

382-th determinant is 272

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 666

Trace down to try to hit S4(K(N)), of dimension 155

Plus lifts, plus nonlifts, minus dimension: {50, 80, 25}

Target plus rank, target minus rank: 130 25

Dimensions of savedTargetMats: {382, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1335-6800.ma, Getting it

 ... done making G2.  Dimensions of Grits: {36, 72676}

Determining indices: {1, 2, 3, 4, 5, 6, 9, 10, 17, 18, 25, 26, 27, 28, 41, 42, 43, 44,
  69, 70, 81, 82, 89, 90, 101, 102, 117, 118, 137, 138, 161, 162, 177, 179, 217, 237}

Atkin-Lehner truncation: 248

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.
```

```
ALspacesDims = {{{1, 1, 1}, 16}, {{1, -1, -1}, 10}, {{-1, 1, -1}, 7}, {{-1, -1, 1}, 3}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1335-6800-to-267-272.ma exists, reading it

Found file Grits-2-1335-6800.ma, Getting it

Grit dimensions: {36, 72676}

Viable file found: GG-Grits-2-1335-6800.ma-6800-mod-12347.ma

New best file: GG-Grits-2-1335-6800.ma-6800-mod-12347.ma

Dimensions of G2G2: {666, 72676}

Dimensions of fDown = {666, 382}

Rank of fDown mod pp = 128

Dimensions and rank of plus space fDown = {666, 382}  128

Dimensions and rank of minus space fDown = {666, 382}  0

Rank of plus basis attempt = 130 and dimension of S_4(K(267))^+ = 130

Rank of minus basis attempt = 0 and dimension of S_4(K(267))^- = 25
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 28

Found file Grits-2-267-272.ma, Getting it

Viable file found: GG-Grits-2-267-22032.ma-22032-mod-12347.ma

New best file: GG-Grits-2-267-22032.ma-22032-mod-12347.ma

Viable file found: GG-Grits-2-267-4400.ma-4400-mod-12347.ma

New best file: GG-Grits-2-267-4400.ma-4400-mod-12347.ma

Dimensions of G2G2: {28, 382}

Rank of plus basis attempt = 130 and dimension of S_4(K(267))^+ = 130

Rank of minus basis attempt = 0 and dimension of S_4(K(267))^- = 25
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 272 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 22032

Hecke spreads: 1

Pre-Hecke expansions will have length 373448

Found file Grits-2-267-22032.ma, Getting it
```

```
Viable file found: GG-Grits-2-267-22032.ma-22032-mod-12347.ma

New best file: GG-Grits-2-267-22032.ma-22032-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 373 448

Called ShortenVecs with G2G2T[{}]

 input dimensions {28, 373 448}, output dimensions {28, 373 448}

Length, max det of input vectors to ParaHeckeOp: 373 448, 22 032 (check: 373 448, 22 032)

Desired length, max det of output vectors: 382, 272

Quotient of the max dets should be 81

Smaller max det is the minimal max det 272
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 382, 272

Rank of T[G2G2]^+/- = (28,8)

Rank of plus basis attempt = 130 and dimension of S_4(K(267))^+ = 130

Rank of minus basis attempt = 8 and dimension of S_4(K(267))^- = 25


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-267-500.ma}

Dimensions of BPcoeffMatProven: {12, 1092}

Dimensions of join: {20, 382}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 130 and dimension of S_4(K(267))^+ = 130

Rank of minus basis attempt = 19 and dimension of S_4(K(267))^- = 25


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}
```

```
Found a plus basis of dimension 130

Didn't find a minus basis


N = 273

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 0}

lift dimension of S_4(K(273))^+ : 48

nonlift dimension of S_4(K(273))^+ : 93

dimension of S_4(K(273))^- : 19

Ibukiyama-Kitayama dimension of S_4(K(273)) : 160


STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(273))^+ = 141

Rank of minus basis attempt = 0 and dimension of S_4(K(273))^- = 19

Initial short vector length: 350

Have vectors of length 358: long enough

Determinant shell containing the vectors of length 350 : 264

Short vector length is 358, and the corresponding determinant is 264

Vectors fill out a determinant shell, as they should


STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-273-270.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,273}^{cusp}), which is 48

Rank of plus basis attempt = 48 and dimension of S_4(K(273))^+ = 141

Rank of minus basis attempt = 0 and dimension of S_4(K(273))^- = 19


STEP 2: TRACE DOWN

N = 273  q = 5  Nq = 1365

358-th determinant is 264

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 465

Trace down to try to hit S4(K(N)), of dimension 160

Plus lifts, plus nonlifts, minus dimension: {48, 93, 19}

Target plus rank, target minus rank: 141 19

Dimensions of savedTargetMats: {358, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.
```

```
Making G2 for S_2(K(Nq))...

Found file Grits-2-1365-6600.ma, Getting it

 ... done making G2.  Dimensions of Grits: {30, 79 896}

Determining indices: {1, 2, 5, 6, 7, 8, 21, 22, 29, 30, 31, 32, 53,
  54, 55, 56, 57, 58, 59, 93, 94, 109, 111, 149, 201, 202, 225, 226, 227, 281}

Atkin-Lehner truncation: 312

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims =
 {{{1, 1, 1, 1}, 9}, {{1, 1, -1, -1}, 4}, {{1, -1, 1, -1}, 4}, {{1, -1, -1, 1}, 2},
  {{-1, 1, 1, -1}, 4}, {{-1, 1, -1, 1}, 3}, {{-1, -1, 1, 1}, 2}, {{-1, -1, -1, -1}, 2}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1365-6600-to-273-264.ma exists, reading it

Found file Grits-2-1365-6600.ma, Getting it

Grit dimensions: {30, 79 896}

Viable file found: GG-Grits-2-1365-7791.ma-7791-mod-12347.ma

New best file: GG-Grits-2-1365-7791.ma-7791-mod-12347.ma

Dimensions of G2G2: {465, 103 432}

Dimensions of fDown = {465, 358}

Rank of fDown mod pp = 135

Dimensions and rank of plus space fDown = {465, 358}  135

Dimensions and rank of minus space fDown = {465, 358}  0

Rank of plus basis attempt = 141 and dimension of S_4(K(273))^+ = 141

Rank of minus basis attempt = 0 and dimension of S_4(K(273))^- = 19


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 21

Found file Grits-2-273-264.ma, Getting it

Viable file found: GG-Grits-2-273-21384.ma-21384-mod-12347.ma

New best file: GG-Grits-2-273-21384.ma-21384-mod-12347.ma

Viable file found: GG-Grits-2-273-264.ma-264-mod-12347.ma

New best file: GG-Grits-2-273-264.ma-264-mod-12347.ma

Dimensions of G2G2: {21, 358}

Rank of plus basis attempt = 141 and dimension of S_4(K(273))^+ = 141

Rank of minus basis attempt = 0 and dimension of S_4(K(273))^- = 19
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 264 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 21 384

Hecke spreads: 1

Pre-Hecke expansions will have length 426 064

Found file Grits-2-273-21384.ma, Getting it

Viable file found: GG-Grits-2-273-21384.ma-21384-mod-12347.ma

New best file: GG-Grits-2-273-21384.ma-21384-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 426 064

Called ShortenVecs with G2G2T[{}]

 input dimensions {21, 426 064}, output dimensions {21, 426 064}

Length, max det of input vectors to ParaHeckeOp: 426 064, 21 384 (check: 426 064, 21 384)

Desired length, max det of output vectors: 358, 264

Quotient of the max dets should be 81

Smaller max det is the minimal max det 264
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 358, 264

Rank of T[G2G2]^+/- = (21,10)

Rank of plus basis attempt = 141 and dimension of S_4(K(273))^+ = 141

Rank of minus basis attempt = 10 and dimension of S_4(K(273))^- = 19
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-273-500.ma}
Dimensions of BPcoeffMatProven: {7, 1068}
Dimensions of join: {17, 358}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 141 and dimension of S_4(K(273))^+ = 141
Rank of minus basis attempt = 17 and dimension of S_4(K(273))^- = 19

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Found a plus basis of dimension 141
Didn't find a minus basis

N = 274
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}
lift dimension of S_4(K(274))^+ : 52
nonlift dimension of S_4(K(274))^+ : 112
dimension of S_4(K(274))^- : 27
Ibukiyama-Kitayama dimension of S_4(K(274)) : 191

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(274))^+ = 164
Rank of minus basis attempt = 0 and dimension of S_4(K(274))^- = 27
Initial short vector length: 346
Have vectors of length 356: long enough
Determinant shell containing the vectors of length 346 : 224
Short vector length is 356, and the corresponding determinant is 224
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-274-230.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,274}^{cusp}), which is 52
Rank of plus basis attempt = 52 and dimension of S_4(K(274))^+ = 164
Rank of minus basis attempt = 0 and dimension of S_4(K(274))^- = 27

STEP 2: TRACE DOWN
```

```
N = 274  q = 5  Nq = 1370

356-th determinant is 224

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 820

Trace down to try to hit S4(K(N)), of dimension 191

Plus lifts, plus nonlifts, minus dimension: {52, 112, 27}

Target plus rank, target minus rank: 164 27

Dimensions of savedTargetMats: {356, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1370-5600.ma, Getting it

 ... done making G2.  Dimensions of Grits: {40, 61754}

Determining indices: {1, 2, 3, 4, 7, 8, 13, 14, 19, 20, 21, 22, 35, 36, 47, 48, 55, 63, 64, 77,
  78, 89, 95, 96, 97, 98, 135, 139, 155, 156, 163, 164, 189, 190, 191, 217, 218, 249, 289, 439}

Atkin-Lehner truncation: 474

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 21}, {{1, -1, -1}, 13}, {{-1, 1, -1}, 4}, {{-1, -1, 1}, 2}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1370-5600-to-274-224.ma exists, reading it

Found file Grits-2-1370-5600.ma, Getting it

Grit dimensions: {40, 61754}

Viable file found: GG-Grits-2-1370-5600.ma-5600-mod-12347.ma

New best file: GG-Grits-2-1370-5600.ma-5600-mod-12347.ma

Dimensions of G2G2: {820, 61754}

Dimensions of fDown = {820, 356}

Rank of fDown mod pp = 159

Dimensions and rank of plus space fDown = {820, 356}  159

Dimensions and rank of minus space fDown = {820, 356}  0

Rank of plus basis attempt = 162 and dimension of S_4(K(274))^+ = 164

Rank of minus basis attempt = 0 and dimension of S_4(K(274))^- = 27
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 28

Found file Grits-2-274-224.ma, Getting it

Viable file found: GG-Grits-2-274-14336.ma-14336-mod-12347.ma
```

```
New best file: GG-Grits-2-274-14336.ma-14336-mod-12347.ma

Viable file found: GG-Grits-2-274-224.ma-224-mod-12347.ma

New best file: GG-Grits-2-274-224.ma-224-mod-12347.ma

Viable file found: GG-Grits-2-274-896.ma-896-mod-12347.ma

Dimensions of G2G2: {28, 356}

Rank of plus basis attempt = 162 and dimension of S_4(K(274))^+ = 164

Rank of minus basis attempt = 0 and dimension of S_4(K(274))^- = 27
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 2 Hecke spreads, the last of which dilates determinants by 64

Need to multiply the minimal max det 224 by the largest det contraction factor
 64 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 14 336

Hecke spreads: 2

Pre-Hecke expansions will have length 219 643

Found file Grits-2-274-14336.ma, Getting it

Viable file found: GG-Grits-2-274-14336.ma-14336-mod-12347.ma

New best file: GG-Grits-2-274-14336.ma-14336-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 4

{{2, 2}, {2, 1}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 219 643

Called ShortenVecs with G2G2T[{}]

 input dimensions {28, 219 643}, output dimensions {28, 219 643}

Length, max det of input vectors to ParaHeckeOp: 219 643, 14 336 (check: 219 643, 14 336)

Desired length, max det of output vectors: 3135, 896

Quotient of the max dets should be 16

Smaller max det is the minimal max det 224
  times the biggest postpended det inverse dilation factor
 4 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]
```

```
Making abstract Hecke formula at a squared bad prime
Using Ralf's formula
Substituting given paramodular forms in abstract formula
Hecke spread 1 length and max det should be as desired: 3135, 896
Rank of T[G2G2]^+/- = (28,19)
Rank of plus basis attempt = 162 and dimension of S_4(K(274))^+ = 164
Rank of minus basis attempt = 19 and dimension of S_4(K(274))^- = 27

Hecke spread 2 out of 2
Is Hecke spread 2 present?  False
DoOneHecke called {},{{2, 2}, {2, 1}}
{tp,tpdel} = {2, 2}
DoOneHecke called {{2, 2}},{{2, 1}}
{tp,tpdel} = {2, 1}
Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 3135
Called ShortenVecs with G2G2T[{{2, 2}}]
 input dimensions {28, 3135}, output dimensions {28, 3135}
Length, max det of input vectors to ParaHeckeOp: 3135, 896 (check: 3135, 896)
Desired length, max det of output vectors: 356, 224
Quotient of the max dets should be 4
Smaller max det is the minimal max det 224
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}
About to compute G2G2T[{{2, 2}, {2, 1}}]
Making abstract Hecke formula at a bad prime
Using Ralf's formula
Substituting given paramodular forms in abstract formula
Hecke spread 2 length and max det should be as desired: 356, 224
Rank of T[G2G2]^+/- = (28,19)
Rank of plus basis attempt = 162 and dimension of S_4(K(274))^+ = 164
Rank of minus basis attempt = 19 and dimension of S_4(K(274))^- = 27

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-274-400.ma}
Dimensions of BPcoeffMatProven: {6, 861}
Dimensions of join: {25, 356}
Minus space span attempt may contain non-numerical entries
```

Rank of plus basis attempt = 162 and dimension of S_4(K(274))^+ = 164

Rank of minus basis attempt = 24 and dimension of S_4(K(274))^- = 27

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}

Didn't find a plus basis

Didn't find a minus basis

N = 278

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 0}

lift dimension of S_4(K(278))^+ : 51

nonlift dimension of S_4(K(278))^+ : 91

dimension of S_4(K(278))^- : 43

Ibukiyama-Kitayama dimension of S_4(K(278)) : 185

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(278))^+ = 142

Rank of minus basis attempt = 0 and dimension of S_4(K(278))^- = 43

Initial short vector length: 395

Have vectors of length 429: long enough

Determinant shell containing the vectors of length 395 : 232

Short vector length is 395, and the corresponding determinant is 232

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-278-240.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,278}^{cusp}), which is 51

Rank of plus basis attempt = 51 and dimension of S_4(K(278))^+ = 142

Rank of minus basis attempt = 0 and dimension of S_4(K(278))^- = 43

STEP 2: TRACE DOWN

N = 278 q = 7 Nq = 1946

395-th determinant is 232

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 1596

Trace down to try to hit S4(K(N)), of dimension 185

```
Plus lifts, plus nonlifts, minus dimension: {51, 91, 43}

Target plus rank, target minus rank: 142 43

Dimensions of savedTargetMats: {395, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1946-11368.ma, Getting it

 ... done making G2.  Dimensions of Grits: {56, 171890}

Determining indices: {1, 2, 5, 6, 9, 10, 21, 25, 26, 33, 34, 45, 49, 50, 51, 52, 73, 74, 75, 76,
  93, 94, 105, 106, 121, 122, 123, 124, 153, 154, 173, 174, 181, 182, 201, 202, 229, 230,
  231, 232, 265, 266, 321, 322, 349, 369, 393, 394, 429, 430, 441, 442, 457, 693, 694, 721}

Atkin-Lehner truncation: 768

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 26}, {{1, -1, -1}, 21}, {{-1, 1, -1}, 5}, {{-1, -1, 1}, 4}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1946-11368-to-278-232.ma
  exists, reading it

Found file Grits-2-1946-11368.ma, Getting it

Grit dimensions: {56, 171890}

Viable file found: GG-Grits-2-1946-11368.ma-11368-mod-12347.ma

New best file: GG-Grits-2-1946-11368.ma-11368-mod-12347.ma

Dimensions of G2G2: {1596, 171890}

Dimensions of fDown = {1596, 395}

Rank of fDown mod pp = 137

Dimensions and rank of plus space fDown = {1596, 395}  137

Dimensions and rank of minus space fDown = {1596, 395}  0

Rank of plus basis attempt = 140 and dimension of S_4(K(278))^+ = 142

Rank of minus basis attempt = 0 and dimension of S_4(K(278))^- = 43


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 21

Found file Grits-2-278-232.ma, Getting it

Viable file found: GG-Grits-2-278-33408.ma-33408-mod-12347.ma

New best file: GG-Grits-2-278-33408.ma-33408-mod-12347.ma

Viable file found: GG-Grits-2-278-8784.ma-8784-mod-12347.ma

New best file: GG-Grits-2-278-8784.ma-8784-mod-12347.ma
```

```
Dimensions of G2G2: {21, 395}

Rank of plus basis attempt = 140 and dimension of S_4(K(278))^+ = 142

Rank of minus basis attempt = 0 and dimension of S_4(K(278))^- = 43


STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 2 Hecke spreads, the last of which dilates determinants by 64

Need to multiply the minimal max det 232 by the largest det contraction factor
 64 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 14 848

Hecke spreads: 2

Pre-Hecke expansions will have length 231 073

Found file Grits-2-278-14848.ma, Getting it

Viable file found: GG-Grits-2-278-33408.ma-33408-mod-12347.ma

New best file: GG-Grits-2-278-33408.ma-33408-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 4

{{2, 2}, {2, 1}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 790 190

Called ShortenVecs with G2G2T[{}]

 input dimensions {21, 790 190}, output dimensions {21, 231 073}

Length, max det of input vectors to ParaHeckeOp: 231 073, 14 848 (check: 231 073, 14 848)

Desired length, max det of output vectors: 3244, 928

Quotient of the max dets should be 16

Smaller max det is the minimal max det 232
  times the biggest postpended det inverse dilation factor
 4 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 3244, 928
```

```
Rank of T[G2G2]^+/- = (21,16)

Rank of plus basis attempt = 140 and dimension of S_4(K(278))^+ = 142

Rank of minus basis attempt = 16 and dimension of S_4(K(278))^- = 43


Hecke spread 2 out of 2

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 3244

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {21, 3244}, output dimensions {21, 3244}

Length, max det of input vectors to ParaHeckeOp: 3244, 928 (check: 3244, 928)

Desired length, max det of output vectors: 395, 232

Quotient of the max dets should be 4

Smaller max det is the minimal max det 232
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 395, 232

Rank of T[G2G2]^+/- = (21,16)

Rank of plus basis attempt = 140 and dimension of S_4(K(278))^+ = 142

Rank of minus basis attempt = 25 and dimension of S_4(K(278))^- = 43


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-278-Extended-rank-26-mod-12347-A.ma}

Dimensions of BPcoeffMatProven: {26, 1000}

Dimensions of join: {51, 395}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 140 and dimension of S_4(K(278))^+ = 142

Rank of minus basis attempt = 41 and dimension of S_4(K(278))^- = 43


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
```

```
Found files: {}

Didn't find a plus basis
Didn't find a minus basis

N = 282
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}
lift dimension of S_4(K(282))^+ : 49
nonlift dimension of S_4(K(282))^+ : 104
dimension of S_4(K(282))^- : 44
Ibukiyama-Kitayama dimension of S_4(K(282)) : 197

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(282))^+ = 153
Rank of minus basis attempt = 0 and dimension of S_4(K(282))^- = 44
Initial short vector length: 456
Have vectors of length 474: long enough
Determinant shell containing the vectors of length 456 : 255
Short vector length is 474, and the corresponding determinant is 255
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-282-260.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,282}^{cusp}), which is 49
Rank of plus basis attempt = 49 and dimension of S_4(K(282))^+ = 153
Rank of minus basis attempt = 0 and dimension of S_4(K(282))^- = 44

STEP 2: TRACE DOWN
N = 282  q = 5  Nq = 1410
474-th determinant is 255
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 528
Trace down to try to hit S4(K(N)), of dimension 197
Plus lifts, plus nonlifts, minus dimension: {49, 104, 44}
Target plus rank, target minus rank: 153 44
Dimensions of savedTargetMats: {474, 3}
Making ordered good sigs for source space S_4(K(Nq))...
```

```
 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1410-6375.ma, Getting it

 ... done making G2.  Dimensions of Grits: {32, 96744}

Determining indices: {1, 2, 5, 6, 9, 10, 11, 12, 25, 26, 27, 28, 49, 57, 58,
  77, 78, 79, 80, 101, 102, 125, 129, 130, 157, 158, 181, 182, 213, 214, 215, 317}

Atkin-Lehner truncation: 328

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1, 1}, 14}, {{1, 1, -1, -1}, 6},
  {{1, -1, 1, -1}, 5}, {{1, -1, -1, 1}, 3}, {{-1, 1, 1, -1}, 3}, {{-1, 1, -1, 1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1410-6375-to-282-255.ma exists, reading it

Found file Grits-2-1410-6375.ma, Getting it

Grit dimensions: {32, 96744}

Viable file found: GG-Grits-2-1410-6375.ma-6375-mod-12347.ma

New best file: GG-Grits-2-1410-6375.ma-6375-mod-12347.ma

Dimensions of G2G2: {528, 96744}

Dimensions of fDown = {528, 474}

Rank of fDown mod pp = 150

Dimensions and rank of plus space fDown = {528, 474}  150

Dimensions and rank of minus space fDown = {528, 474}  0

Rank of plus basis attempt = 151 and dimension of S_4(K(282))^+ = 153

Rank of minus basis attempt = 0 and dimension of S_4(K(282))^- = 44


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 15

Found file Grits-2-282-255.ma, Getting it

Viable file found: GG-Grits-2-282-1020.ma-1020-mod-12347.ma

New best file: GG-Grits-2-282-1020.ma-1020-mod-12347.ma

Viable file found: GG-Grits-2-282-16320.ma-16320-mod-12347.ma

Viable file found: GG-Grits-2-282-36720.ma-36720-mod-12347.ma

Viable file found: GG-Grits-2-282-684.ma-684-mod-12347.ma

New best file: GG-Grits-2-282-684.ma-684-mod-12347.ma

Dimensions of G2G2: {15, 474}

Rank of plus basis attempt = 151 and dimension of S_4(K(282))^+ = 153
```

```
Rank of minus basis attempt = 0 and dimension of S_4(K(282))^- = 44
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 4 Hecke spreads, the last of which dilates determinants by 144

Need to multiply the minimal max det 255 by the largest det contraction factor
 144 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 36 720

Hecke spreads: 4

Pre-Hecke expansions will have length 1 218 918

Found file Grits-2-282-36720.ma, Getting it

Viable file found: GG-Grits-2-282-36720.ma-36720-mod-12347.ma

New best file: GG-Grits-2-282-36720.ma-36720-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 9

{{2, 2}, {2, 1}} 1

{{3, 2}} 1

{{2, 2}, {3, 1}} 1

Hecke spread 1 out of 4

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 1 218 918

Called ShortenVecs with G2G2T[{}]

 input dimensions {15, 1 218 918}, output dimensions {15, 1 218 918}

Length, max det of input vectors to ParaHeckeOp: 1 218 918, 36 720 (check: 1 218 918, 36 720)

Desired length, max det of output vectors: 17 912, 2295

Quotient of the max dets should be 16

Smaller max det is the minimal max det 255
  times the biggest postpended det inverse dilation factor
 9 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 17 912, 2295
```

```
Rank of T[G2G2]^+/- = (15,10)

Rank of plus basis attempt = 151 and dimension of S_4(K(282))^+ = 153

Rank of minus basis attempt = 10 and dimension of S_4(K(282))^- = 44


Hecke spread 2 out of 4

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 17912

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {15, 17912}, output dimensions {15, 4794}

Length, max det of input vectors to ParaHeckeOp: 4794, 1020 (check: 4794, 1020)

Desired length, max det of output vectors: 474, 255

Quotient of the max dets should be 4

Smaller max det is the minimal max det 255
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 474, 255

Rank of T[G2G2]^+/- = (15,10)

Rank of plus basis attempt = 151 and dimension of S_4(K(282))^+ = 153

Rank of minus basis attempt = 17 and dimension of S_4(K(282))^- = 44


Hecke spread 3 out of 4

Is Hecke spread 3 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 1218918

Called ShortenVecs with G2G2T[{}]

 input dimensions {15, 1218918}, output dimensions {15, 509312}

Length, max det of input vectors to ParaHeckeOp: 509312, 20655 (check: 509312, 20655)

Desired length, max det of output vectors: 474, 255
```

```
Quotient of the max dets should be 81

Smaller max det is the minimal max det 255
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 3 length and max det should be as desired: 474, 255

Rank of T[G2G2]^+/- = (15,7)

Rank of plus basis attempt = 151 and dimension of S_4(K(282))^+ = 153

Rank of minus basis attempt = 24 and dimension of S_4(K(282))^- = 44


Hecke spread 4 out of 4

Is Hecke spread 4 present?  False

DoOneHecke called {},{{2, 2}, {3, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{3, 1}}

{tp,tpdel} = {3, 1}

Need to compute G2G2T[{{2, 2}, {3, 1}}] having G2G2T[{{2, 2}}] vectors to length 17 912

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {15, 17 912}, output dimensions {15, 17 912}

Length, max det of input vectors to ParaHeckeOp: 17 912, 2295 (check: 17 912, 2295)

Desired length, max det of output vectors: 474, 255

Quotient of the max dets should be 9

Smaller max det is the minimal max det 255
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {3, 1}}

About to compute G2G2T[{{2, 2}, {3, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 4 length and max det should be as desired: 474, 255

Rank of T[G2G2]^+/- = (15,10)

Rank of plus basis attempt = 151 and dimension of S_4(K(282))^+ = 153

Rank of minus basis attempt = 27 and dimension of S_4(K(282))^- = 44
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-282-Extended-rank-23-mod-12347-A.ma}
Dimensions of BPcoeffMatProven: {23, 1000}
Dimensions of join: {50, 474}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 151 and dimension of S_4(K(282))^+ = 153
Rank of minus basis attempt = 41 and dimension of S_4(K(282))^- = 44

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Didn't find a plus basis
Didn't find a minus basis

N = 285
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 1, 0}
lift dimension of S_4(K(285))^+ : 47
nonlift dimension of S_4(K(285))^+ : 97
dimension of S_4(K(285))^- : 25
Ibukiyama-Kitayama dimension of S_4(K(285)) : 169

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(285))^+ = 144
Rank of minus basis attempt = 0 and dimension of S_4(K(285))^- = 25
Initial short vector length: 359
Have vectors of length 368: long enough
Determinant shell containing the vectors of length 359 : 236
Short vector length is 360, and the corresponding determinant is 236
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-285-240.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,285}^{cusp}), which is 47
Rank of plus basis attempt = 47 and dimension of S_4(K(285))^+ = 144
Rank of minus basis attempt = 0 and dimension of S_4(K(285))^- = 25

STEP 2: TRACE DOWN
```

```
N = 285  q = 7  Nq = 1995

360-th determinant is 236

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 1275

Trace down to try to hit S4(K(N)), of dimension 169

Plus lifts, plus nonlifts, minus dimension: {47, 97, 25}

Target plus rank, target minus rank: 144 25

Dimensions of savedTargetMats: {360, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1995-11564.ma, Getting it

 ... done making G2.  Dimensions of Grits: {50, 186736}

Determining indices: {1, 2, 3, 4, 17, 18, 19, 20, 21, 22, 23, 24, 41, 42, 49, 50,
  57, 58, 59, 60, 61, 62, 63, 64, 121, 122, 137, 138, 139, 140, 185, 186, 209, 210,
  211, 212, 213, 214, 305, 306, 329, 330, 331, 332, 361, 362, 553, 554, 555, 625}

Atkin-Lehner truncation: 648

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims =
 {{{1, 1, 1, 1}, 14}, {{1, 1, -1, -1}, 7}, {{1, -1, 1, -1}, 8}, {{1, -1, -1, 1}, 5},
  {{-1, 1, 1, -1}, 5}, {{-1, 1, -1, 1}, 4}, {{-1, -1, 1, 1}, 4}, {{-1, -1, -1, -1}, 3}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1995-11564-to-285-236.ma
  exists, reading it

Found file Grits-2-1995-11564.ma, Getting it

Grit dimensions: {50, 186736}

Viable file found: GG-Grits-2-1995-11564.ma-11564-mod-12347.ma

New best file: GG-Grits-2-1995-11564.ma-11564-mod-12347.ma

Dimensions of G2G2: {1275, 186736}

Dimensions of fDown = {1275, 360}

Rank of fDown mod pp = 143

Dimensions and rank of plus space fDown = {1275, 360}  143

Dimensions and rank of minus space fDown = {1275, 360}  0

Rank of plus basis attempt = 143 and dimension of S_4(K(285))^+ = 144

Rank of minus basis attempt = 0 and dimension of S_4(K(285))^- = 25
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 6

Found file Grits-2-285-236.ma, Getting it

Viable file found: GG-Grits-2-285-19116.ma-19116-mod-12347.ma

New best file: GG-Grits-2-285-19116.ma-19116-mod-12347.ma

Viable file found: GG-Grits-2-285-236.ma-236-mod-12347.ma

New best file: GG-Grits-2-285-236.ma-236-mod-12347.ma

Viable file found: GG-Grits-2-285-76464.ma-76464-mod-12347.ma

Viable file found: GG-Grits-2-285-944.ma-944-mod-12347.ma

Dimensions of G2G2: {6, 360}

Rank of plus basis attempt = 143 and dimension of S_4(K(285))^+ = 144

Rank of minus basis attempt = 0 and dimension of S_4(K(285))^- = 25


STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 236 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 19 116

Hecke spreads: 1

Pre-Hecke expansions will have length 369 332

Found file Grits-2-285-19116.ma, Getting it

Viable file found: GG-Grits-2-285-19116.ma-19116-mod-12347.ma

New best file: GG-Grits-2-285-19116.ma-19116-mod-12347.ma

Viable file found: GG-Grits-2-285-76464.ma-76464-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 369 332

Called ShortenVecs with G2G2T[{}]

 input dimensions {6, 369 332}, output dimensions {6, 369 332}

Length, max det of input vectors to ParaHeckeOp: 369 332, 19 116 (check: 369 332, 19 116)

Desired length, max det of output vectors: 360, 236
```

```
Quotient of the max dets should be 81

Smaller max det is the minimal max det 236
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}

About to compute G2G2T[{{3, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 360, 236

Rank of T[G2G2]^+/- = (6,4)

Rank of plus basis attempt = 143 and dimension of S_4(K(285))^+ = 144

Rank of minus basis attempt = 4 and dimension of S_4(K(285))^- = 25


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-285-500-5-0.ma}

Dimensions of BPcoeffMatProven: {22, 1138}

Dimensions of join: {26, 360}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 143 and dimension of S_4(K(285))^+ = 144

Rank of minus basis attempt = 24 and dimension of S_4(K(285))^- = 25


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Didn't find a plus basis

Didn't find a minus basis


N = 286

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 15}

lift dimension of S_4(K(286))^+ : 48

nonlift dimension of S_4(K(286))^+ : 113

dimension of S_4(K(286))^- : 28

Ibukiyama-Kitayama dimension of S_4(K(286)) : 189


STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(286))^+ = 161

Rank of minus basis attempt = 0 and dimension of S_4(K(286))^- = 28
```

```
Initial short vector length: 479
Have vectors of length 492: long enough
Determinant shell containing the vectors of length 479 : 296
Adding determinant shell 1
Adding determinant shell 2
Adding determinant shell 3
Adding determinant shell 4
Adding determinant shell 5
Adding determinant shell 6
Adding determinant shell 7
Adding determinant shell 8
Adding determinant shell 9
Adding determinant shell 10
Adding determinant shell 11
Adding determinant shell 12
Adding determinant shell 13
Adding determinant shell 14
Adding determinant shell 15
Short vector length is 978, and the corresponding determinant is 428
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-286-428.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,286}^{cusp}), which is 48
Rank of plus basis attempt = 48 and dimension of S_4(K(286))^+ = 161
Rank of minus basis attempt = 0 and dimension of S_4(K(286))^- = 28

STEP 2: TRACE DOWN
N = 286  q = 5  Nq = 1430
978-th determinant is 428
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 595
Trace down to try to hit S4(K(N)), of dimension 189
Plus lifts, plus nonlifts, minus dimension: {48, 113, 28}
Target plus rank, target minus rank: 161 28
Dimensions of savedTargetMats: {978, 3}
Making ordered good sigs for source space S_4(K(Nq))...
```

```
 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1430-10700.ma, Getting it

 ... done making G2.  Dimensions of Grits: {34, 182 520}

Determining indices: {1, 2, 3, 4, 17, 18, 21, 22, 29, 30, 31, 32, 33, 34, 35, 36, 69,
  71, 101, 102, 103, 104, 125, 126, 133, 134, 157, 158, 189, 190, 209, 210, 257, 321}

Atkin-Lehner truncation: 344

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims =
 {{{1, 1, 1, 1}, 12}, {{1, 1, -1, -1}, 7}, {{1, -1, 1, -1}, 4}, {{1, -1, -1, 1}, 4},
  {{-1, 1, 1, -1}, 2}, {{-1, 1, -1, 1}, 2}, {{-1, -1, 1, 1}, 2}, {{-1, -1, -1, -1}, 1}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1430-10700-to-286-428.ma
  exists, reading it

Found file Grits-2-1430-10700.ma, Getting it

Grit dimensions: {34, 182 520}

Viable file found: GG-Grits-2-1430-14196.ma-14196-mod-12347.ma

New best file: GG-Grits-2-1430-14196.ma-14196-mod-12347.ma

Dimensions of G2G2: {595, 284 664}

Dimensions of fDown = {595, 978}

Rank of fDown mod pp = 157

Dimensions and rank of plus space fDown = {595, 978}  157

Dimensions and rank of minus space fDown = {595, 978}  0

Rank of plus basis attempt = 157 and dimension of S_4(K(286))^+ = 161

Rank of minus basis attempt = 0 and dimension of S_4(K(286))^- = 28


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 6

Found file Grits-2-286-428.ma, Getting it

Viable file found: GG-Grits-2-286-1184.ma-1184-mod-12347.ma

New best file: GG-Grits-2-286-1184.ma-1184-mod-12347.ma

Viable file found: GG-Grits-2-286-61632.ma-61632-mod-12347.ma

Viable file found: GG-Grits-2-286-9984.ma-9984-mod-12347.ma

Dimensions of G2G2: {6, 978}

Rank of plus basis attempt = 157 and dimension of S_4(K(286))^+ = 161

Rank of minus basis attempt = 0 and dimension of S_4(K(286))^- = 28
```

STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 2 Hecke spreads, the last of which dilates determinants by 64

Need to multiply the minimal max det 428 by the largest det contraction factor
 64 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 27 392

Hecke spreads: 2

Pre-Hecke expansions will have length 670 376

Found file Grits-2-286-27392.ma, Getting it

Viable file found: GG-Grits-2-286-61632.ma-61632-mod-12347.ma

New best file: GG-Grits-2-286-61632.ma-61632-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 4

{{2, 2}, {2, 1}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present? False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 2 292 504

Called ShortenVecs with G2G2T[{}]

 input dimensions {6, 2 292 504}, output dimensions {6, 670 376}

Length, max det of input vectors to ParaHeckeOp: 670 376, 27 392 (check: 670 376, 27 392)

Desired length, max det of output vectors: 9188, 1712

Quotient of the max dets should be 16

Smaller max det is the minimal max det 428
  times the biggest postpended det inverse dilation factor
 4 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 9188, 1712

Rank of T[G2G2]^+/- = (6,4)

Rank of plus basis attempt = 157 and dimension of S_4(K(286))^+ = 161

Rank of minus basis attempt = 4 and dimension of S_4(K(286))^- = 28

```
Hecke spread 2 out of 2

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 9188

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {6, 9188}, output dimensions {6, 9188}

Length, max det of input vectors to ParaHeckeOp: 9188, 1712 (check: 9188, 1712)

Desired length, max det of output vectors: 978, 428

Quotient of the max dets should be 4

Smaller max det is the minimal max det 428
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}
About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 978, 428

Rank of T[G2G2]^+/- = (6,4)

Rank of plus basis attempt = 157 and dimension of S_4(K(286))^+ = 161

Rank of minus basis attempt = 8 and dimension of S_4(K(286))^- = 28


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-286-500-5-0.ma}

Dimensions of BPcoeffMatProven: {22, 1212}

Dimensions of join: {30, 978}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 157 and dimension of S_4(K(286))^+ = 161

Rank of minus basis attempt = 27 and dimension of S_4(K(286))^- = 28


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {BPinfo-files/BPplusInfo-4-286-800-merge.ma}

(from inside plusInfo file) Dimensions of JSglobalBasisAttempt: {157, 978}

(from inside plusInfo file) Rank of JSglobalBasisAttempt: 157
```

(from inside plusInfo file) Dimensions of BPcoeffMatProven: {60, 2616}

(from inside plusInfo file) Rank of BPcoeffMatProven's numerical columns: 60

(from inside plusInfo file) Dimensions of BPcoeffMatProven after join: {217, 978}

(from inside plusInfo file) Dimensions of numerical BPcoeffMatProven after join: {217, 271}

(from inside plusInfo file) Rank of numerical BPcoeffMatProven after join: 161

New dimensions of JSglobalBasisAttempt: {217, 978}

Plus space span attempt may contain non-numerical entries

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 161 and dimension of S_4(K(286))^+ = 161

Rank of minus basis attempt = 27 and dimension of S_4(K(286))^- = 28

Found a plus basis of dimension 161

Didn't find a minus basis

N = 287

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 5}

lift dimension of S_4(K(287))^+ : 50

nonlift dimension of S_4(K(287))^+ : 84

dimension of S_4(K(287))^- : 28

Ibukiyama-Kitayama dimension of S_4(K(287)) : 162

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(287))^+ = 134

Rank of minus basis attempt = 0 and dimension of S_4(K(287))^- = 28

Initial short vector length: 297

Have vectors of length 310: long enough

Determinant shell containing the vectors of length 297 : 248

Adding determinant shell 1

Adding determinant shell 2

Adding determinant shell 3

Adding determinant shell 4

Adding determinant shell 5

Short vector length is 416, and the corresponding determinant is 279

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-287-279.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,287}^{cusp}), which is 50

Rank of plus basis attempt = 50 and dimension of S_4(K(287))^+ = 134

Rank of minus basis attempt = 0 and dimension of S_4(K(287))^- = 28
```

STEP 2: TRACE DOWN

```
N = 287  q = 5  Nq = 1435

416-th determinant is 279

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 1035

Trace down to try to hit S4(K(N)), of dimension 162

Plus lifts, plus nonlifts, minus dimension: {50, 84, 28}

Target plus rank, target minus rank: 134 28

Dimensions of savedTargetMats: {416, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1435-6975.ma, Getting it

 ... done making G2.  Dimensions of Grits: {45, 65436}

Determining indices:
 {1, 2, 5, 6, 7, 8, 17, 18, 21, 22, 23, 24, 33, 34, 45, 46, 53, 54, 57, 58, 61, 62, 81, 82, 83, 84,
  105, 106, 107, 108, 125, 126, 137, 138, 139, 177, 178, 189, 205, 206, 217, 241, 242, 317, 318}

Atkin-Lehner truncation: 340

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 18}, {{1, -1, -1}, 13}, {{-1, 1, -1}, 8}, {{-1, -1, 1}, 6}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1435-6975-to-287-279.ma exists, reading it

Found file Grits-2-1435-6975.ma, Getting it

Grit dimensions: {45, 65436}

Viable file found: GG-Grits-2-1435-6975.ma-6975-mod-12347.ma

New best file: GG-Grits-2-1435-6975.ma-6975-mod-12347.ma

Dimensions of G2G2: {1035, 65436}

Dimensions of fDown = {1035, 416}

Rank of fDown mod pp = 134

Dimensions and rank of plus space fDown = {1035, 416}  134

Dimensions and rank of minus space fDown = {1035, 416}  0
```

```
Rank of plus basis attempt = 134 and dimension of S_4(K(287))^+ = 134
Rank of minus basis attempt = 0 and dimension of S_4(K(287))^- = 28

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS
Pairs from dim(J_{2,N}^{cusp}: 10
Found file Grits-2-287-279.ma, Getting it
Viable file found: GG-Grits-2-287-17856.ma-17856-mod-12347.ma
New best file: GG-Grits-2-287-17856.ma-17856-mod-12347.ma
Viable file found: GG-Grits-2-287-33759.ma-33759-mod-12347.ma
Dimensions of G2G2: {10, 416}
Rank of plus basis attempt = 134 and dimension of S_4(K(287))^+ = 134
Rank of minus basis attempt = 0 and dimension of S_4(K(287))^- = 28

STEP 4: HECKE SPREAD
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-287-500.ma}
Dimensions of BPcoeffMatProven: {25, 884}
Dimensions of join: {25, 416}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 134 and dimension of S_4(K(287))^+ = 134
Rank of minus basis attempt = 25 and dimension of S_4(K(287))^- = 28

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Found a plus basis of dimension 134
Didn't find a minus basis

N = 290
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}
lift dimension of S_4(K(290))^+ : 50
nonlift dimension of S_4(K(290))^+ : 103
dimension of S_4(K(290))^- : 45
```

```
Ibukiyama-Kitayama dimension of S_4(K(290)) : 198

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(290))^+ = 153
Rank of minus basis attempt = 0 and dimension of S_4(K(290))^- = 45
Initial short vector length: 420
Have vectors of length 474: long enough
Determinant shell containing the vectors of length 420 : 239
Short vector length is 474, and the corresponding determinant is 239
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-290-240.ma, Getting it
Rank of plus basis attempt should match dim(J_{4,290}^{cusp}), which is 50
Rank of plus basis attempt = 50 and dimension of S_4(K(290))^+ = 153
Rank of minus basis attempt = 0 and dimension of S_4(K(290))^- = 45

STEP 2: TRACE DOWN
N = 290  q = 7  Nq = 2030
474-th determinant is 239
Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 1326
Trace down to try to hit S4(K(N)), of dimension 198
Plus lifts, plus nonlifts, minus dimension: {50, 103, 45}
Target plus rank, target minus rank: 153 45
Dimensions of savedTargetMats: {474, 3}
Making ordered good sigs for source space S_4(K(Nq))...
 ... done making ordered good sigs.
Making G2 for S_2(K(Nq))...
Found file Grits-2-2030-11711.ma, Getting it
 ... done making G2.  Dimensions of Grits: {51, 212660}
Determining indices: {1, 2, 5, 6, 7, 8, 13, 14, 33, 34, 35, 36, 73, 74, 75, 76, 77, 78,
  79, 80, 137, 138, 139, 140, 153, 165, 166, 189, 190, 201, 202, 217, 218, 219, 220,
  221, 222, 223, 224, 289, 290, 291, 292, 321, 322, 369, 370, 441, 442, 443, 489}
Atkin-Lehner truncation: 540
Beginning ALSignaturesAndDims...
 ... done with ALSignaturesAndDims.
```

```
ALspacesDims =
 {{{1, 1, 1, 1}, 16}, {{1, 1, -1, -1}, 11}, {{1, -1, 1, -1}, 8}, {{1, -1, -1, 1}, 6},
  {{-1, 1, 1, -1}, 3}, {{-1, 1, -1, 1}, 3}, {{-1, -1, 1, 1}, 2}, {{-1, -1, -1, -1}, 2}}

Need 8 AL signatures; have the signatures {{-1, -1, -1, -1}, {-1, -1, 1, 1}, {-1, 1, -1, 1},
  {-1, 1, 1, -1}, {1, -1, -1, 1}, {1, -1, 1, -1}, {1, 1, -1, -1}, {1, 1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-2030-11711-to-290-239.ma
  exists, reading it

Found file Grits-2-2030-11711.ma, Getting it

Grit dimensions: {51, 212660}

Viable file found: GG-Grits-2-2030-11711.ma-11711-mod-12347.ma

New best file: GG-Grits-2-2030-11711.ma-11711-mod-12347.ma

Dimensions of G2G2: {1326, 212660}

Dimensions of fDown = {1326, 474}

Rank of fDown mod pp = 150

Dimensions and rank of plus space fDown = {1326, 474}  150

Dimensions and rank of minus space fDown = {1326, 474}  0

Rank of plus basis attempt = 152 and dimension of S_4(K(290))^+ = 153

Rank of minus basis attempt = 0 and dimension of S_4(K(290))^- = 45


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 15

Found file Grits-2-290-239.ma, Getting it

Viable file found: GG-Grits-2-290-15296.ma-15296-mod-12347.ma

New best file: GG-Grits-2-290-15296.ma-15296-mod-12347.ma

Viable file found: GG-Grits-2-290-18944.ma-18944-mod-12347.ma

Viable file found: GG-Grits-2-290-34416.ma-34416-mod-12347.ma

Dimensions of G2G2: {15, 474}

Rank of plus basis attempt = 152 and dimension of S_4(K(290))^+ = 153

Rank of minus basis attempt = 0 and dimension of S_4(K(290))^- = 45


STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 3 Hecke spreads, the last of which dilates determinants by 144

Need to multiply the minimal max det 239 by the largest det contraction factor
 144 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 34416

Hecke spreads: 3
```

```
Pre-Hecke expansions will have length 1 003 786

Found file Grits-2-290-34416.ma, Getting it

Viable file found: GG-Grits-2-290-34416.ma-34416-mod-12347.ma

New best file: GG-Grits-2-290-34416.ma-34416-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 9

{{2, 2}, {2, 1}} 1

{{2, 2}, {3, 1}} 1

Hecke spread 1 out of 3

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 1 003 786

Called ShortenVecs with G2G2T[{}]

 input dimensions {15, 1 003 786}, output dimensions {15, 1 003 786}

Length, max det of input vectors to ParaHeckeOp: 1 003 786, 34 416 (check: 1 003 786, 34 416)

Desired length, max det of output vectors: 13 964, 2151

Quotient of the max dets should be 16

Smaller max det is the minimal max det 239
  times the biggest postpended det inverse dilation factor
 9 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 13 964, 2151

Rank of T[G2G2]^+/- = (15,15)

Rank of plus basis attempt = 152 and dimension of S_4(K(290))^+ = 153

Rank of minus basis attempt = 15 and dimension of S_4(K(290))^- = 45


Hecke spread 2 out of 3

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}
```

```
Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 13964

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {15, 13964}, output dimensions {15, 3996}

Length, max det of input vectors to ParaHeckeOp: 3996, 956 (check: 3996, 956)

Desired length, max det of output vectors: 474, 239

Quotient of the max dets should be 4

Smaller max det is the minimal max det 239
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}

About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 474, 239

Rank of T[G2G2]^+/- = (15,15)

Rank of plus basis attempt = 152 and dimension of S_4(K(290))^+ = 153

Rank of minus basis attempt = 29 and dimension of S_4(K(290))^- = 45


Hecke spread 3 out of 3

Is Hecke spread 3 present?  False

DoOneHecke called {},{{2, 2}, {3, 1}}

{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{3, 1}}

{tp,tpdel} = {3, 1}

Need to compute G2G2T[{{2, 2}, {3, 1}}] having G2G2T[{{2, 2}}] vectors to length 13964

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {15, 13964}, output dimensions {15, 13964}

Length, max det of input vectors to ParaHeckeOp: 13964, 2151 (check: 13964, 2151)

Desired length, max det of output vectors: 474, 239

Quotient of the max dets should be 9

Smaller max det is the minimal max det 239
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {3, 1}}

About to compute G2G2T[{{2, 2}, {3, 1}}]

Making abstract Hecke formula

Substituting given paramodular forms in abstract formula

Hecke spread 3 length and max det should be as desired: 474, 239

Rank of T[G2G2]^+/- = (15,15)
```

```
Rank of plus basis attempt = 152 and dimension of S_4(K(290))^+ = 153
Rank of minus basis attempt = 30 and dimension of S_4(K(290))^- = 45

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-290-507-5-0.ma}
Dimensions of BPcoeffMatProven: {24, 1384}
Dimensions of join: {54, 474}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 152 and dimension of S_4(K(290))^+ = 153
Rank of minus basis attempt = 43 and dimension of S_4(K(290))^- = 45

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Didn't find a plus basis
Didn't find a minus basis

N = 291
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 1, 1}
lift dimension of S_4(K(291))^+ : 55
nonlift dimension of S_4(K(291))^+ : 106
dimension of S_4(K(291))^- : 27
Ibukiyama-Kitayama dimension of S_4(K(291)) : 188

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(291))^+ = 161
Rank of minus basis attempt = 0 and dimension of S_4(K(291))^- = 27
Initial short vector length: 341
Have vectors of length 344: long enough
Determinant shell containing the vectors of length 341 : 227
Adding determinant shell 1
Short vector length is 353, and the corresponding determinant is 243
Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS
Found file Grits-4-291-243.ma, Getting it
```

```
Rank of plus basis attempt should match dim(J_{4,291}^{cusp}), which is 55

Rank of plus basis attempt = 55 and dimension of S_4(K(291))^+ = 161

Rank of minus basis attempt = 0 and dimension of S_4(K(291))^- = 27
```

STEP 2: TRACE DOWN

```
N = 291  q = 5  Nq = 1455

353-th determinant is 243

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 741

Trace down to try to hit S4(K(N)), of dimension 188

Plus lifts, plus nonlifts, minus dimension: {55, 106, 27}

Target plus rank, target minus rank: 161 27

Dimensions of savedTargetMats: {353, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1455-6075.ma, Getting it

 ... done making G2.  Dimensions of Grits: {38, 60904}

Determining indices: {1, 2, 3, 4, 5, 6, 9, 10, 13, 14, 21, 22, 23, 24, 37, 41, 42, 57, 58, 69,
  70, 81, 82, 83, 84, 121, 122, 137, 138, 161, 162, 181, 182, 183, 184, 221, 273, 277}

Atkin-Lehner truncation: 300

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 17}, {{1, -1, -1}, 10}, {{-1, 1, -1}, 7}, {{-1, -1, 1}, 4}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1455-6075-to-291-243.ma exists, reading it

Found file Grits-2-1455-6075.ma, Getting it

Grit dimensions: {38, 60904}

Viable file found: GG-Grits-2-1455-6075.ma-6075-mod-12347.ma

New best file: GG-Grits-2-1455-6075.ma-6075-mod-12347.ma

Dimensions of G2G2: {741, 60904}

Dimensions of fDown = {741, 353}

Rank of fDown mod pp = 156

Dimensions and rank of plus space fDown = {741, 353}  156

Dimensions and rank of minus space fDown = {741, 353}  0

Rank of plus basis attempt = 161 and dimension of S_4(K(291))^+ = 161

Rank of minus basis attempt = 0 and dimension of S_4(K(291))^- = 27
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 36

Found file Grits-2-291-243.ma, Getting it

Viable file found: GG-Grits-2-291-19683.ma-19683-mod-12347.ma

New best file: GG-Grits-2-291-19683.ma-19683-mod-12347.ma

Viable file found: GG-Grits-2-291-4608.ma-4608-mod-12347.ma

New best file: GG-Grits-2-291-4608.ma-4608-mod-12347.ma

Dimensions of G2G2: {36, 353}

Rank of plus basis attempt = 161 and dimension of S_4(K(291))^+ = 161

Rank of minus basis attempt = 0 and dimension of S_4(K(291))^- = 27
```

STEP 4: HECKE SPREAD

```
J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads

Preparing for 1 Hecke spreads, the last of which dilates determinants by 81

Need to multiply the minimal max det 243 by the largest det contraction factor
 81 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 19 683

Hecke spreads: 1

Pre-Hecke expansions will have length 314 703

Found file Grits-2-291-19683.ma, Getting it

Viable file found: GG-Grits-2-291-19683.ma-19683-mod-12347.ma

New best file: GG-Grits-2-291-19683.ma-19683-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{3, 2}} 1

Hecke spread 1 out of 1

Is Hecke spread 1 present?  False

DoOneHecke called {},{{3, 2}}

{tp,tpdel} = {3, 2}

Need to compute G2G2T[{{3, 2}}] having G2G2T[{}] vectors to length 314 703

Called ShortenVecs with G2G2T[{}]

 input dimensions {36, 314 703}, output dimensions {36, 314 703}

Length, max det of input vectors to ParaHeckeOp: 314 703, 19 683 (check: 314 703, 19 683)

Desired length, max det of output vectors: 353, 243

Quotient of the max dets should be 81
```

```
Smaller max det is the minimal max det 243
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{3, 2}}
About to compute G2G2T[{{3, 2}}]
Making abstract Hecke formula at a squared bad prime
Using Ralf's formula
Substituting given paramodular forms in abstract formula
Hecke spread 1 length and max det should be as desired: 353, 243
Rank of T[G2G2]^+/- = (35,13)
Rank of plus basis attempt = 161 and dimension of S_4(K(291))^+ = 161
Rank of minus basis attempt = 13 and dimension of S_4(K(291))^- = 27

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-291-500.ma}
Dimensions of BPcoeffMatProven: {8, 1097}
Dimensions of join: {21, 353}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 161 and dimension of S_4(K(291))^+ = 161
Rank of minus basis attempt = 20 and dimension of S_4(K(291))^- = 27

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS
Found files: {}

Found a plus basis of dimension 161
Didn't find a minus basis

N = 295
{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {7, 0, 0}
lift dimension of S_4(K(295))^+ : 54
nonlift dimension of S_4(K(295))^+ : 113
dimension of S_4(K(295))^- : 22
Ibukiyama-Kitayama dimension of S_4(K(295)) : 189

STEP 0: SET UP
Rank of plus basis attempt = 0 and dimension of S_4(K(295))^+ = 167
Rank of minus basis attempt = 0 and dimension of S_4(K(295))^- = 22
Initial short vector length: 312
```

```
Have vectors of length 345: long enough

Determinant shell containing the vectors of length 312 : 231

Short vector length is 331, and the corresponding determinant is 231

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-295-240.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,295}^{cusp}), which is 54

Rank of plus basis attempt = 54 and dimension of S_4(K(295))^+ = 167

Rank of minus basis attempt = 0 and dimension of S_4(K(295))^- = 22
```

STEP 2: TRACE DOWN

```
N = 295  q = 7  Nq = 2065

331-th determinant is 231

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 2346

Trace down to try to hit S4(K(N)), of dimension 189

Plus lifts, plus nonlifts, minus dimension: {54, 113, 22}

Target plus rank, target minus rank: 167 22

Dimensions of savedTargetMats: {331, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-2065-11319.ma, Getting it

 ... done making G2.  Dimensions of Grits: {68, 134 982}

Determining indices: {1, 2, 3, 4, 9, 10, 11, 12, 21, 22, 25, 26, 33, 34, 41, 42, 47, 48,
  51, 52, 53, 54, 75, 76, 77, 78, 107, 108, 111, 112, 113, 114, 135, 136, 137, 138,
  155, 156, 171, 172, 183, 184, 191, 192, 219, 220, 221, 222, 251, 252, 267, 268,
  291, 292, 315, 316, 339, 340, 341, 342, 363, 364, 371, 373, 403, 404, 463, 527}

Atkin-Lehner truncation: 574

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 26}, {{1, -1, -1}, 18}, {{-1, 1, -1}, 14}, {{-1, -1, 1}, 10}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-2065-11319-to-295-231.ma
  exists, reading it

Found file Grits-2-2065-11319.ma, Getting it

Grit dimensions: {68, 134 982}
```

```
Viable file found: GG-Grits-2-2065-11319.ma-11319-mod-12347.ma
New best file: GG-Grits-2-2065-11319.ma-11319-mod-12347.ma
Dimensions of G2G2: {2346, 134 982}
Dimensions of fDown = {2346, 331}
Rank of fDown mod pp = 167
Dimensions and rank of plus space fDown = {2346, 331}  167
Dimensions and rank of minus space fDown = {2346, 331}  0
Rank of plus basis attempt = 167 and dimension of S_4(K(295))^+ = 167
Rank of minus basis attempt = 0 and dimension of S_4(K(295))^- = 22
```

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

```
Pairs from dim(J_{2,N}^{cusp}: 21
Found file Grits-2-295-231.ma, Getting it
Viable file found: GG-Grits-2-295-17199.ma-17199-mod-12347.ma
New best file: GG-Grits-2-295-17199.ma-17199-mod-12347.ma
Dimensions of G2G2: {21, 331}
Rank of plus basis attempt = 167 and dimension of S_4(K(295))^+ = 167
Rank of minus basis attempt = 0 and dimension of S_4(K(295))^- = 22
```

STEP 4: HECKE SPREAD

```
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0
```

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

```
Found files: {BPinfo-files/BPminusInfo-4-295-500.ma}
Dimensions of BPcoeffMatProven: {17, 956}
Dimensions of join: {17, 331}
Minus space span attempt may contain non-numerical entries
Rank of plus basis attempt = 167 and dimension of S_4(K(295))^+ = 167
Rank of minus basis attempt = 17 and dimension of S_4(K(295))^- = 22
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Found a plus basis of dimension 167
Didn't find a minus basis
```

N = 298

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}

lift dimension of S_4(K(298))^+ : 57

nonlift dimension of S_4(K(298))^+ : 128

dimension of S_4(K(298))^- : 38

Ibukiyama-Kitayama dimension of S_4(K(298)) : 223

STEP 0: SET UP

Rank of plus basis attempt = 0 and dimension of S_4(K(298))^+ = 185

Rank of minus basis attempt = 0 and dimension of S_4(K(298))^- = 38

Initial short vector length: 428

Have vectors of length 442: long enough

Determinant shell containing the vectors of length 428 : 263

Short vector length is 436, and the corresponding determinant is 263

Vectors fill out a determinant shell, as they should

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

Found file Grits-4-298-270.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,298}^{cusp}), which is 57

Rank of plus basis attempt = 57 and dimension of S_4(K(298))^+ = 185

Rank of minus basis attempt = 0 and dimension of S_4(K(298))^- = 38

STEP 2: TRACE DOWN

N = 298 q = 5 Nq = 1490

436-th determinant is 263

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 1081

Trace down to try to hit S4(K(N)), of dimension 223

Plus lifts, plus nonlifts, minus dimension: {57, 128, 38}

Target plus rank, target minus rank: 185 38

Dimensions of savedTargetMats: {436, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1490-6575.ma, Getting it

 ... done making G2. Dimensions of Grits: {46, 78 312}

```
Determining indices: {1, 2, 3, 4, 9, 11, 12, 15, 16, 17, 18, 27, 28,
  33, 34, 35, 36, 49, 50, 63, 64, 75, 85, 86, 101, 102, 121, 127, 128, 139, 140,
  151, 152, 153, 154, 191, 219, 231, 232, 257, 258, 289, 290, 313, 327, 328}

Atkin-Lehner truncation: 378

Beginning ALSignaturesAndDims...

 ... done with ALSignaturesAndDims.

ALspacesDims = {{{1, 1, 1}, 23}, {{1, -1, -1}, 15}, {{-1, 1, -1}, 5}, {{-1, -1, 1}, 3}}

Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}

Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1490-6575-to-298-263.ma exists, reading it

Found file Grits-2-1490-6575.ma, Getting it

Grit dimensions: {46, 78 312}

Viable file found: GG-Grits-2-1490-6575.ma-6575-mod-12347.ma

New best file: GG-Grits-2-1490-6575.ma-6575-mod-12347.ma

Dimensions of G2G2: {1081, 78 312}

Dimensions of fDown = {1081, 436}

Rank of fDown mod pp = 180

Dimensions and rank of plus space fDown = {1081, 436}  180

Dimensions and rank of minus space fDown = {1081, 436}  0

Rank of plus basis attempt = 182 and dimension of S_4(K(298))^+ = 185

Rank of minus basis attempt = 0 and dimension of S_4(K(298))^- = 38


STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS

Pairs from dim(J_{2,N}^{cusp}: 36

Found file Grits-2-298-263.ma, Getting it

Viable file found: GG-Grits-2-298-16807.ma-16807-mod-12347.ma

New best file: GG-Grits-2-298-16807.ma-16807-mod-12347.ma

Viable file found: GG-Grits-2-298-16832.ma-16832-mod-12347.ma

Viable file found: GG-Grits-2-298-8100.ma-8100-mod-12347.ma

New best file: GG-Grits-2-298-8100.ma-8100-mod-12347.ma

Dimensions of G2G2: {36, 436}

Rank of plus basis attempt = 182 and dimension of S_4(K(298))^+ = 185

Rank of minus basis attempt = 0 and dimension of S_4(K(298))^- = 38


STEP 4: HECKE SPREAD

J2 nonzero, and don't have plus space basis or extra Hecke spreads were specified

Found custom table of Hecke spreads
```

```
Preparing for 2 Hecke spreads, the last of which dilates determinants by 64

Need to multiply the minimal max det 263 by the largest det contraction factor
 64 under Hecke spreads to get the pre-Hecke max det

Need pre-Hecke expansions up to determinant 16 832

Hecke spreads: 2

Pre-Hecke expansions will have length 280 272

Found file Grits-2-298-16832.ma, Getting it

Viable file found: GG-Grits-2-298-16832.ma-16832-mod-12347.ma

New best file: GG-Grits-2-298-16832.ma-16832-mod-12347.ma

Hecke operators to spread by, biggest det contraction
  factor that each can be followed by as a head of itself or another

{{2, 2}} 4

{{2, 2}, {2, 1}} 1

Hecke spread 1 out of 2

Is Hecke spread 1 present?  False

DoOneHecke called {},{{2, 2}}

{tp,tpdel} = {2, 2}

Need to compute G2G2T[{{2, 2}}] having G2G2T[{}] vectors to length 280 272

Called ShortenVecs with G2G2T[{}]

 input dimensions {36, 280 272}, output dimensions {36, 280 272}

Length, max det of input vectors to ParaHeckeOp: 280 272, 16 832 (check: 280 272, 16 832)

Desired length, max det of output vectors: 3950, 1052

Quotient of the max dets should be 16

Smaller max det is the minimal max det 263
  times the biggest postpended det inverse dilation factor
 4 for the Hecke operator{{2, 2}}

About to compute G2G2T[{{2, 2}}]

Making abstract Hecke formula at a squared bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 1 length and max det should be as desired: 3950, 1052

Rank of T[G2G2]^+/- = (36,21)

Rank of plus basis attempt = 182 and dimension of S_4(K(298))^+ = 185

Rank of minus basis attempt = 21 and dimension of S_4(K(298))^- = 38


Hecke spread 2 out of 2

Is Hecke spread 2 present?  False

DoOneHecke called {},{{2, 2}, {2, 1}}
```

```
{tp,tpdel} = {2, 2}

DoOneHecke called {{2, 2}},{{2, 1}}

{tp,tpdel} = {2, 1}

Need to compute G2G2T[{{2, 2}, {2, 1}}] having G2G2T[{{2, 2}}] vectors to length 3950

Called ShortenVecs with G2G2T[{{2, 2}}]

 input dimensions {36, 3950}, output dimensions {36, 3950}

Length, max det of input vectors to ParaHeckeOp: 3950, 1052 (check: 3950, 1052)

Desired length, max det of output vectors: 436, 263

Quotient of the max dets should be 4

Smaller max det is the minimal max det 263
  times the biggest postpended det inverse dilation factor
 1 for the Hecke operator{{2, 2}, {2, 1}}
About to compute G2G2T[{{2, 2}, {2, 1}}]

Making abstract Hecke formula at a bad prime

Using Ralf's formula

Substituting given paramodular forms in abstract formula

Hecke spread 2 length and max det should be as desired: 436, 263

Rank of T[G2G2]^+/- = (36,21)

Rank of plus basis attempt = 182 and dimension of S_4(K(298))^+ = 185

Rank of minus basis attempt = 24 and dimension of S_4(K(298))^- = 38


STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS

Found files: {BPinfo-files/BPminusInfo-4-298-Extended-rank-20-mod-12347-D.ma}

Dimensions of BPcoeffMatProven: {20, 1000}

Dimensions of join: {44, 436}

Minus space span attempt may contain non-numerical entries

Rank of plus basis attempt = 182 and dimension of S_4(K(298))^+ = 185

Rank of minus basis attempt = 33 and dimension of S_4(K(298))^- = 38


STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

Found files: {}


Didn't find a plus basis

Didn't find a minus basis


N = 299

{qDown,extraHeckeSpreads,extraShortVectorDetShells} = {5, 0, 0}
```

```
lift dimension of S_4(K(299))^+ : 52

nonlift dimension of S_4(K(299))^+ : 93

dimension of S_4(K(299))^- : 29

Ibukiyama-Kitayama dimension of S_4(K(299)) : 174
```

STEP 0: SET UP

```
Rank of plus basis attempt = 0 and dimension of S_4(K(299))^+ = 145

Rank of minus basis attempt = 0 and dimension of S_4(K(299))^- = 29

Initial short vector length: 506

Have vectors of length 543: long enough

Determinant shell containing the vectors of length 506 : 352

Short vector length is 515, and the corresponding determinant is 352

Vectors fill out a determinant shell, as they should
```

STEP 1: START THE SPANNING ATTEMPT WITH WEIGHT 4 GRITSENKO LIFTS

```
Found file Grits-4-299-360.ma, Getting it

Rank of plus basis attempt should match dim(J_{4,299}^{cusp}), which is 52

Rank of plus basis attempt = 52 and dimension of S_4(K(299))^+ = 145

Rank of minus basis attempt = 0 and dimension of S_4(K(299))^- = 29
```

STEP 2: TRACE DOWN

```
N = 299  q = 5  Nq = 1495

515-th determinant is 352

Products from Grit(J_{2,Nq}^{cusp}) in S_4(K(Nq))^+ = 861

Trace down to try to hit S4(K(N)), of dimension 174

Plus lifts, plus nonlifts, minus dimension: {52, 93, 29}

Target plus rank, target minus rank: 145 29

Dimensions of savedTargetMats: {515, 3}

Making ordered good sigs for source space S_4(K(Nq))...

 ... done making ordered good sigs.

Making G2 for S_2(K(Nq))...

Found file Grits-2-1495-8800.ma, Getting it

 ... done making G2.  Dimensions of Grits: {41, 88344}

Determining indices:
 {1, 2, 5, 6, 7, 8, 13, 14, 15, 16, 29, 30, 31, 32, 49, 50, 53, 54, 61, 62, 69, 70, 71, 72,
  109, 110, 121, 122, 123, 124, 173, 177, 178, 179, 180, 213, 214, 215, 321, 322, 323}

Atkin-Lehner truncation: 348
```

```
Beginning ALSignaturesAndDims...
 ... done with ALSignaturesAndDims.
ALspacesDims = {{{1, 1, 1}, 14}, {{1, -1, -1}, 10}, {{-1, 1, -1}, 9}, {{-1, -1, 1}, 8}}
Need 4 AL signatures; have the signatures {{-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}, {1, 1, 1}}
Abstract TraceDown list file
 abstractTraceDownList/abstractTraceDownList-4-1495-8800-to-299-352.ma exists, reading it
Found file Grits-2-1495-8800.ma, Getting it
Grit dimensions: {41, 88 344}
Viable file found: GG-Grits-2-1495-8800.ma-8800-mod-12347.ma
New best file: GG-Grits-2-1495-8800.ma-8800-mod-12347.ma
Dimensions of G2G2: {861, 88 344}
Dimensions of fDown = {861, 515}
Rank of fDown mod pp = 144
Dimensions and rank of plus space fDown = {861, 515}  144
Dimensions and rank of minus space fDown = {861, 515}  0
Rank of plus basis attempt = 145 and dimension of S_4(K(299))^+ = 145
Rank of minus basis attempt = 0 and dimension of S_4(K(299))^- = 29

STEP 3: ADD THE PRODUCTS OF PAIRS OF WEIGHT 2 GRITSENKO LIFTS
Pairs from dim(J_{2,N}^{cusp}: 10
Found file Grits-2-299-352.ma, Getting it
Viable file found: GG-Grits-2-299-23895.ma-23895-mod-12347.ma
New best file: GG-Grits-2-299-23895.ma-23895-mod-12347.ma
Viable file found: GG-Grits-2-299-29500.ma-29500-mod-12347.ma
Dimensions of G2G2: {10, 515}
Rank of plus basis attempt = 145 and dimension of S_4(K(299))^+ = 145
Rank of minus basis attempt = 0 and dimension of S_4(K(299))^- = 29

STEP 4: HECKE SPREAD
J2=0, or already have plus space basis and no extra Hecke spreads were specified
Hecke spreads: 0

STEP 5: BORCHERDS PRODUCT MINUS SPACE VECTORS
Found files: {BPinfo-files/BPminusInfo-4-299-500-5-0.ma}
Dimensions of BPcoeffMatProven: {26, 797}
Dimensions of join: {26, 515}
Minus space span attempt may contain non-numerical entries
```

```
Rank of plus basis attempt = 145 and dimension of S_4(K(299))^+ = 145
Rank of minus basis attempt = 26 and dimension of S_4(K(299))^- = 29
```

STEP 6: BORCHERDS PRODUCT PLUS SPACE VECTORS

```
Found files: {}

Found a plus basis of dimension 145
Didn't find a minus basis
```