```
globalNMin = 62, globalNMax = 300
New computations will use products in H_4(N,n,1) and H_4(N,n,n)+ for n <= 4

N = 62
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {11, 11}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {1, 1}
dim(J_{2,N}^{cusp}) = 0
The file reports a plus basis attempt of dimensions {11, 29}
and a minus basis attempt of dimensions {1, 29}
ShortVectorFinalDet = 48
ShortVectorLength = 29

Entering H4Nddindices with d = 1
 Number of indices is 27
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 11 - 11 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 11)
H4Ndd(1,+) says that S_2(K(N))=0

N = 65
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {13, 13}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {0, 0}
dim(J_{2,N}^{cusp}) = 1
The file reports a plus basis attempt of dimensions {13, 28}
and a minus basis attempt of dimensions {0}
ShortVectorFinalDet = 55
ShortVectorLength = 28

Entering H4Nddindices with d = 1
 Number of indices is 24
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 13 - 12 = 1, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 13)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 13 - 12 - 0 = 1, needs to be less than 2
 (Full rank of plus and minus basis attempts mod 12347 are 13 and 0)
```

```
H4Nd1(1) says that S_2(K(N))=Grit(J_{2,N}^{cusp})
```

N = 66

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {11, 11}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {0, 0}
dim(J_{2,N}^{cusp}) = 0
The file reports a plus basis attempt of dimensions {11, 28}
and a minus basis attempt of dimensions {0}
ShortVectorFinalDet = 44
ShortVectorLength = 28

Entering H4Nddindices with d = 1
 Number of indices is 26
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 11 - 11 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 11)
H4Ndd(1,+) says that S_2(K(N))=0
```

N = 69

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {14, 14}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {0, 0}
dim(J_{2,N}^{cusp}) = 0
The file reports a plus basis attempt of dimensions {14, 29}
and a minus basis attempt of dimensions {0}
ShortVectorFinalDet = 56
ShortVectorLength = 29

Entering H4Nddindices with d = 1
 Number of indices is 27
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 14 - 14 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 14)
H4Ndd(1,+) says that S_2(K(N))=0
```

N = 70

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {14, 14}
```

```
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {0, 0}
dim(J_{2,N}^{cusp}) = 0
The file reports a plus basis attempt of dimensions {14, 28}
and a minus basis attempt of dimensions {0}
ShortVectorFinalDet = 55
ShortVectorLength = 28

Entering H4Nddindices with d = 1
 Number of indices is 28
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 14 - 14 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 14)
H4Ndd(1,+) says that S_2(K(N))=0
```

N = 74

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {16, 16}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {1, 1}
dim(J_{2,N}^{cusp}) = 1
The file reports a plus basis attempt of dimensions {16, 39}
and a minus basis attempt of dimensions {1, 39}
ShortVectorFinalDet = 48
ShortVectorLength = 39

Entering H4Nddindices with d = 1
 Number of indices is 31
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 16 - 15 = 1, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 16)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 17 - 15 - 1 = 1, needs to be less than 2
 (Full rank of plus and minus basis attempts mod 12 347 are 16 and 1)
H4Nd1(1) says that S_2(K(N))=Grit(J_{2,N}^{cusp})
```

N = 77

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {15, 15}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {0, 0}
```

```
dim(J_{2,N}^{cusp}) = 1
The file reports a plus basis attempt of dimensions {15, 23}
and a minus basis attempt of dimensions {0}
ShortVectorFinalDet = 55
ShortVectorLength = 23

Entering H4Nddindices with d = 1
 Number of indices is 22
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 15 - 14 = 1, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 15)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 15 - 14 - 0 = 1, needs to be less than 2
 (Full rank of plus and minus basis attempts mod 12347 are 15 and 0)
H4Nd1(1) says that S_2(K(N))=Grit(J_{2,N}^{cusp})
```

N = 78

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {14, 14}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {1, 1}
dim(J_{2,N}^{cusp}) = 0
The file reports a plus basis attempt of dimensions {14, 36}
and a minus basis attempt of dimensions {1, 36}
ShortVectorFinalDet = 56
ShortVectorLength = 36

Entering H4Nddindices with d = 1
 Number of indices is 34
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 14 - 14 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 14)
H4Ndd(1,+) says that S_2(K(N))=0
```

N = 82

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {22, 22}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {1, 1}
dim(J_{2,N}^{cusp}) = 1
```

```
The file reports a plus basis attempt of dimensions {22, 44}
and a minus basis attempt of dimensions {1, 44}
ShortVectorFinalDet = 64
ShortVectorLength = 44

Entering H4Nddindices with d = 1
 Number of indices is 38
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 22 - 21 = 1, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 22)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 23 - 21 - 1 = 1, needs to be less than 2
 (Full rank of plus and minus basis attempts mod 12347 are 22 and 1)
H4Nd1(1) says that S_2(K(N))=Grit(J_{2,N}^{cusp})

N = 85
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {24, 24}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {0, 0}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {24, 34}
and a minus basis attempt of dimensions {0}
ShortVectorFinalDet = 60
ShortVectorLength = 34

Entering H4Nddindices with d = 1
 Number of indices is 26
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 24 - 21 = 3, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 24)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 24 - 21 - 0 = 3, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12347 are 24 and 0)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 24 - 0 - 21 = 3, needs to be less than 3
 (Full rank of plus basis attempt mod 12347 is 24)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
```

```
 Rigorous upper bound of dim(H_4(N,n,1))^- is 24 - 24 - 0 = 0, needs to be less than 2
 (Full rank of minus basis attempt mod 12347 is 0)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 33
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 24 - 24 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 24)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 86
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {20, 20}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {1, 1}
dim(J_{2,N}^{cusp}) = 1
The file reports a plus basis attempt of dimensions {20, 62}
and a minus basis attempt of dimensions {1, 62}
ShortVectorFinalDet = 71
ShortVectorLength = 62

Entering H4Nddindices with d = 1
 Number of indices is 51
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 20 - 19 = 1, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 20)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 21 - 19 - 1 = 1, needs to be less than 2
 (Full rank of plus and minus basis attempts mod 12347 are 20 and 1)
H4Nd1(1) says that S_2(K(N))=Grit(J_{2,N}^{cusp})

N = 87
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {19, 19}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {1, 1}
dim(J_{2,N}^{cusp}) = 0
The file reports a plus basis attempt of dimensions {19, 50}
and a minus basis attempt of dimensions {1, 50}
```

```
ShortVectorFinalDet = 71
ShortVectorLength = 50

Entering H4Nddindices with d = 1
 Number of indices is 50
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 19 - 19 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 19)
H4Ndd(1,+) says that S_2(K(N))=0
```

N = 91

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {26, 26}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {0, 0}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {26, 46}
and a minus basis attempt of dimensions {0}
ShortVectorFinalDet = 68
ShortVectorLength = 46

Entering H4Nddindices with d = 1
 Number of indices is 36
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 26 - 23 = 3, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 26)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 26 - 23 - 0 = 3, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12347 are 26 and 0)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 26 - 0 - 23 = 3, needs to be less than 3
 (Full rank of plus basis attempt mod 12347 is 26)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 26 - 26 - 0 = 0, needs to be less than 2
 (Full rank of minus basis attempt mod 12347 is 0)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
```

```
 Number of indices is 44
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 26 - 26 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 26)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 93
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {24, 24}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {1, 1}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {24, 45}
and a minus basis attempt of dimensions {1, 45}
ShortVectorFinalDet = 75
ShortVectorLength = 45

Entering H4Nddindices with d = 1
 Number of indices is 34
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 24 - 21 = 3, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 24)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 25 - 21 - 1 = 3, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12347 are 24 and 1)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 25 - 1 - 21 = 3, needs to be less than 3
 (Full rank of plus basis attempt mod 12347 is 24)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 25 - 24 - 1 = 0, needs to be less than 2
 (Full rank of minus basis attempt mod 12347 is 1)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 42
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 24 - 24 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 24)
```

```
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1
```

N = 94

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {27, 27}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {1, 1}
dim(J_{2,N}^{cusp}) = 0
The file reports a plus basis attempt of dimensions {27, 55}
and a minus basis attempt of dimensions {1, 55}
ShortVectorFinalDet = 76
ShortVectorLength = 55

Entering H4Nddindices with d = 1
 Number of indices is 51
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 27 - 27 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 27)
H4Ndd(1,+) says that S_2(K(N))=0
```

N = 95

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {21, 21}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {1, 1}
dim(J_{2,N}^{cusp}) = 0
The file reports a plus basis attempt of dimensions {21, 47}
and a minus basis attempt of dimensions {1, 47}
ShortVectorFinalDet = 71
ShortVectorLength = 47

Entering H4Nddindices with d = 1
 Number of indices is 45
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 21 - 21 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 21)
H4Ndd(1,+) says that S_2(K(N))=0
```

N = 102

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {24, 24}
```

```
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {2, 2}
dim(J_{2,N}^{cusp}) = 1
The file reports a plus basis attempt of dimensions {24, 62}
and a minus basis attempt of dimensions {2, 62}
ShortVectorFinalDet = 84
ShortVectorLength = 62

Entering H4Nddindices with d = 1
 Number of indices is 54
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 24 - 23 = 1, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 24)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 26 - 17 - 2 = 7, needs to be less than 2
 (Full rank of plus and minus basis attempts mod 12 347 are 24 and 2)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 26 - 2 - 23 = 1, needs to be less than 2
 (Full rank of plus basis attempt mod 12 347 is 24)
H4Nd1(1,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 26 - 24 - 2 = 0, needs to be less than 1
 (Full rank of minus basis attempt mod 12 347 is 2)
H4Nd1(1,-) says that S_2(K(N))^-=0

N = 105
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {24, 24}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {0, 0}
dim(J_{2,N}^{cusp}) = 0
The file reports a plus basis attempt of dimensions {24, 62}
and a minus basis attempt of dimensions {0}
ShortVectorFinalDet = 96
ShortVectorLength = 62

Entering H4Nddindices with d = 1
 Number of indices is 54
Entering H4Nddplus routine with d = 1
```

```
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 24 - 24 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 24)
H4Ndd(1,+) says that S_2(K(N))=0

N = 106
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {34, 34}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {1, 1}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {34, 68}
and a minus basis attempt of dimensions {1, 68}
ShortVectorFinalDet = 68
ShortVectorLength = 68

Entering H4Nddindices with d = 1
 Number of indices is 51
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 34 - 30 = 4, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 34)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 35 - 30 - 1 = 4, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12347 are 34 and 1)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 35 - 1 - 30 = 4, needs to be less than 3
 (Full rank of plus basis attempt mod 12347 is 34)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 35 - 34 - 1 = 0, needs to be less than 2
 (Full rank of minus basis attempt mod 12347 is 1)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 64
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 34 - 34 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 34)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1
```

N = 110

{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {25, 25}

{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {3, 3}

dim(J_{2,N}^{cusp}) = 0

The file reports a plus basis attempt of dimensions {25, 72}

and a minus basis attempt of dimensions {3, 72}

ShortVectorFinalDet = 84

ShortVectorLength = 72

Entering H4Nddindices with d = 1

Number of indices is 72

Entering H4Nddplus routine with d = 1

Rigorous upper bound of dim(H_4(N,n,n))^+ is 25 - 25 = 0, needs to be 0

(Full rank of plus basis attempt mod 12 347 is 25)

H4Ndd(1,+) says that S_2(K(N))=0

N = 111

{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {32, 32}

{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {1, 1}

dim(J_{2,N}^{cusp}) = 1

The file reports a plus basis attempt of dimensions {32, 69}

and a minus basis attempt of dimensions {1, 69}

ShortVectorFinalDet = 84

ShortVectorLength = 69

Entering H4Nddindices with d = 1

Number of indices is 60

Entering H4Nddplus routine with d = 1

Rigorous upper bound of dim(H_4(N,n,n))^+ is 32 - 31 = 1, needs to be 0

(Full rank of plus basis attempt mod 12 347 is 32)

Entering H4Nd1 routine with {d,sgn} = {1, 0}

Rigorous upper bound of dim(H_4(N,n,1)) is 33 - 31 - 1 = 1, needs to be less than 2

(Full rank of plus and minus basis attempts mod 12 347 are 32 and 1)

H4Nd1(1) says that S_2(K(N))=Grit(J_{2,N}^{cusp})

N = 114

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {30, 30}

{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {2, 2}

dim(J_{2,N}^{cusp}) = 1

The file reports a plus basis attempt of dimensions {30, 78}

and a minus basis attempt of dimensions {2, 78}

ShortVectorFinalDet = 84

ShortVectorLength = 78


Entering H4Nddindices with d = 1

 Number of indices is 64

Entering H4Nddplus routine with d = 1

 Rigorous upper bound of dim(H_4(N,n,n))^+ is 30 - 28 = 2, needs to be 0

 (Full rank of plus basis attempt mod 12 347 is 30)

Entering H4Nd1 routine with {d,sgn} = {1, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 32 - 28 - 2 = 2, needs to be less than 2

 (Full rank of plus and minus basis attempts mod 12 347 are 30 and 2)

Entering H4Nd1 routine with {d,sgn} = {1, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is 32 - 2 - 28 = 2, needs to be less than 2

 (Full rank of plus basis attempt mod 12 347 is 30)

Entering H4Nd1 routine with {d,sgn} = {1, -1}

 Rigorous upper bound of dim(H_4(N,n,1))^- is 32 - 30 - 2 = 0, needs to be less than 1

 (Full rank of minus basis attempt mod 12 347 is 2)

H4Nd1(1,-) says that S_2(K(N))^-=0


Entering H4Nddindices with d = 2

 Number of indices is 75

Entering H4Nddplus routine with d = 2

 Rigorous upper bound of dim(H_4(N,n,n))^+ is 30 - 30 = 0, needs to be 0

 (Full rank of plus basis attempt mod 12 347 is 30)

H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1


N = 115

{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {35, 35}

{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {1, 1}

dim(J_{2,N}^{cusp}) = 2

The file reports a plus basis attempt of dimensions {35, 66}
```

```
and a minus basis attempt of dimensions {1, 66}
ShortVectorFinalDet = 84
ShortVectorLength = 66

Entering H4Nddindices with d = 1
 Number of indices is 56
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 35 - 32 = 3, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 35)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 36 - 25 - 1 = 10, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12347 are 35 and 1)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 36 - 1 - 32 = 3, needs to be less than 3
 (Full rank of plus basis attempt mod 12347 is 35)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 36 - 35 - 1 = 0, needs to be less than 2
 (Full rank of minus basis attempt mod 12347 is 1)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 64
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 35 - 35 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 35)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 118
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {38, 38}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {3, 3}
dim(J_{2,N}^{cusp}) = 1
The file reports a plus basis attempt of dimensions {38, 85}
and a minus basis attempt of dimensions {3, 85}
ShortVectorFinalDet = 96
ShortVectorLength = 85
```

```
Entering H4Nddindices with d = 1
 Number of indices is 75
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 38 - 37 = 1, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 38)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 41 - 27 - 3 = 11, needs to be less than 2
 (Full rank of plus and minus basis attempts mod 12347 are 38 and 3)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 41 - 3 - 37 = 1, needs to be less than 2
 (Full rank of plus basis attempt mod 12347 is 38)
H4Nd1(1,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 41 - 38 - 3 = 0, needs to be less than 1
 (Full rank of minus basis attempt mod 12347 is 3)
H4Nd1(1,-) says that S_2(K(N))^-=0

N = 119
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {31, 31}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {1, 1}
dim(J_{2,N}^{cusp}) = 0
The file reports a plus basis attempt of dimensions {31, 76}
and a minus basis attempt of dimensions {1, 76}
ShortVectorFinalDet = 103
ShortVectorLength = 76

Entering H4Nddindices with d = 1
 Number of indices is 74
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 31 - 31 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 31)
H4Ndd(1,+) says that S_2(K(N))=0

N = 122
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {35, 35}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {6, 6}
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dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {35, 94}
and a minus basis attempt of dimensions {6, 94}
ShortVectorFinalDet = 95
ShortVectorLength = 94

Entering H4Nddindices with d = 1
 Number of indices is 75
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 35 - 32 = 3, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 35)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 41 - 23 - 6 = 12, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12347 are 35 and 6)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 41 - 6 - 32 = 3, needs to be less than 3
 (Full rank of plus basis attempt mod 12347 is 35)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 41 - 35 - 6 = 0, needs to be less than 2
 (Full rank of minus basis attempt mod 12347 is 6)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 88
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 35 - 35 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 35)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 123
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {34, 34}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {3, 3}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {34, 80}
and a minus basis attempt of dimensions {3, 80}
ShortVectorFinalDet = 92
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ShortVectorLength = 80

Entering H4Nddindices with d = 1
 Number of indices is 64
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 34 - 31 = 3, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 34)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 37 - 25 - 3 = 9, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12347 are 34 and 3)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 37 - 3 - 31 = 3, needs to be less than 3
 (Full rank of plus basis attempt mod 12347 is 34)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 37 - 34 - 3 = 0, needs to be less than 2
 (Full rank of minus basis attempt mod 12347 is 3)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 74
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 34 - 34 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 34)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 129
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {43, 43}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {1, 1}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {43, 93}
and a minus basis attempt of dimensions {1, 93}
ShortVectorFinalDet = 108
ShortVectorLength = 93

Entering H4Nddindices with d = 1
 Number of indices is 70
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Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 43 - 40 = 3, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 43)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 44 - 40 - 1 = 3, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12 347 are 43 and 1)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 44 - 1 - 40 = 3, needs to be less than 3
 (Full rank of plus basis attempt mod 12 347 is 43)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 44 - 43 - 1 = 0, needs to be less than 2
 (Full rank of minus basis attempt mod 12 347 is 1)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 87
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 43 - 43 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 43)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 130
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {43, 43}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {3, 3}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {43, 96}
and a minus basis attempt of dimensions {3, 96}
ShortVectorFinalDet = 100
ShortVectorLength = 96

Entering H4Nddindices with d = 1
 Number of indices is 80
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 43 - 40 = 3, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 43)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
```

```
 Rigorous upper bound of dim(H_4(N,n,1)) is 46 - 40 - 3 = 3, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12347 are 43 and 3)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 46 - 3 - 40 = 3, needs to be less than 3
 (Full rank of plus basis attempt mod 12347 is 43)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 46 - 43 - 3 = 0, needs to be less than 2
 (Full rank of minus basis attempt mod 12347 is 3)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 90
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 43 - 43 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 43)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 133
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {48, 48}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {1, 1}
dim(J_{2,N}^{cusp}) = 4
The file reports a plus basis attempt of dimensions {48, 69}
and a minus basis attempt of dimensions {1, 69}
ShortVectorFinalDet = 103
ShortVectorLength = 69

Entering H4Nddindices with d = 1
 Number of indices is 53
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 48 - 38 = 10, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 48)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 49 - 30 - 1 = 18, needs to be less than 5
 (Full rank of plus and minus basis attempts mod 12347 are 48 and 1)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 49 - 1 - 38 = 10, needs to be less than 5
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```
 (Full rank of plus basis attempt mod 12347 is 48)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 49 - 48 - 1 = 0, needs to be less than 4
 (Full rank of minus basis attempt mod 12347 is 1)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 65
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 48 - 48 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 48)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 134
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {42, 42}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {5, 5}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {42, 129}
and a minus basis attempt of dimensions {5, 129}
ShortVectorFinalDet = 112
ShortVectorLength = 129

Entering H4Nddindices with d = 1
 Number of indices is 99
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 42 - 39 = 3, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 42)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 47 - 29 - 5 = 13, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12347 are 42 and 5)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 47 - 5 - 39 = 3, needs to be less than 3
 (Full rank of plus basis attempt mod 12347 is 42)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 47 - 42 - 5 = 0, needs to be less than 2
 (Full rank of minus basis attempt mod 12347 is 5)
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H4Nd1(1,-) says that S_2(K(N))^-=0


Entering H4Nddindices with d = 2
 Number of indices is 123
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 42 - 42 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 42)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1


N = 138
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {41, 41}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {7, 7}
dim(J_{2,N}^{cusp}) = 1
The file reports a plus basis attempt of dimensions {41, 118}
and a minus basis attempt of dimensions {7, 118}
ShortVectorFinalDet = 111
ShortVectorLength = 118


Entering H4Nddindices with d = 1
 Number of indices is 104
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 41 - 40 = 1, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 41)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 48 - 26 - 7 = 15, needs to be less than 2
 (Full rank of plus and minus basis attempts mod 12347 are 41 and 7)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 48 - 7 - 40 = 1, needs to be less than 2
 (Full rank of plus basis attempt mod 12347 is 41)
H4Nd1(1,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 48 - 41 - 7 = 0, needs to be less than 1
 (Full rank of minus basis attempt mod 12347 is 7)
H4Nd1(1,-) says that S_2(K(N))^-=0


N = 141
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{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {47, 47}

{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {2, 2}

dim(J_{2,N}^{cusp}) = 2

The file reports a plus basis attempt of dimensions {47, 105}

and a minus basis attempt of dimensions {2, 105}

ShortVectorFinalDet = 116

ShortVectorLength = 105


Entering H4Nddindices with d = 1

 Number of indices is 81

Entering H4Nddplus routine with d = 1

 Rigorous upper bound of dim(H_4(N,n,n))^+ is 47 - 44 = 3, needs to be 0

 (Full rank of plus basis attempt mod 12 347 is 47)

Entering H4Nd1 routine with {d,sgn} = {1, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 49 - 31 - 2 = 16, needs to be less than 3

 (Full rank of plus and minus basis attempts mod 12 347 are 47 and 2)

Entering H4Nd1 routine with {d,sgn} = {1, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is 49 - 2 - 44 = 3, needs to be less than 3

 (Full rank of plus basis attempt mod 12 347 is 47)

Entering H4Nd1 routine with {d,sgn} = {1, -1}

 Rigorous upper bound of dim(H_4(N,n,1))^- is 49 - 47 - 2 = 0, needs to be less than 2

 (Full rank of minus basis attempt mod 12 347 is 2)

H4Nd1(1,-) says that S_2(K(N))^-=0


Entering H4Nddindices with d = 2

 Number of indices is 97

Entering H4Nddplus routine with d = 2

 Rigorous upper bound of dim(H_4(N,n,n))^+ is 47 - 47 = 0, needs to be 0

 (Full rank of plus basis attempt mod 12 347 is 47)

H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1


N = 142

{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {52, 52}

{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {5, 5}

dim(J_{2,N}^{cusp}) = 2

The file reports a plus basis attempt of dimensions {52, 109}
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and a minus basis attempt of dimensions {5, 109}
ShortVectorFinalDet = 112
ShortVectorLength = 109

Entering H4Nddindices with d = 1
 Number of indices is 91
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 52 - 48 = 4, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 52)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 57 - 34 - 5 = 18, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12347 are 52 and 5)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 57 - 5 - 48 = 4, needs to be less than 3
 (Full rank of plus basis attempt mod 12347 is 52)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 57 - 52 - 5 = 0, needs to be less than 2
 (Full rank of minus basis attempt mod 12347 is 5)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 102
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 52 - 52 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 52)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 143
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {40, 40}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {4, 4}
dim(J_{2,N}^{cusp}) = 1
The file reports a plus basis attempt of dimensions {40, 154}
and a minus basis attempt of dimensions {4, 146}
ShortVectorFinalDet = 156
ShortVectorLength = 154
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Entering H4Nddindices with d = 1
 Number of indices is 146
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 40 - 39 = 1, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 40)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 44 - 33 - 4 = 7, needs to be less than 2
 (Full rank of plus and minus basis attempts mod 12 347 are 40 and 4)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 44 - 4 - 39 = 1, needs to be less than 2
 (Full rank of plus basis attempt mod 12 347 is 40)
H4Nd1(1,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 44 - 40 - 4 = 0, needs to be less than 1
 (Full rank of minus basis attempt mod 12 347 is 4)
H4Nd1(1,-) says that S_2(K(N))^-=0

N = 145
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {56, 56}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {1, 1}
dim(J_{2,N}^{cusp}) = 3
The file reports a plus basis attempt of dimensions {56, 96}
and a minus basis attempt of dimensions {1, 96}
ShortVectorFinalDet = 111
ShortVectorLength = 96

Entering H4Nddindices with d = 1
 Number of indices is 74
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 56 - 50 = 6, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 56)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 57 - 34 - 1 = 22, needs to be less than 4
 (Full rank of plus and minus basis attempts mod 12 347 are 56 and 1)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 57 - 1 - 50 = 6, needs to be less than 4
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```
 (Full rank of plus basis attempt mod 12347 is 56)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 57 - 56 - 1 = 0, needs to be less than 3
 (Full rank of minus basis attempt mod 12347 is 1)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 89
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 56 - 56 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 56)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 146
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {49, 49}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {8, 8}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {49, 126}
and a minus basis attempt of dimensions {8, 126}
ShortVectorFinalDet = 111
ShortVectorLength = 126

Entering H4Nddindices with d = 1
 Number of indices is 96
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 49 - 46 = 3, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 49)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 57 - 30 - 8 = 19, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12347 are 49 and 8)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 57 - 8 - 46 = 3, needs to be less than 3
 (Full rank of plus basis attempt mod 12347 is 49)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 57 - 49 - 8 = 0, needs to be less than 2
 (Full rank of minus basis attempt mod 12347 is 8)
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H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 114
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 49 - 49 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 49)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 154
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {55, 55}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {3, 3}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {55, 942}
and a minus basis attempt of dimensions {3, 112}
ShortVectorFinalDet = 400
ShortVectorLength = 942

Entering H4Nddindices with d = 1
 Number of indices is 580
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 55 - 46 = 9, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 55)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 58 - 34 - 3 = 21, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12347 are 55 and 3)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 58 - 3 - 46 = 9, needs to be less than 3
 (Full rank of plus basis attempt mod 12347 is 55)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 58 - 55 - 3 = 0, needs to be less than 2
 (Full rank of minus basis attempt mod 12347 is 3)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 714
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Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 55 - 55 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 55)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 155
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {48, 48}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {5, 5}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {48, 117}
and a minus basis attempt of dimensions {5, 117}
ShortVectorFinalDet = 119
ShortVectorLength = 117

Entering H4Nddindices with d = 1
 Number of indices is 97
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 48 - 45 = 3, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 48)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 53 - 33 - 5 = 15, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12347 are 48 and 5)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 53 - 5 - 45 = 3, needs to be less than 3
 (Full rank of plus basis attempt mod 12347 is 48)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 53 - 48 - 5 = 0, needs to be less than 2
 (Full rank of minus basis attempt mod 12347 is 5)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 114
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 48 - 48 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 48)
```

```
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1
```

N = 158

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {54, 54}

{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {10, 10}

dim(J_{2,N}^{cusp}) = 3

The file reports a plus basis attempt of dimensions {54, 139}

and a minus basis attempt of dimensions {10, 139}

ShortVectorFinalDet = 127

ShortVectorLength = 139


Entering H4Nddindices with d = 1

 Number of indices is 107

Entering H4Nddplus routine with d = 1

 Rigorous upper bound of dim(H_4(N,n,n))^+ is 54 - 48 = 6, needs to be 0

 (Full rank of plus basis attempt mod 12 347 is 54)

Entering H4Nd1 routine with {d,sgn} = {1, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 64 - 30 - 9 = 25, needs to be less than 4

 (Full rank of plus and minus basis attempts mod 12 347 are 54 and 10)

Entering H4Nd1 routine with {d,sgn} = {1, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is 64 - 10 - 48 = 6, needs to be less than 4

 (Full rank of plus basis attempt mod 12 347 is 54)

Entering H4Nd1 routine with {d,sgn} = {1, -1}

 Rigorous upper bound of dim(H_4(N,n,1))^- is 64 - 54 - 9 = 1, needs to be less than 3

 (Full rank of minus basis attempt mod 12 347 is 10)

H4Nd1(1,-) says that S_2(K(N))^-=0


Entering H4Nddindices with d = 2

 Number of indices is 129

Entering H4Nddplus routine with d = 2

 Rigorous upper bound of dim(H_4(N,n,n))^+ is 54 - 54 = 0, needs to be 0

 (Full rank of plus basis attempt mod 12 347 is 54)

H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1
```

N = 159

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {55, 55}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {4, 4}
dim(J_{2,N}^{cusp}) = 1
The file reports a plus basis attempt of dimensions {55, 146}
and a minus basis attempt of dimensions {4, 146}
ShortVectorFinalDet = 135
ShortVectorLength = 146

Entering H4Nddindices with d = 1
 Number of indices is 132
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 55 - 54 = 1, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 55)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 59 - 40 - 4 = 15, needs to be less than 2
 (Full rank of plus and minus basis attempts mod 12 347 are 55 and 4)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 59 - 4 - 54 = 1, needs to be less than 2
 (Full rank of plus basis attempt mod 12 347 is 55)
H4Nd1(1,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 59 - 55 - 4 = 0, needs to be less than 1
 (Full rank of minus basis attempt mod 12 347 is 4)
H4Nd1(1,-) says that S_2(K(N))^-=0

N = 161
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {55, 55}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {2, 2}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {55, 110}
and a minus basis attempt of dimensions {2, 110}
ShortVectorFinalDet = 136
ShortVectorLength = 110

Entering H4Nddindices with d = 1
 Number of indices is 94
```

```
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 55 - 52 = 3, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 55)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 57 - 34 - 2 = 21, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12 347 are 55 and 2)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 57 - 2 - 52 = 3, needs to be less than 3
 (Full rank of plus basis attempt mod 12 347 is 55)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 57 - 55 - 2 = 0, needs to be less than 2
 (Full rank of minus basis attempt mod 12 347 is 2)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 106
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 55 - 55 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 55)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 165
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {55, 55}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {4, 4}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {55, 136}
and a minus basis attempt of dimensions {4, 136}
ShortVectorFinalDet = 140
ShortVectorLength = 136

Entering H4Nddindices with d = 1
 Number of indices is 120
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 55 - 52 = 3, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 55)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
```

```
 Rigorous upper bound of dim(H_4(N,n,1)) is 59 - 35 - 4 = 20, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12 347 are 55 and 4)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 59 - 4 - 52 = 3, needs to be less than 3
 (Full rank of plus basis attempt mod 12 347 is 55)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 59 - 55 - 4 = 0, needs to be less than 2
 (Full rank of minus basis attempt mod 12 347 is 4)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 132
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 55 - 55 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 55)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 166
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {68, 69}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {6, 6}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {68, 164}
and a minus basis attempt of dimensions {6, 164}
ShortVectorFinalDet = 140
ShortVectorLength = 164

Entering H4Nd1indices with d = 1
 Number of indices is 137
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 75 - 42 - 6 = 27, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12 347 are 68 and 6)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 75 - 6 - 64 = 5, needs to be less than 3
 (Full rank of plus basis attempt mod 12 347 is 68)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 75 - 68 - 6 = 1, needs to be less than 2
```

```
 (Full rank of minus basis attempt mod 12347 is 6)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 145
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 75 - 47 - 6 = 22, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12347 are 68 and 6)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 75 - 6 - 67 = 2, needs to be less than 3
 (Full rank of plus basis attempt mod 12347 is 68)
H4Nd1(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})

N = 170
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {59, 59}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {11, 11}
dim(J_{2,N}^{cusp}) = 3
The file reports a plus basis attempt of dimensions {59, 158}
and a minus basis attempt of dimensions {11, 158}
ShortVectorFinalDet = 135
ShortVectorLength = 158

Entering H4Nddindices with d = 1
 Number of indices is 126
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 59 - 53 = 6, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 59)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 70 - 34 - 11 = 25, needs to be less than 4
 (Full rank of plus and minus basis attempts mod 12347 are 59 and 11)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 70 - 11 - 53 = 6, needs to be less than 4
 (Full rank of plus basis attempt mod 12347 is 59)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 70 - 59 - 11 = 0, needs to be less than 3
 (Full rank of minus basis attempt mod 12347 is 11)
```

```
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 146
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 59 - 59 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 59)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 174
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {65, 65}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {10, 10}
dim(J_{2,N}^{cusp}) = 1
The file reports a plus basis attempt of dimensions {65, 650}
and a minus basis attempt of dimensions {10, 288}
ShortVectorFinalDet = 296
ShortVectorLength = 650

Entering H4Nddindices with d = 1
 Number of indices is 412
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 65 - 54 = 11, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 65)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 75 - 40 - 10 = 25, needs to be less than 2
 (Full rank of plus and minus basis attempts mod 12347 are 65 and 10)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 75 - 10 - 54 = 11, needs to be less than 2
 (Full rank of plus basis attempt mod 12347 is 65)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 75 - 65 - 10 = 0, needs to be less than 1
 (Full rank of minus basis attempt mod 12347 is 10)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 490
```

```
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 65 - 65 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 65)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 177
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {69, 69}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {5, 5}
dim(J_{2,N}^{cusp}) = 4
The file reports a plus basis attempt of dimensions {69, 139}
and a minus basis attempt of dimensions {5, 139}
ShortVectorFinalDet = 152
ShortVectorLength = 139

Entering H4Nddindices with d = 1
 Number of indices is 101
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 69 - 58 = 11, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 69)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 74 - 42 - 5 = 27, needs to be less than 5
 (Full rank of plus and minus basis attempts mod 12 347 are 69 and 5)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 74 - 5 - 58 = 11, needs to be less than 5
 (Full rank of plus basis attempt mod 12 347 is 69)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 74 - 69 - 5 = 0, needs to be less than 4
 (Full rank of minus basis attempt mod 12 347 is 5)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 128
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 69 - 69 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 69)
```

H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 178

{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {76, 76}

{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {11, 11}

dim(J_{2,N}^{cusp}) = 3

The file reports a plus basis attempt of dimensions {76, 164}

and a minus basis attempt of dimensions {11, 164}

ShortVectorFinalDet = 144

ShortVectorLength = 164

Entering H4Nddindices with d = 1

 Number of indices is 136

Entering H4Nddplus routine with d = 1

 Rigorous upper bound of dim(H_4(N,n,n))^+ is 76 - 68 = 8, needs to be 0

 (Full rank of plus basis attempt mod 12 347 is 76)

Entering H4Nd1 routine with {d,sgn} = {1, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 87 - 43 - 11 = 33, needs to be less than 4

 (Full rank of plus and minus basis attempts mod 12 347 are 76 and 11)

Entering H4Nd1 routine with {d,sgn} = {1, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is 87 - 11 - 68 = 8, needs to be less than 4

 (Full rank of plus basis attempt mod 12 347 is 76)

Entering H4Nd1 routine with {d,sgn} = {1, -1}

 Rigorous upper bound of dim(H_4(N,n,1))^- is 87 - 76 - 11 = 0, needs to be less than 3

 (Full rank of minus basis attempt mod 12 347 is 11)

H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2

 Number of indices is 152

Entering H4Nddplus routine with d = 2

 Rigorous upper bound of dim(H_4(N,n,n))^+ is 76 - 75 = 1, needs to be 0

 (Full rank of plus basis attempt mod 12 347 is 76)

Entering H4Nd1indices with d = 2

 Number of indices is 143

Entering H4Nd1 routine with {d,sgn} = {2, 0}

```
 Rigorous upper bound of dim(H_4(N,n,1)) is 87 - 48 - 11 = 28, needs to be less than 4
 (Full rank of plus and minus basis attempts mod 12 347 are 76 and 11)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 87 - 11 - 73 = 3, needs to be less than 4
 (Full rank of plus basis attempt mod 12 347 is 76)
H4Nd1(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})

N = 182
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {61, 61}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {13, 13}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {61, 160}
and a minus basis attempt of dimensions {13, 160}
ShortVectorFinalDet = 152
ShortVectorLength = 160

Entering H4Nddindices with d = 1
 Number of indices is 146
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 61 - 58 = 3, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 61)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 74 - 38 - 13 = 23, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12 347 are 61 and 13)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 74 - 13 - 58 = 3, needs to be less than 3
 (Full rank of plus basis attempt mod 12 347 is 61)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 74 - 61 - 13 = 0, needs to be less than 2
 (Full rank of minus basis attempt mod 12 347 is 13)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 155
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 61 - 61 = 0, needs to be 0
```

```
 (Full rank of plus basis attempt mod 12347 is 61)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 183
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {70, 70}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {10, 10}
dim(J_{2,N}^{cusp}) = 3
The file reports a plus basis attempt of dimensions {70, 160}
and a minus basis attempt of dimensions {10, 160}
ShortVectorFinalDet = 147
ShortVectorLength = 160

Entering H4Nddindices with d = 1
 Number of indices is 126
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 70 - 63 = 7, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 70)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 80 - 39 - 9 = 32, needs to be less than 4
 (Full rank of plus and minus basis attempts mod 12347 are 70 and 10)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 80 - 10 - 63 = 7, needs to be less than 4
 (Full rank of plus basis attempt mod 12347 is 70)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 80 - 70 - 9 = 1, needs to be less than 3
 (Full rank of minus basis attempt mod 12347 is 10)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 146
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 70 - 70 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 70)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1
```

```
N = 185
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {72, 72}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {6, 6}
dim(J_{2,N}^{cusp}) = 4
The file reports a plus basis attempt of dimensions {72, 152}
and a minus basis attempt of dimensions {6, 152}
ShortVectorFinalDet = 155
ShortVectorLength = 152

Entering H4Nddindices with d = 1
 Number of indices is 120
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 72 - 62 = 10, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 72)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 78 - 40 - 6 = 32, needs to be less than 5
 (Full rank of plus and minus basis attempts mod 12 347 are 72 and 6)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 78 - 6 - 62 = 10, needs to be less than 5
 (Full rank of plus basis attempt mod 12 347 is 72)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 78 - 72 - 6 = 0, needs to be less than 4
 (Full rank of minus basis attempt mod 12 347 is 6)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 144
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 72 - 72 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 72)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 186
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {72, 73}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {12, 12}
dim(J_{2,N}^{cusp}) = 2
```

```
The file reports a plus basis attempt of dimensions {72, 202}
and a minus basis attempt of dimensions {12, 202}
ShortVectorFinalDet = 135
ShortVectorLength = 202

Entering H4Nd1indices with d = 1
 Number of indices is 160
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 85 - 38 - 12 = 35, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12 347 are 72 and 12)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 85 - 12 - 67 = 6, needs to be less than 3
 (Full rank of plus basis attempt mod 12 347 is 72)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 85 - 72 - 12 = 1, needs to be less than 2
 (Full rank of minus basis attempt mod 12 347 is 12)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 170
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 85 - 45 - 12 = 28, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12 347 are 72 and 12)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 85 - 12 - 71 = 2, needs to be less than 3
 (Full rank of plus basis attempt mod 12 347 is 72)
H4Nd1(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})

N = 187
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {74, 74}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {6, 6}
dim(J_{2,N}^{cusp}) = 5
The file reports a plus basis attempt of dimensions {74, 218}
and a minus basis attempt of dimensions {6, 218}
ShortVectorFinalDet = 208
ShortVectorLength = 218
```

```
Entering H4Nddindices with d = 1
 Number of indices is 164
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 74 - 59 = 15, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 74)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 80 - 47 - 6 = 27, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12 347 are 74 and 6)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 80 - 6 - 59 = 15, needs to be less than 6
 (Full rank of plus basis attempt mod 12 347 is 74)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 80 - 74 - 6 = 0, needs to be less than 5
 (Full rank of minus basis attempt mod 12 347 is 6)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 206
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 74 - 74 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 74)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 190
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {79, 80}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {9, 9}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {79, 202}
and a minus basis attempt of dimensions {9, 192}
ShortVectorFinalDet = 160
ShortVectorLength = 202

Entering H4Nd1indices with d = 1
 Number of indices is 172
Entering H4Nd1 routine with {d,sgn} = {1, 0}
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 Rigorous upper bound of dim(H_4(N,n,1)) is 89 - 43 - 9 = 37, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12 347 are 79 and 9)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 89 - 9 - 76 = 4, needs to be less than 3
 (Full rank of plus basis attempt mod 12 347 is 79)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 89 - 79 - 9 = 1, needs to be less than 2
 (Full rank of minus basis attempt mod 12 347 is 9)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 180
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 89 - 49 - 9 = 31, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12 347 are 79 and 9)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 89 - 9 - 79 = 1, needs to be less than 3
 (Full rank of plus basis attempt mod 12 347 is 79)
H4Nd1(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})

N = 194
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {79, 79}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {17, 17}
dim(J_{2,N}^{cusp}) = 3
The file reports a plus basis attempt of dimensions {79, 202}
and a minus basis attempt of dimensions {17, 202}
ShortVectorFinalDet = 151
ShortVectorLength = 202

Entering H4Nddindices with d = 1
 Number of indices is 156
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 79 - 71 = 8, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 79)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 96 - 45 - 16 = 35, needs to be less than 4
```

```
 (Full rank of plus and minus basis attempts mod 12347 are 79 and 17)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 96 - 17 - 71 = 8, needs to be less than 4
 (Full rank of plus basis attempt mod 12347 is 79)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 96 - 79 - 16 = 1, needs to be less than 3
 (Full rank of minus basis attempt mod 12347 is 17)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 183
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 79 - 79 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 79)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 195
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {69, 69}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {10, 10}
dim(J_{2,N}^{cusp}) = 1
The file reports a plus basis attempt of dimensions {69, 192}
and a minus basis attempt of dimensions {10, 192}
ShortVectorFinalDet = 159
ShortVectorLength = 192

Entering H4Nddindices with d = 1
 Number of indices is 176
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 69 - 68 = 1, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 69)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 79 - 42 - 10 = 27, needs to be less than 2
 (Full rank of plus and minus basis attempts mod 12347 are 69 and 10)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 79 - 10 - 68 = 1, needs to be less than 2
 (Full rank of plus basis attempt mod 12347 is 69)
```

```
H4Nd1(1,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 79 - 69 - 10 = 0, needs to be less than 1
 (Full rank of minus basis attempt mod 12 347 is 10)
H4Nd1(1,-) says that S_2(K(N))^-=0
```

N = 201

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {91, 91}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {6, 6}
dim(J_{2,N}^{cusp}) = 5
The file reports a plus basis attempt of dimensions {91, 182}
and a minus basis attempt of dimensions {6, 182}
ShortVectorFinalDet = 168
ShortVectorLength = 182

Entering H4Nddindices with d = 1
 Number of indices is 130
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 91 - 73 = 18, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 91)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 97 - 45 - 6 = 46, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12 347 are 91 and 6)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 97 - 6 - 73 = 18, needs to be less than 6
 (Full rank of plus basis attempt mod 12 347 is 91)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 97 - 91 - 6 = 0, needs to be less than 5
 (Full rank of minus basis attempt mod 12 347 is 6)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 169
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 91 - 91 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 91)
```

```
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 202

{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {93, 94}

{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {15, 15}

dim(J_{2,N}^{cusp}) = 4

The file reports a plus basis attempt of dimensions {93, 232}

and a minus basis attempt of dimensions {15, 232}

ShortVectorFinalDet = 183

ShortVectorLength = 232

Entering H4Nd1indices with d = 1

 Number of indices is 181

Entering H4Nd1 routine with {d,sgn} = {1, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 109 - 49 - 15 = 45, needs to be less than 5

 (Full rank of plus and minus basis attempts mod 12 347 are 93 and 15)

Entering H4Nd1 routine with {d,sgn} = {1, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is 109 - 15 - 82 = 12, needs to be less than 5

 (Full rank of plus basis attempt mod 12 347 is 93)

Entering H4Nd1 routine with {d,sgn} = {1, -1}

 Rigorous upper bound of dim(H_4(N,n,1))^- is 109 - 93 - 15 = 1, needs to be less than 4

 (Full rank of minus basis attempt mod 12 347 is 15)

H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2

 Number of indices is 194

Entering H4Nd1 routine with {d,sgn} = {2, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 109 - 58 - 15 = 36, needs to be less than 5

 (Full rank of plus and minus basis attempts mod 12 347 are 93 and 15)

Entering H4Nd1 routine with {d,sgn} = {2, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is 109 - 15 - 91 = 3, needs to be less than 5

 (Full rank of plus basis attempt mod 12 347 is 93)

H4Nd1(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})

N = 203

{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {73, 73}
```

```
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {11, 11}

dim(J_{2,N}^{cusp}) = 4

The file reports a plus basis attempt of dimensions {73, 217}

and a minus basis attempt of dimensions {11, 217}

ShortVectorFinalDet = 199

ShortVectorLength = 217


Entering H4Nddindices with d = 1

 Number of indices is 174

Entering H4Nddplus routine with d = 1

 Rigorous upper bound of dim(H_4(N,n,n))^+ is 73 - 63 = 10, needs to be 0

 (Full rank of plus basis attempt mod 12 347 is 73)

Entering H4Nd1 routine with {d,sgn} = {1, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 84 - 46 - 11 = 27, needs to be less than 5

 (Full rank of plus and minus basis attempts mod 12 347 are 73 and 11)

Entering H4Nd1 routine with {d,sgn} = {1, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is 84 - 11 - 63 = 10, needs to be less than 5

 (Full rank of plus basis attempt mod 12 347 is 73)

Entering H4Nd1 routine with {d,sgn} = {1, -1}

 Rigorous upper bound of dim(H_4(N,n,1))^- is 84 - 73 - 11 = 0, needs to be less than 4

 (Full rank of minus basis attempt mod 12 347 is 11)

H4Nd1(1,-) says that S_2(K(N))^-=0


Entering H4Nddindices with d = 2

 Number of indices is 203

Entering H4Nddplus routine with d = 2

 Rigorous upper bound of dim(H_4(N,n,n))^+ is 73 - 73 = 0, needs to be 0

 (Full rank of plus basis attempt mod 12 347 is 73)

H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1


N = 205

{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {97, 97}

{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {6, 6}

dim(J_{2,N}^{cusp}) = 5

The file reports a plus basis attempt of dimensions {97, 178}

and a minus basis attempt of dimensions {6, 178}
```

```
ShortVectorFinalDet = 159
ShortVectorLength = 178

Entering H4Nddindices with d = 1
 Number of indices is 126
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 97 - 79 = 18, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 97)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 103 - 49 - 6 = 48, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12 347 are 97 and 6)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 103 - 6 - 79 = 18, needs to be less than 6
 (Full rank of plus basis attempt mod 12 347 is 97)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 103 - 97 - 6 = 0, needs to be less than 5
 (Full rank of minus basis attempt mod 12 347 is 6)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 157
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 97 - 96 = 1, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 97)

Entering H4Nd1indices with d = 2
 Number of indices is 142
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 103 - 61 - 6 = 36, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12 347 are 97 and 6)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 103 - 6 - 93 = 4, needs to be less than 6
 (Full rank of plus basis attempt mod 12 347 is 97)
H4Nd1(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})

N = 206
```

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {85, 86}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {19, 19}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {85, 231}
and a minus basis attempt of dimensions {19, 231}
ShortVectorFinalDet = 172
ShortVectorLength = 231

Entering H4Nd1indices with d = 1
 Number of indices is 195
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 105 - 45 - 17 = 43, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12 347 are 85 and 19)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 105 - 19 - 82 = 4, needs to be less than 3
 (Full rank of plus basis attempt mod 12 347 is 85)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 105 - 85 - 17 = 3, needs to be less than 2
 (Full rank of minus basis attempt mod 12 347 is 19)

Entering H4Nd1indices with d = 2
 Number of indices is 206
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 105 - 53 - 19 = 33, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12 347 are 85 and 19)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 105 - 19 - 85 = 1, needs to be less than 3
 (Full rank of plus basis attempt mod 12 347 is 85)
H4Nd1(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})
Entering H4Nd1 routine with {d,sgn} = {2, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 105 - 85 - 19 = 1, needs to be less than 2
 (Full rank of minus basis attempt mod 12 347 is 19)
H4Nd1(2,-) says that Jacobi restriction out to index
 1 gives a weight 2 minus space upper bound

N = 209
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {84, 84}
```

```
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {6, 6}
dim(J_{2,N}^{cusp}) = 3
The file reports a plus basis attempt of dimensions {84, 157}
and a minus basis attempt of dimensions {6, 157}
ShortVectorFinalDet = 164
ShortVectorLength = 157

Entering H4Nddindices with d = 1
 Number of indices is 133
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 84 - 78 = 6, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 84)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 90 - 51 - 6 = 33, needs to be less than 4
 (Full rank of plus and minus basis attempts mod 12 347 are 84 and 6)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 90 - 6 - 78 = 6, needs to be less than 4
 (Full rank of plus basis attempt mod 12 347 is 84)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 90 - 84 - 6 = 0, needs to be less than 3
 (Full rank of minus basis attempt mod 12 347 is 6)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 150
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 84 - 84 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 84)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 210
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {77, 77}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {15, 15}
dim(J_{2,N}^{cusp}) = 1
The file reports a plus basis attempt of dimensions {77, 240}
and a minus basis attempt of dimensions {15, 240}
```

```
ShortVectorFinalDet = 164
ShortVectorLength = 240

Entering H4Nddindices with d = 1
 Number of indices is 232
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 77 - 76 = 1, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 77)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 92 - 41 - 15 = 36, needs to be less than 2
 (Full rank of plus and minus basis attempts mod 12 347 are 77 and 15)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 92 - 15 - 76 = 1, needs to be less than 2
 (Full rank of plus basis attempt mod 12 347 is 77)
H4Nd1(1,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 92 - 77 - 15 = 0, needs to be less than 1
 (Full rank of minus basis attempt mod 12 347 is 15)
H4Nd1(1,-) says that S_2(K(N))^-=0

N = 213
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {91, 91}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {13, 13}
dim(J_{2,N}^{cusp}) = 4
The file reports a plus basis attempt of dimensions {91, 195}
and a minus basis attempt of dimensions {13, 195}
ShortVectorFinalDet = 176
ShortVectorLength = 195

Entering H4Nddindices with d = 1
 Number of indices is 153
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 91 - 79 = 12, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 91)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 104 - 49 - 13 = 42, needs to be less than 5
```

```
 (Full rank of plus and minus basis attempts mod 12 347 are 91 and 13)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 104 - 13 - 79 = 12, needs to be less than 5
 (Full rank of plus basis attempt mod 12 347 is 91)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 104 - 91 - 13 = 0, needs to be less than 4
 (Full rank of minus basis attempt mod 12 347 is 13)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 176
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 91 - 91 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 91)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 214
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {105, 107}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {12, 12}
dim(J_{2,N}^{cusp}) = 5
The file reports a plus basis attempt of dimensions {105, 238}
and a minus basis attempt of dimensions {12, 238}
ShortVectorFinalDet = 175
ShortVectorLength = 238

Entering H4Nd1indices with d = 1
 Number of indices is 167
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 119 - 52 - 12 = 55, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12 347 are 105 and 12)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 119 - 12 - 86 = 21, needs to be less than 6
 (Full rank of plus basis attempt mod 12 347 is 105)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 119 - 105 - 12 = 2, needs to be less than 5
 (Full rank of minus basis attempt mod 12 347 is 12)
```

```
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 189
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 119 - 65 - 12 = 42, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12 347 are 105 and 12)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 119 - 12 - 103 = 4, needs to be less than 6
 (Full rank of plus basis attempt mod 12 347 is 105)
H4Nd1(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})

N = 215
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {83, 83}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {14, 14}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {83, 200}
and a minus basis attempt of dimensions {14, 200}
ShortVectorFinalDet = 175
ShortVectorLength = 200

Entering H4Nddindices with d = 1
 Number of indices is 182
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 83 - 80 = 3, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 83)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 97 - 51 - 14 = 32, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12 347 are 83 and 14)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 97 - 14 - 80 = 3, needs to be less than 3
 (Full rank of plus basis attempt mod 12 347 is 83)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 97 - 83 - 14 = 0, needs to be less than 2
 (Full rank of minus basis attempt mod 12 347 is 14)
H4Nd1(1,-) says that S_2(K(N))^-=0
```

```
Entering H4Nddindices with d = 2
 Number of indices is 191
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 83 - 83 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 83)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 217
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {107, 107}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {6, 6}
dim(J_{2,N}^{cusp}) = 6
The file reports a plus basis attempt of dimensions {107, 181}
and a minus basis attempt of dimensions {6, 181}
ShortVectorFinalDet = 192
ShortVectorLength = 181

Entering H4Nddindices with d = 1
 Number of indices is 127
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 107 - 84 = 23, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 107)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 113 - 51 - 6 = 56, needs to be less than 7
 (Full rank of plus and minus basis attempts mod 12 347 are 107 and 6)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 113 - 6 - 84 = 23, needs to be less than 7
 (Full rank of plus basis attempt mod 12 347 is 107)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 113 - 107 - 6 = 0, needs to be less than 6
 (Full rank of minus basis attempt mod 12 347 is 6)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 159
Entering H4Nddplus routine with d = 2
```

```
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 107 - 107 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 107)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 218
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {96, 96}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {22, 23}
dim(J_{2,N}^{cusp}) = 6
The file reports a plus basis attempt of dimensions {96, 234}
and a minus basis attempt of dimensions {22, 234}
ShortVectorFinalDet = 175
ShortVectorLength = 234

Entering H4Nd1indices with d = 1
 Number of indices is 153
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 119 - 46 - 20 = 53, needs to be less than 7
 (Full rank of plus and minus basis attempts mod 12 347 are 96 and 22)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 119 - 22 - 74 = 23, needs to be less than 7
 (Full rank of plus basis attempt mod 12 347 is 96)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 119 - 96 - 20 = 3, needs to be less than 6
 (Full rank of minus basis attempt mod 12 347 is 22)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 176
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 119 - 57 - 22 = 40, needs to be less than 7
 (Full rank of plus and minus basis attempts mod 12 347 are 96 and 22)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 119 - 22 - 94 = 3, needs to be less than 7
 (Full rank of plus basis attempt mod 12 347 is 96)
H4Nd1(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})
```

```
N = 219
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {98, 98}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {13, 13}
dim(J_{2,N}^{cusp}) = 5
The file reports a plus basis attempt of dimensions {98, 224}
and a minus basis attempt of dimensions {13, 224}
ShortVectorFinalDet = 171
ShortVectorLength = 224

Entering H4Nddindices with d = 1
 Number of indices is 154
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 98 - 79 = 19, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 98)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 111 - 52 - 13 = 46, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12347 are 98 and 13)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 111 - 13 - 79 = 19, needs to be less than 6
 (Full rank of plus basis attempt mod 12347 is 98)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 111 - 98 - 13 = 0, needs to be less than 5
 (Full rank of minus basis attempt mod 12347 is 13)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 200
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 98 - 98 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12347 is 98)
H4Ndd(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp}) and
  dim(S_2(K(N))^-) is bounded above by Jacobi restriction out to index 1

N = 221
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {95, 95}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {6, 9}
dim(J_{2,N}^{cusp}) = 5
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The file reports a plus basis attempt of dimensions {95, 228}
and a minus basis attempt of dimensions {6, 228}
ShortVectorFinalDet = 196
ShortVectorLength = 228

Entering H4Nd1indices with d = 1
 Number of indices is 179
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 104 - 52 - 6 = 46, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12 347 are 95 and 6)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 104 - 6 - 79 = 19, needs to be less than 6
 (Full rank of plus basis attempt mod 12 347 is 95)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 104 - 95 - 6 = 3, needs to be less than 5
 (Full rank of minus basis attempt mod 12 347 is 6)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 196
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 104 - 64 - 6 = 34, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12 347 are 95 and 6)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 104 - 6 - 90 = 8, needs to be less than 6
 (Full rank of plus basis attempt mod 12 347 is 95)

Entering H4Nd1indices with d = 3
 Number of indices is 207
Entering H4Nd1 routine with {d,sgn} = {3, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 104 - 64 - 6 = 34, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12 347 are 95 and 6)
Entering H4Nd1 routine with {d,sgn} = {3, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 104 - 6 - 94 = 4, needs to be less than 6
 (Full rank of plus basis attempt mod 12 347 is 95)
H4Nd1(3,+) says that Jacobi restriction out to index
 2 gives a weight 2 plus space upper bound
```

```
N = 222
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {96, 96}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {24, 24}
dim(J_{2,N}^{cusp}) = 1
The file reports a plus basis attempt of dimensions {96, 824}
and a minus basis attempt of dimensions {24, 824}
ShortVectorFinalDet = 344
ShortVectorLength = 824

Entering H4Nddindices with d = 1
 Number of indices is 538
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 96 - 69 = 27, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 96)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 120 - 69 - 24 = 27, needs to be less than 2
 (Full rank of plus and minus basis attempts mod 12 347 are 96 and 24)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 120 - 24 - 69 = 27, needs to be less than 2
 (Full rank of plus basis attempt mod 12 347 is 96)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 120 - 96 - 24 = 0, needs to be less than 1
 (Full rank of minus basis attempt mod 12 347 is 24)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nddindices with d = 2
 Number of indices is 611
Entering H4Nddplus routine with d = 2
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 96 - 93 = 3, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 96)

Entering H4Nd1indices with d = 2
 Number of indices is 569
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 120 - 84 - 24 = 12, needs to be less than 2
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 (Full rank of plus and minus basis attempts mod 12 347 are 96 and 24)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 120 - 24 - 84 = 12, needs to be less than 2
 (Full rank of plus basis attempt mod 12 347 is 96)

Entering H4Nddindices with d = 3
 Number of indices is 682
Entering H4Nddplus routine with d = 3
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 96 - 96 = 0, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 96)
H4Ndd(3+) says that dim(S_2(K(N))^+) and
   dim(S_2(K(N))^-) are bounded above by Jacobi restriction out to index 2

N = 226
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {117, 118}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {12, 16}
dim(J_{2,N}^{cusp}) = 6
The file reports a plus basis attempt of dimensions {117, 260}
and a minus basis attempt of dimensions {12, 260}
ShortVectorFinalDet = 176
ShortVectorLength = 260

Entering H4Nd1indices with d = 1
 Number of indices is 168
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 134 - 90 - 12 = 32, needs to be less than 7
 (Full rank of plus and minus basis attempts mod 12 347 are 117 and 12)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 134 - 12 - 90 = 32, needs to be less than 7
 (Full rank of plus basis attempt mod 12 347 is 117)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 134 - 117 - 12 = 5, needs to be less than 6
 (Full rank of minus basis attempt mod 12 347 is 12)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 193
```

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Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 134 - 110 - 12 = 12, needs to be less than 7
 (Full rank of plus and minus basis attempts mod 12 347 are 117 and 12)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 134 - 12 - 110 = 12, needs to be less than 7
 (Full rank of plus basis attempt mod 12 347 is 117)

Entering H4Nd1indices with d = 3
 Number of indices is 222
Entering H4Nd1 routine with {d,sgn} = {3, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 134 - 117 - 12 = 5, needs to be less than 7
 (Full rank of plus and minus basis attempts mod 12 347 are 117 and 12)
H4Nd1(3) says that Jacobi restriction out to index 2 gives a weight 2 space upper bound

N = 230
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {96, 97}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {25, 25}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {96, 318}
and a minus basis attempt of dimensions {25, 318}
ShortVectorFinalDet = 191
ShortVectorLength = 318

Entering H4Nd1indices with d = 1
 Number of indices is 272
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 122 - 61 - 24 = 37, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12 347 are 96 and 25)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 122 - 25 - 93 = 4, needs to be less than 3
 (Full rank of plus basis attempt mod 12 347 is 96)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 122 - 96 - 24 = 2, needs to be less than 2
 (Full rank of minus basis attempt mod 12 347 is 25)

Entering H4Nd1indices with d = 2
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 Number of indices is 280
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 122 - 68 - 24 = 30, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12 347 are 96 and 25)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 122 - 25 - 96 = 1, needs to be less than 3
 (Full rank of plus basis attempt mod 12 347 is 96)
H4Nd1(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})
Entering H4Nd1 routine with {d,sgn} = {2, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 122 - 96 - 24 = 2, needs to be less than 2
 (Full rank of minus basis attempt mod 12 347 is 25)

Entering H4Nd1indices with d = 3
 Number of indices is 295
Entering H4Nd1 routine with {d,sgn} = {3, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 122 - 72 - 24 = 26, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12 347 are 96 and 25)
Entering H4Nd1 routine with {d,sgn} = {3, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 122 - 96 - 24 = 2, needs to be less than 2
 (Full rank of minus basis attempt mod 12 347 is 25)

Entering H4Nd1indices with d = 4
 Number of indices is 309
Entering H4Nd1 routine with {d,sgn} = {4, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 122 - 73 - 25 = 24, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12 347 are 96 and 25)
Entering H4Nd1 routine with {d,sgn} = {4, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 122 - 96 - 25 = 1, needs to be less than 2
 (Full rank of minus basis attempt mod 12 347 is 25)
H4Nd1(4,-) says that Jacobi restriction out to index
 3 gives a weight 2 minus space upper bound

N = 231
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {95, 95}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {11, 11}
dim(J_{2,N}^{cusp}) = 1
The file reports a plus basis attempt of dimensions {95, 232}
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and a minus basis attempt of dimensions {11, 232}
ShortVectorFinalDet = 195
ShortVectorLength = 232

Entering H4Nddindices with d = 1
 Number of indices is 220
Entering H4Nddplus routine with d = 1
 Rigorous upper bound of dim(H_4(N,n,n))^+ is 95 - 94 = 1, needs to be 0
 (Full rank of plus basis attempt mod 12 347 is 95)
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 106 - 52 - 11 = 43, needs to be less than 2
 (Full rank of plus and minus basis attempts mod 12 347 are 95 and 11)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 106 - 11 - 94 = 1, needs to be less than 2
 (Full rank of plus basis attempt mod 12 347 is 95)
H4Nd1(1,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 106 - 95 - 11 = 0, needs to be less than 1
 (Full rank of minus basis attempt mod 12 347 is 11)
H4Nd1(1,-) says that S_2(K(N))^-=0

N = 235
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {114, 114}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {7, 11}
dim(J_{2,N}^{cusp}) = 6
The file reports a plus basis attempt of dimensions {114, 222}
and a minus basis attempt of dimensions {7, 222}
ShortVectorFinalDet = 180
ShortVectorLength = 222

Entering H4Nd1indices with d = 1
 Number of indices is 148
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 125 - 52 - 7 = 66, needs to be less than 7
 (Full rank of plus and minus basis attempts mod 12 347 are 114 and 7)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
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 Rigorous upper bound of dim(H_4(N,n,1))^+ is 125 - 7 - 86 = 32, needs to be less than 7
 (Full rank of plus basis attempt mod 12 347 is 114)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 125 - 114 - 7 = 4, needs to be less than 6
 (Full rank of minus basis attempt mod 12 347 is 7)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 167
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 125 - 67 - 7 = 51, needs to be less than 7
 (Full rank of plus and minus basis attempts mod 12 347 are 114 and 7)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 125 - 7 - 103 = 15, needs to be less than 7
 (Full rank of plus basis attempt mod 12 347 is 114)

Entering H4Nd1indices with d = 3
 Number of indices is 201
Entering H4Nd1 routine with {d,sgn} = {3, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 125 - 67 - 7 = 51, needs to be less than 7
 (Full rank of plus and minus basis attempts mod 12 347 are 114 and 7)
Entering H4Nd1 routine with {d,sgn} = {3, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 125 - 7 - 114 = 4, needs to be less than 7
 (Full rank of plus basis attempt mod 12 347 is 114)
H4Nd1(3,+) says that Jacobi restriction out to index
 2 gives a weight 2 plus space upper bound

N = 237
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {112, 112}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {16, 19}
dim(J_{2,N}^{cusp}) = 5
The file reports a plus basis attempt of dimensions {112, 360}
and a minus basis attempt of dimensions {16, 360}
ShortVectorFinalDet = 236
ShortVectorLength = 360

Entering H4Nd1indices with d = 1
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 Number of indices is 240
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 131 - 72 - 16 = 43, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12 347 are 112 and 16)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 131 - 16 - 97 = 18, needs to be less than 6
 (Full rank of plus basis attempt mod 12 347 is 112)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 131 - 112 - 16 = 3, needs to be less than 5
 (Full rank of minus basis attempt mod 12 347 is 16)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 267
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 131 - 87 - 16 = 28, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12 347 are 112 and 16)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 131 - 16 - 112 = 3, needs to be less than 6
 (Full rank of plus basis attempt mod 12 347 is 112)
H4Nd1(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})

N = 238
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {113, 114}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {20, 20}
dim(J_{2,N}^{cusp}) = 2
The file reports a plus basis attempt of dimensions {113, 298}
and a minus basis attempt of dimensions {20, 298}
ShortVectorFinalDet = 220
ShortVectorLength = 298

Entering H4Nd1indices with d = 1
 Number of indices is 258
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 134 - 72 - 20 = 42, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12 347 are 113 and 20)
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Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 134 - 20 - 110 = 4, needs to be less than 3
 (Full rank of plus basis attempt mod 12 347 is 113)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 134 - 113 - 20 = 1, needs to be less than 2
 (Full rank of minus basis attempt mod 12 347 is 20)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 268
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 134 - 80 - 20 = 34, needs to be less than 3
 (Full rank of plus and minus basis attempts mod 12 347 are 113 and 20)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 134 - 20 - 113 = 1, needs to be less than 3
 (Full rank of plus basis attempt mod 12 347 is 113)
H4Nd1(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})

N = 246
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {121, 124}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {25, 25}
dim(J_{2,N}^{cusp}) = 4
The file reports a plus basis attempt of dimensions {121, 358}
and a minus basis attempt of dimensions {25, 358}
ShortVectorFinalDet = 207
ShortVectorLength = 358

Entering H4Nd1indices with d = 1
 Number of indices is 258
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 149 - 108 - 25 = 16, needs to be less than 5
 (Full rank of plus and minus basis attempts mod 12 347 are 121 and 25)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 149 - 25 - 109 = 15, needs to be less than 5
 (Full rank of plus basis attempt mod 12 347 is 121)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
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 Rigorous upper bound of dim(H_4(N,n,1))^- is 149 - 121 - 25 = 3, needs to be less than 4
 (Full rank of minus basis attempt mod 12 347 is 25)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 278
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 149 - 121 - 25 = 3, needs to be less than 5
 (Full rank of plus and minus basis attempts mod 12 347 are 121 and 25)
H4Nd1(2) says that S_2(K(N))=Grit(J_{2,N}^{cusp})

N = 247
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {124, 124}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {7, 13}
dim(J_{2,N}^{cusp}) = 7
The file reports a plus basis attempt of dimensions {124, 288}
and a minus basis attempt of dimensions {7, 288}
ShortVectorFinalDet = 231
ShortVectorLength = 288

Entering H4Nd1indices with d = 1
 Number of indices is 212
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 137 - 63 - 7 = 67, needs to be less than 8
 (Full rank of plus and minus basis attempts mod 12 347 are 124 and 7)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 137 - 7 - 96 = 34, needs to be less than 8
 (Full rank of plus basis attempt mod 12 347 is 124)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 137 - 124 - 7 = 6, needs to be less than 7
 (Full rank of minus basis attempt mod 12 347 is 7)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 235
Entering H4Nd1 routine with {d,sgn} = {2, 0}
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 Rigorous upper bound of dim(H_4(N,n,1)) is 137 - 78 - 7 = 52, needs to be less than 8
 (Full rank of plus and minus basis attempts mod 12 347 are 124 and 7)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 137 - 7 - 117 = 13, needs to be less than 8
 (Full rank of plus basis attempt mod 12 347 is 124)

Entering H4Nd1indices with d = 3
 Number of indices is 252
Entering H4Nd1 routine with {d,sgn} = {3, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 137 - 78 - 7 = 52, needs to be less than 8
 (Full rank of plus and minus basis attempts mod 12 347 are 124 and 7)
Entering H4Nd1 routine with {d,sgn} = {3, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 137 - 7 - 124 = 6, needs to be less than 8
 (Full rank of plus basis attempt mod 12 347 is 124)
H4Nd1(3,+) says that Jacobi restriction out to index
 2 gives a weight 2 plus space upper bound

N = 249
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {126, 127}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {10, 12}
dim(J_{2,N}^{cusp}) = 5
The file reports a plus basis attempt of dimensions {126, 263}
and a minus basis attempt of dimensions {10, 263}
ShortVectorFinalDet = 212
ShortVectorLength = 263

Entering H4Nd1indices with d = 1
 Number of indices is 187
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 139 - 74 - 10 = 55, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12 347 are 126 and 10)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 139 - 10 - 104 = 25, needs to be less than 6
 (Full rank of plus basis attempt mod 12 347 is 126)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 139 - 126 - 10 = 3, needs to be less than 5
```

```
 (Full rank of minus basis attempt mod 12347 is 10)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 208
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 139 - 87 - 10 = 42, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12347 are 126 and 10)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 139 - 10 - 120 = 9, needs to be less than 6
 (Full rank of plus basis attempt mod 12347 is 126)

Entering H4Nd1indices with d = 3
 Number of indices is 233
Entering H4Nd1 routine with {d,sgn} = {3, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 139 - 94 - 10 = 35, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12347 are 126 and 10)
Entering H4Nd1 routine with {d,sgn} = {3, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 139 - 10 - 126 = 3, needs to be less than 6
 (Full rank of plus basis attempt mod 12347 is 126)
H4Nd1(3,+) says that Jacobi restriction out to index
 2 gives a weight 2 plus space upper bound

Entering H4Nd1indices with d = 4
 Number of indices is 247
Entering H4Nd1 routine with {d,sgn} = {4, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 139 - 95 - 10 = 34, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12347 are 126 and 10)

N = 253
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {131, 131}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {3, 10}
dim(J_{2,N}^{cusp}) = 8
The file reports a plus basis attempt of dimensions {131, 230}
and a minus basis attempt of dimensions {3, 230}
ShortVectorFinalDet = 228
ShortVectorLength = 230
```

```
Entering H4Nd1indices with d = 1
 Number of indices is 164
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 141 - 62 - 3 = 76, needs to be less than 9
 (Full rank of plus and minus basis attempts mod 12 347 are 131 and 3)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 141 - 3 - 94 = 44, needs to be less than 9
 (Full rank of plus basis attempt mod 12 347 is 131)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 141 - 131 - 3 = 7, needs to be less than 8
 (Full rank of minus basis attempt mod 12 347 is 3)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 190
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 141 - 76 - 3 = 62, needs to be less than 9
 (Full rank of plus and minus basis attempts mod 12 347 are 131 and 3)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 141 - 3 - 118 = 20, needs to be less than 9
 (Full rank of plus basis attempt mod 12 347 is 131)

Entering H4Nd1indices with d = 3
 Number of indices is 207
Entering H4Nd1 routine with {d,sgn} = {3, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 141 - 76 - 3 = 62, needs to be less than 9
 (Full rank of plus and minus basis attempts mod 12 347 are 131 and 3)
Entering H4Nd1 routine with {d,sgn} = {3, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 141 - 3 - 128 = 10, needs to be less than 9
 (Full rank of plus basis attempt mod 12 347 is 131)

Entering H4Nd1indices with d = 4
 Number of indices is 223
Entering H4Nd1 routine with {d,sgn} = {4, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 141 - 76 - 3 = 62, needs to be less than 9
```

```
 (Full rank of plus and minus basis attempts mod 12 347 are 131 and 3)

Entering H4Nd1 routine with {d,sgn} = {4, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is 141 - 3 - 131 = 7, needs to be less than 9

 (Full rank of plus basis attempt mod 12 347 is 131)

H4Nd1(4,+) says that Jacobi restriction out to index
 3 gives a weight 2 plus space upper bound
```

N = 254

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {125, 127}

{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {27, 29}

dim(J_{2,N}^{cusp}) = 5

The file reports a plus basis attempt of dimensions {125, 323}

and a minus basis attempt of dimensions {27, 323}

ShortVectorFinalDet = 207

ShortVectorLength = 323


Entering H4Nd1indices with d = 1

 Number of indices is 229

Entering H4Nd1 routine with {d,sgn} = {1, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 156 - 70 - 25 = 61, needs to be less than 6

 (Full rank of plus and minus basis attempts mod 12 347 are 125 and 27)

Entering H4Nd1 routine with {d,sgn} = {1, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 156 - 27 - 107 = 22, needs to be less than 6

 (Full rank of plus basis attempt mod 12 347 is 125)

Entering H4Nd1 routine with {d,sgn} = {1, -1}

 Rigorous upper bound of dim(H_4(N,n,1))^- is 156 - 125 - 25 = 6, needs to be less than 5

 (Full rank of minus basis attempt mod 12 347 is 27)


Entering H4Nd1indices with d = 2

 Number of indices is 251

Entering H4Nd1 routine with {d,sgn} = {2, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 156 - 82 - 27 = 47, needs to be less than 6

 (Full rank of plus and minus basis attempts mod 12 347 are 125 and 27)

Entering H4Nd1 routine with {d,sgn} = {2, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is 156 - 27 - 124 = 5, needs to be less than 6

 (Full rank of plus basis attempt mod 12 347 is 125)
```

H4Nd1(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})

Entering H4Nd1 routine with {d,sgn} = {2, -1}

Rigorous upper bound of dim(H_4(N,n,1))^- is 156 - 125 - 27 = 4, needs to be less than 5

(Full rank of minus basis attempt mod 12 347 is 27)

H4Nd1(2,-) says that Jacobi restriction out to index
1 gives a weight 2 minus space upper bound

N = 255

{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {114, 114}

{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {21, 22}

dim(J_{2,N}^{cusp}) = 2

The file reports a plus basis attempt of dimensions {114, 298}

and a minus basis attempt of dimensions {21, 298}

ShortVectorFinalDet = 219

ShortVectorLength = 298

Entering H4Nd1indices with d = 1

Number of indices is 270

Entering H4Nd1 routine with {d,sgn} = {1, 0}

Rigorous upper bound of dim(H_4(N,n,1)) is 136 - 73 - 21 = 42, needs to be less than 3

(Full rank of plus and minus basis attempts mod 12 347 are 114 and 21)

Entering H4Nd1 routine with {d,sgn} = {1, 1}

Rigorous upper bound of dim(H_4(N,n,1))^+ is 136 - 21 - 111 = 4, needs to be less than 3

(Full rank of plus basis attempt mod 12 347 is 114)

Entering H4Nd1 routine with {d,sgn} = {1, -1}

Rigorous upper bound of dim(H_4(N,n,1))^- is 136 - 114 - 21 = 1, needs to be less than 2

(Full rank of minus basis attempt mod 12 347 is 21)

H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2

Number of indices is 278

Entering H4Nd1 routine with {d,sgn} = {2, 0}

Rigorous upper bound of dim(H_4(N,n,1)) is 136 - 78 - 21 = 37, needs to be less than 3

(Full rank of plus and minus basis attempts mod 12 347 are 114 and 21)

Entering H4Nd1 routine with {d,sgn} = {2, 1}

Rigorous upper bound of dim(H_4(N,n,1))^+ is 136 - 21 - 114 = 1, needs to be less than 3

(Full rank of plus basis attempt mod 12 347 is 114)

```
H4Nd1(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})

N = 258
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {127, 128}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {34, 35}
dim(J_{2,N}^{cusp}) = 4
The file reports a plus basis attempt of dimensions {127, 358}
and a minus basis attempt of dimensions {34, 358}
ShortVectorFinalDet = 204
ShortVectorLength = 358

Entering H4Nd1indices with d = 1
 Number of indices is 280
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 163 - 70 - 30 = 63, needs to be less than 5
 (Full rank of plus and minus basis attempts mod 12 347 are 127 and 34)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 163 - 34 - 113 = 16, needs to be less than 5
 (Full rank of plus basis attempt mod 12 347 is 127)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 163 - 127 - 30 = 6, needs to be less than 4
 (Full rank of minus basis attempt mod 12 347 is 34)

Entering H4Nd1indices with d = 2
 Number of indices is 295
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 163 - 79 - 32 = 52, needs to be less than 5
 (Full rank of plus and minus basis attempts mod 12 347 are 127 and 34)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 163 - 34 - 126 = 3, needs to be less than 5
 (Full rank of plus basis attempt mod 12 347 is 127)
H4Nd1(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})
Entering H4Nd1 routine with {d,sgn} = {2, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 163 - 127 - 32 = 4, needs to be less than 4
 (Full rank of minus basis attempt mod 12 347 is 34)
```

```
Entering H4Nd1indices with d = 3
 Number of indices is 311
Entering H4Nd1 routine with {d,sgn} = {3, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 163 - 82 - 33 = 48, needs to be less than 5
 (Full rank of plus and minus basis attempts mod 12 347 are 127 and 34)
Entering H4Nd1 routine with {d,sgn} = {3, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 163 - 127 - 33 = 3, needs to be less than 4
 (Full rank of minus basis attempt mod 12 347 is 34)
H4Nd1(3,-) says that Jacobi restriction out to index
 2 gives a weight 2 minus space upper bound
```

N = 259

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {137, 137}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {9, 13}
dim(J_{2,N}^{cusp}) = 7
The file reports a plus basis attempt of dimensions {137, 307}
and a minus basis attempt of dimensions {9, 307}
ShortVectorFinalDet = 243
ShortVectorLength = 307

Entering H4Nd1indices with d = 1
 Number of indices is 218
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 150 - 69 - 9 = 72, needs to be less than 8
 (Full rank of plus and minus basis attempts mod 12 347 are 137 and 9)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 150 - 9 - 107 = 34, needs to be less than 8
 (Full rank of plus basis attempt mod 12 347 is 137)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 150 - 137 - 9 = 4, needs to be less than 7
 (Full rank of minus basis attempt mod 12 347 is 9)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 245
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 150 - 80 - 9 = 61, needs to be less than 8
```

```
 (Full rank of plus and minus basis attempts mod 12 347 are 137 and 9)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 150 - 9 - 128 = 13, needs to be less than 8
 (Full rank of plus basis attempt mod 12 347 is 137)

Entering H4Nd1indices with d = 3
 Number of indices is 272
Entering H4Nd1 routine with {d,sgn} = {3, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 150 - 80 - 9 = 61, needs to be less than 8
 (Full rank of plus and minus basis attempts mod 12 347 are 137 and 9)
Entering H4Nd1 routine with {d,sgn} = {3, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 150 - 9 - 137 = 4, needs to be less than 8
 (Full rank of plus basis attempt mod 12 347 is 137)
H4Nd1(3,+) says that Jacobi restriction out to index
 2 gives a weight 2 plus space upper bound

N = 262
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {141, 144}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {28, 29}
dim(J_{2,N}^{cusp}) = 5
The file reports a plus basis attempt of dimensions {141, 335}
and a minus basis attempt of dimensions {28, 335}
ShortVectorFinalDet = 223
ShortVectorLength = 335

Entering H4Nd1indices with d = 1
 Number of indices is 259
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 173 - 78 - 27 = 68, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12 347 are 141 and 28)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 173 - 28 - 121 = 24, needs to be less than 6
 (Full rank of plus basis attempt mod 12 347 is 141)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 173 - 141 - 27 = 5, needs to be less than 5
 (Full rank of minus basis attempt mod 12 347 is 28)
```

```
Entering H4Nd1indices with d = 2

 Number of indices is 273

Entering H4Nd1 routine with {d,sgn} = {2, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 173 - 91 - 28 = 54, needs to be less than 6

 (Full rank of plus and minus basis attempts mod 12 347 are 141 and 28)

Entering H4Nd1 routine with {d,sgn} = {2, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 173 - 28 - 135 = 10, needs to be less than 6

 (Full rank of plus basis attempt mod 12 347 is 141)

Entering H4Nd1 routine with {d,sgn} = {2, -1}

 Rigorous upper bound of dim(H_4(N,n,1))^- is 173 - 141 - 28 = 4, needs to be less than 5

 (Full rank of minus basis attempt mod 12 347 is 28)

H4Nd1(2,-) says that Jacobi restriction out to index
 1 gives a weight 2 minus space upper bound


Entering H4Nd1indices with d = 3

 Number of indices is 291

Entering H4Nd1 routine with {d,sgn} = {3, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 173 - 100 - 28 = 45, needs to be less than 6

 (Full rank of plus and minus basis attempts mod 12 347 are 141 and 28)

Entering H4Nd1 routine with {d,sgn} = {3, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is 173 - 28 - 141 = 4, needs to be less than 6

 (Full rank of plus basis attempt mod 12 347 is 141)

H4Nd1(3,+) says that Jacobi restriction out to index
 2 gives a weight 2 plus space upper bound


N = 265

{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {151, 151}

{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {5, 11}

dim(J_{2,N}^{cusp}) = 8

The file reports a plus basis attempt of dimensions {151, 358}

and a minus basis attempt of dimensions {5, 358}

ShortVectorFinalDet = 264

ShortVectorLength = 358


Entering H4Nd1indices with d = 1

 Number of indices is 224
```

```
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 162 - 63 - 5 = 94, needs to be less than 9
 (Full rank of plus and minus basis attempts mod 12 347 are 151 and 5)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 162 - 5 - 106 = 51, needs to be less than 9
 (Full rank of plus basis attempt mod 12 347 is 151)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 162 - 151 - 5 = 6, needs to be less than 8
 (Full rank of minus basis attempt mod 12 347 is 5)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 261
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 162 - 87 - 5 = 70, needs to be less than 9
 (Full rank of plus and minus basis attempts mod 12 347 are 151 and 5)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 162 - 5 - 135 = 22, needs to be less than 9
 (Full rank of plus basis attempt mod 12 347 is 151)

Entering H4Nd1indices with d = 3
 Number of indices is 289
Entering H4Nd1 routine with {d,sgn} = {3, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 162 - 87 - 5 = 70, needs to be less than 9
 (Full rank of plus and minus basis attempts mod 12 347 are 151 and 5)
Entering H4Nd1 routine with {d,sgn} = {3, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 162 - 5 - 151 = 6, needs to be less than 9
 (Full rank of plus basis attempt mod 12 347 is 151)
H4Nd1(3,+) says that Jacobi restriction out to index
 2 gives a weight 2 plus space upper bound

N = 266
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {125, 126}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {29, 32}
dim(J_{2,N}^{cusp}) = 4
The file reports a plus basis attempt of dimensions {125, 328}
and a minus basis attempt of dimensions {29, 328}
```

```
ShortVectorFinalDet = 223

ShortVectorLength = 328


Entering H4Nd1indices with d = 1

 Number of indices is 266

Entering H4Nd1 routine with {d,sgn} = {1, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 158 - 114 - 29 = 15, needs to be less than 5

 (Full rank of plus and minus basis attempts mod 12 347 are 125 and 29)

Entering H4Nd1 routine with {d,sgn} = {1, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 158 - 29 - 114 = 15, needs to be less than 5

 (Full rank of plus basis attempt mod 12 347 is 125)


Entering H4Nd1indices with d = 2

 Number of indices is 281

Entering H4Nd1 routine with {d,sgn} = {2, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 158 - 123 - 29 = 6, needs to be less than 5

 (Full rank of plus and minus basis attempts mod 12 347 are 125 and 29)

Entering H4Nd1 routine with {d,sgn} = {2, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is 158 - 29 - 123 = 6, needs to be less than 5

 (Full rank of plus basis attempt mod 12 347 is 125)


Entering H4Nd1indices with d = 3

 Number of indices is 299

Entering H4Nd1 routine with {d,sgn} = {3, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 158 - 125 - 29 = 4, needs to be less than 5

 (Full rank of plus and minus basis attempts mod 12 347 are 125 and 29)

H4Nd1(3) says that Jacobi restriction out to index 2 gives a weight 2 space upper bound


N = 267

{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {130, 130}

{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {19, 25}

dim(J_{2,N}^{cusp}) = 7

The file reports a plus basis attempt of dimensions {130, 382}

and a minus basis attempt of dimensions {19, 382}

ShortVectorFinalDet = 272

ShortVectorLength = 382
```

```
Entering H4Nd1indices with d = 1
 Number of indices is 234
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 155 - 71 - 19 = 65, needs to be less than 8
 (Full rank of plus and minus basis attempts mod 12 347 are 130 and 19)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 155 - 19 - 97 = 39, needs to be less than 8
 (Full rank of plus basis attempt mod 12 347 is 130)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 155 - 130 - 19 = 6, needs to be less than 7
 (Full rank of minus basis attempt mod 12 347 is 19)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 266
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 155 - 90 - 19 = 46, needs to be less than 8
 (Full rank of plus and minus basis attempts mod 12 347 are 130 and 19)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 155 - 19 - 124 = 12, needs to be less than 8
 (Full rank of plus basis attempt mod 12 347 is 130)

Entering H4Nd1indices with d = 3
 Number of indices is 303
Entering H4Nd1 routine with {d,sgn} = {3, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 155 - 100 - 19 = 36, needs to be less than 8
 (Full rank of plus and minus basis attempts mod 12 347 are 130 and 19)
Entering H4Nd1 routine with {d,sgn} = {3, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 155 - 19 - 130 = 6, needs to be less than 8
 (Full rank of plus basis attempt mod 12 347 is 130)
H4Nd1(3,+) says that Jacobi restriction out to index
 2 gives a weight 2 plus space upper bound

N = 273
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {141, 141}
```

```
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {17, 19}

dim(J_{2,N}^{cusp}) = 6

The file reports a plus basis attempt of dimensions {141, 358}

and a minus basis attempt of dimensions {17, 358}

ShortVectorFinalDet = 264

ShortVectorLength = 358


Entering H4Nd1indices with d = 1

 Number of indices is 280

Entering H4Nd1 routine with {d,sgn} = {1, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 160 - 76 - 17 = 67, needs to be less than 7

 (Full rank of plus and minus basis attempts mod 12 347 are 141 and 17)

Entering H4Nd1 routine with {d,sgn} = {1, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 160 - 17 - 120 = 23, needs to be less than 7

 (Full rank of plus basis attempt mod 12 347 is 141)

Entering H4Nd1 routine with {d,sgn} = {1, -1}

 Rigorous upper bound of dim(H_4(N,n,1))^- is 160 - 141 - 17 = 2, needs to be less than 6

 (Full rank of minus basis attempt mod 12 347 is 17)

H4Nd1(1,-) says that S_2(K(N))^-=0


Entering H4Nd1indices with d = 2

 Number of indices is 300

Entering H4Nd1 routine with {d,sgn} = {2, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 160 - 88 - 17 = 55, needs to be less than 7

 (Full rank of plus and minus basis attempts mod 12 347 are 141 and 17)

Entering H4Nd1 routine with {d,sgn} = {2, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is 160 - 17 - 136 = 7, needs to be less than 7

 (Full rank of plus basis attempt mod 12 347 is 141)


Entering H4Nd1indices with d = 3

 Number of indices is 311

Entering H4Nd1 routine with {d,sgn} = {3, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 160 - 94 - 17 = 49, needs to be less than 7

 (Full rank of plus and minus basis attempts mod 12 347 are 141 and 17)

Entering H4Nd1 routine with {d,sgn} = {3, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is 160 - 17 - 141 = 2, needs to be less than 7
```

```
 (Full rank of plus basis attempt mod 12 347 is 141)

H4Nd1(3,+) says that Jacobi restriction out to index
 2 gives a weight 2 plus space upper bound


N = 274

{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {162, 164}

{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {24, 27}

dim(J_{2,N}^{cusp}) = 7

The file reports a plus basis attempt of dimensions {162, 356}

and a minus basis attempt of dimensions {24, 356}

ShortVectorFinalDet = 224

ShortVectorLength = 356


Entering H4Nd1indices with d = 1

 Number of indices is 228

Entering H4Nd1 routine with {d,sgn} = {1, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 191 - 79 - 23 = 89, needs to be less than 8

 (Full rank of plus and minus basis attempts mod 12 347 are 162 and 24)

Entering H4Nd1 routine with {d,sgn} = {1, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 191 - 24 - 124 = 43, needs to be less than 8

 (Full rank of plus basis attempt mod 12 347 is 162)

Entering H4Nd1 routine with {d,sgn} = {1, -1}

 Rigorous upper bound of dim(H_4(N,n,1))^- is 191 - 162 - 23 = 6, needs to be less than 7

 (Full rank of minus basis attempt mod 12 347 is 24)

H4Nd1(1,-) says that S_2(K(N))^-=0


Entering H4Nd1indices with d = 2

 Number of indices is 258

Entering H4Nd1 routine with {d,sgn} = {2, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 191 - 96 - 23 = 72, needs to be less than 8

 (Full rank of plus and minus basis attempts mod 12 347 are 162 and 24)

Entering H4Nd1 routine with {d,sgn} = {2, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 191 - 24 - 149 = 18, needs to be less than 8

 (Full rank of plus basis attempt mod 12 347 is 162)


Entering H4Nd1indices with d = 3
```

```
 Number of indices is 296
Entering H4Nd1 routine with {d,sgn} = {3, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 191 - 105 - 24 = 62, needs to be less than 8
 (Full rank of plus and minus basis attempts mod 12 347 are 162 and 24)
Entering H4Nd1 routine with {d,sgn} = {3, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 191 - 24 - 162 = 5, needs to be less than 8
 (Full rank of plus basis attempt mod 12 347 is 162)
H4Nd1(3,+) says that Jacobi restriction out to index
 2 gives a weight 2 plus space upper bound
```

N = 278

```
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {140, 142}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {41, 43}
dim(J_{2,N}^{cusp}) = 6
The file reports a plus basis attempt of dimensions {140, 395}
and a minus basis attempt of dimensions {41, 395}
ShortVectorFinalDet = 232
ShortVectorLength = 395

Entering H4Nd1indices with d = 1
 Number of indices is 275
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 185 - 116 - 40 = 29, needs to be less than 7
 (Full rank of plus and minus basis attempts mod 12 347 are 140 and 41)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 185 - 41 - 116 = 28, needs to be less than 7
 (Full rank of plus basis attempt mod 12 347 is 140)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 185 - 140 - 40 = 5, needs to be less than 6
 (Full rank of minus basis attempt mod 12 347 is 41)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 297
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 185 - 136 - 41 = 8, needs to be less than 7
```

```
 (Full rank of plus and minus basis attempts mod 12 347 are 140 and 41)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 185 - 41 - 136 = 8, needs to be less than 7
 (Full rank of plus basis attempt mod 12 347 is 140)

Entering H4Nd1indices with d = 3
 Number of indices is 329
Entering H4Nd1 routine with {d,sgn} = {3, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 185 - 140 - 41 = 4, needs to be less than 7
 (Full rank of plus and minus basis attempts mod 12 347 are 140 and 41)
H4Nd1(3) says that Jacobi restriction out to index 2 gives a weight 2 space upper bound

N = 282
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {151, 153}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {41, 44}
dim(J_{2,N}^{cusp}) = 5
The file reports a plus basis attempt of dimensions {151, 474}
and a minus basis attempt of dimensions {41, 474}
ShortVectorFinalDet = 255
ShortVectorLength = 474

Entering H4Nd1indices with d = 1
 Number of indices is 332
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 197 - 134 - 41 = 22, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12 347 are 151 and 41)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 197 - 41 - 134 = 22, needs to be less than 6
 (Full rank of plus basis attempt mod 12 347 is 151)

Entering H4Nd1indices with d = 2
 Number of indices is 357
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 197 - 151 - 41 = 5, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12 347 are 151 and 41)
H4Nd1(2) says that S_2(K(N))=Grit(J_{2,N}^{cusp})
```

N = 285

{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {143, 144}

{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {24, 25}

dim(J_{2,N}^{cusp}) = 3

The file reports a plus basis attempt of dimensions {143, 360}

and a minus basis attempt of dimensions {24, 360}

ShortVectorFinalDet = 236

ShortVectorLength = 360

Entering H4Nd1indices with d = 1

Number of indices is 312

Entering H4Nd1 routine with {d,sgn} = {1, 0}

Rigorous upper bound of dim(H_4(N,n,1)) is 169 - 95 - 24 = 50, needs to be less than 4

(Full rank of plus and minus basis attempts mod 12 347 are 143 and 24)

Entering H4Nd1 routine with {d,sgn} = {1, 1}

Rigorous upper bound of dim(H_4(N,n,1))^+ is 169 - 24 - 136 = 9, needs to be less than 4

(Full rank of plus basis attempt mod 12 347 is 143)

Entering H4Nd1 routine with {d,sgn} = {1, -1}

Rigorous upper bound of dim(H_4(N,n,1))^- is 169 - 143 - 24 = 2, needs to be less than 3

(Full rank of minus basis attempt mod 12 347 is 24)

H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2

Number of indices is 324

Entering H4Nd1 routine with {d,sgn} = {2, 0}

Rigorous upper bound of dim(H_4(N,n,1)) is 169 - 104 - 24 = 41, needs to be less than 4

(Full rank of plus and minus basis attempts mod 12 347 are 143 and 24)

Entering H4Nd1 routine with {d,sgn} = {2, 1}

Rigorous upper bound of dim(H_4(N,n,1))^+ is 169 - 24 - 142 = 3, needs to be less than 4

(Full rank of plus basis attempt mod 12 347 is 143)

H4Nd1(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})

N = 286

{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {161, 161}

{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {27, 28}

```
dim(J_{2,N}^{cusp}) = 3

The file reports a plus basis attempt of dimensions {161, 978}

and a minus basis attempt of dimensions {27, 978}

ShortVectorFinalDet = 428

ShortVectorLength = 978


Entering H4Nd1indices with d = 1

 Number of indices is 694

Entering H4Nd1 routine with {d,sgn} = {1, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 189 - 106 - 27 = 56, needs to be less than 4

 (Full rank of plus and minus basis attempts mod 12 347 are 161 and 27)

Entering H4Nd1 routine with {d,sgn} = {1, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 189 - 27 - 125 = 37, needs to be less than 4

 (Full rank of plus basis attempt mod 12 347 is 161)

Entering H4Nd1 routine with {d,sgn} = {1, -1}

 Rigorous upper bound of dim(H_4(N,n,1))^- is 189 - 161 - 27 = 1, needs to be less than 3

 (Full rank of minus basis attempt mod 12 347 is 27)

H4Nd1(1,-) says that S_2(K(N))^-=0


Entering H4Nd1indices with d = 2

 Number of indices is 735

Entering H4Nd1 routine with {d,sgn} = {2, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 189 - 133 - 27 = 29, needs to be less than 4

 (Full rank of plus and minus basis attempts mod 12 347 are 161 and 27)

Entering H4Nd1 routine with {d,sgn} = {2, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 189 - 27 - 146 = 16, needs to be less than 4

 (Full rank of plus basis attempt mod 12 347 is 161)


Entering H4Nd1indices with d = 3

 Number of indices is 770

Entering H4Nd1 routine with {d,sgn} = {3, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 189 - 147 - 27 = 15, needs to be less than 4

 (Full rank of plus and minus basis attempts mod 12 347 are 161 and 27)

Entering H4Nd1 routine with {d,sgn} = {3, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is 189 - 27 - 160 = 2, needs to be less than 4
```

```
 (Full rank of plus basis attempt mod 12347 is 161)
H4Nd1(3,+) says that Jacobi restriction out to index
 2 gives a weight 2 plus space upper bound

N = 287
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {134, 134}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {25, 28}
dim(J_{2,N}^{cusp}) = 4
The file reports a plus basis attempt of dimensions {134, 416}
and a minus basis attempt of dimensions {25, 416}
ShortVectorFinalDet = 279
ShortVectorLength = 416

Entering H4Nd1indices with d = 1
 Number of indices is 336
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 162 - 92 - 24 = 46, needs to be less than 5
 (Full rank of plus and minus basis attempts mod 12347 are 134 and 25)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 162 - 25 - 124 = 13, needs to be less than 5
 (Full rank of plus basis attempt mod 12347 is 134)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 162 - 134 - 24 = 4, needs to be less than 4
 (Full rank of minus basis attempt mod 12347 is 25)

Entering H4Nd1indices with d = 2
 Number of indices is 351
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 162 - 101 - 24 = 37, needs to be less than 5
 (Full rank of plus and minus basis attempts mod 12347 are 134 and 25)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 162 - 25 - 134 = 3, needs to be less than 5
 (Full rank of plus basis attempt mod 12347 is 134)
H4Nd1(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})
Entering H4Nd1 routine with {d,sgn} = {2, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 162 - 134 - 24 = 4, needs to be less than 4
```

```
 (Full rank of minus basis attempt mod 12 347 is 25)


Entering H4Nd1indices with d = 3

 Number of indices is 384

Entering H4Nd1 routine with {d,sgn} = {3, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 162 - 103 - 25 = 34, needs to be less than 5

 (Full rank of plus and minus basis attempts mod 12 347 are 134 and 25)

Entering H4Nd1 routine with {d,sgn} = {3, -1}

 Rigorous upper bound of dim(H_4(N,n,1))^- is 162 - 134 - 25 = 3, needs to be less than 4

 (Full rank of minus basis attempt mod 12 347 is 25)

H4Nd1(3,-) says that Jacobi restriction out to index
 2 gives a weight 2 minus space upper bound


N = 290

{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {152, 153}

{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {43, 45}

dim(J_{2,N}^{cusp}) = 5

The file reports a plus basis attempt of dimensions {152, 474}

and a minus basis attempt of dimensions {43, 474}

ShortVectorFinalDet = 239

ShortVectorLength = 474


Entering H4Nd1indices with d = 1

 Number of indices is 370

Entering H4Nd1 routine with {d,sgn} = {1, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 198 - 72 - 39 = 87, needs to be less than 6

 (Full rank of plus and minus basis attempts mod 12 347 are 152 and 43)

Entering H4Nd1 routine with {d,sgn} = {1, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 198 - 43 - 136 = 19, needs to be less than 6

 (Full rank of plus basis attempt mod 12 347 is 152)

Entering H4Nd1 routine with {d,sgn} = {1, -1}

 Rigorous upper bound of dim(H_4(N,n,1))^- is 198 - 152 - 39 = 7, needs to be less than 5

 (Full rank of minus basis attempt mod 12 347 is 43)


Entering H4Nd1indices with d = 2

 Number of indices is 391
```

```
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 198 - 86 - 43 = 69, needs to be less than 6
 (Full rank of plus and minus basis attempts mod 12 347 are 152 and 43)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 198 - 43 - 150 = 5, needs to be less than 6
 (Full rank of plus basis attempt mod 12 347 is 152)
H4Nd1(2,+) says that S_2(K(N))^+=Grit(J_{2,N}^{cusp})
Entering H4Nd1 routine with {d,sgn} = {2, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 198 - 152 - 43 = 3, needs to be less than 5
 (Full rank of minus basis attempt mod 12 347 is 43)
H4Nd1(2,-) says that Jacobi restriction out to index
 1 gives a weight 2 minus space upper bound

N = 291
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {161, 161}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {20, 27}
dim(J_{2,N}^{cusp}) = 8
The file reports a plus basis attempt of dimensions {161, 353}
and a minus basis attempt of dimensions {20, 353}
ShortVectorFinalDet = 243
ShortVectorLength = 353

Entering H4Nd1indices with d = 1
 Number of indices is 230
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 188 - 79 - 20 = 89, needs to be less than 9
 (Full rank of plus and minus basis attempts mod 12 347 are 161 and 20)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 188 - 20 - 118 = 50, needs to be less than 9
 (Full rank of plus basis attempt mod 12 347 is 161)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 188 - 161 - 20 = 7, needs to be less than 8
 (Full rank of minus basis attempt mod 12 347 is 20)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
```

```
 Number of indices is 263
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 188 - 98 - 20 = 70, needs to be less than 9
 (Full rank of plus and minus basis attempts mod 12 347 are 161 and 20)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 188 - 20 - 146 = 22, needs to be less than 9
 (Full rank of plus basis attempt mod 12 347 is 161)

Entering H4Nd1indices with d = 3
 Number of indices is 291
Entering H4Nd1 routine with {d,sgn} = {3, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 188 - 103 - 20 = 65, needs to be less than 9
 (Full rank of plus and minus basis attempts mod 12 347 are 161 and 20)
Entering H4Nd1 routine with {d,sgn} = {3, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 188 - 20 - 159 = 9, needs to be less than 9
 (Full rank of plus basis attempt mod 12 347 is 161)

Entering H4Nd1indices with d = 4
 Number of indices is 327
Entering H4Nd1 routine with {d,sgn} = {4, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 188 - 109 - 20 = 59, needs to be less than 9
 (Full rank of plus and minus basis attempts mod 12 347 are 161 and 20)
Entering H4Nd1 routine with {d,sgn} = {4, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 188 - 20 - 161 = 7, needs to be less than 9
 (Full rank of plus basis attempt mod 12 347 is 161)
H4Nd1(4,+) says that Jacobi restriction out to index
 3 gives a weight 2 plus space upper bound

N = 295
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {167, 167}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {17, 22}
dim(J_{2,N}^{cusp}) = 6
The file reports a plus basis attempt of dimensions {167, 331}
and a minus basis attempt of dimensions {17, 331}
ShortVectorFinalDet = 231
ShortVectorLength = 331
```

```
Entering H4Nd1indices with d = 1

 Number of indices is 237

Entering H4Nd1 routine with {d,sgn} = {1, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 189 - 88 - 17 = 84, needs to be less than 7

 (Full rank of plus and minus basis attempts mod 12 347 are 167 and 17)

Entering H4Nd1 routine with {d,sgn} = {1, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 189 - 17 - 136 = 36, needs to be less than 7

 (Full rank of plus basis attempt mod 12 347 is 167)

Entering H4Nd1 routine with {d,sgn} = {1, -1}

 Rigorous upper bound of dim(H_4(N,n,1))^- is 189 - 167 - 17 = 5, needs to be less than 6

 (Full rank of minus basis attempt mod 12 347 is 17)

H4Nd1(1,-) says that S_2(K(N))^-=0


Entering H4Nd1indices with d = 2

 Number of indices is 258

Entering H4Nd1 routine with {d,sgn} = {2, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 189 - 101 - 17 = 71, needs to be less than 7

 (Full rank of plus and minus basis attempts mod 12 347 are 167 and 17)

Entering H4Nd1 routine with {d,sgn} = {2, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 189 - 17 - 155 = 17, needs to be less than 7

 (Full rank of plus basis attempt mod 12 347 is 167)


Entering H4Nd1indices with d = 3

 Number of indices is 297

Entering H4Nd1 routine with {d,sgn} = {3, 0}

 Rigorous upper bound of dim(H_4(N,n,1)) is 189 - 111 - 17 = 61, needs to be less than 7

 (Full rank of plus and minus basis attempts mod 12 347 are 167 and 17)

Entering H4Nd1 routine with {d,sgn} = {3, 1}

 Rigorous upper bound of dim(H_4(N,n,1))^+ is 189 - 17 - 167 = 5, needs to be less than 7

 (Full rank of plus basis attempt mod 12 347 is 167)

H4Nd1(3,+) says that Jacobi restriction out to index
 2 gives a weight 2 plus space upper bound


Entering H4Nd1indices with d = 4

 Number of indices is 325
```

```
Entering H4Nd1 routine with {d,sgn} = {4, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 189 - 112 - 17 = 60, needs to be less than 7
 (Full rank of plus and minus basis attempts mod 12 347 are 167 and 17)

N = 298
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {182, 185}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {33, 38}
dim(J_{2,N}^{cusp}) = 8
The file reports a plus basis attempt of dimensions {182, 436}
and a minus basis attempt of dimensions {33, 436}
ShortVectorFinalDet = 263
ShortVectorLength = 436

Entering H4Nd1indices with d = 1
 Number of indices is 287
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 223 - 136 - 33 = 54, needs to be less than 9
 (Full rank of plus and minus basis attempts mod 12 347 are 182 and 33)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 223 - 33 - 141 = 49, needs to be less than 9
 (Full rank of plus basis attempt mod 12 347 is 182)

Entering H4Nd1indices with d = 2
 Number of indices is 316
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 223 - 163 - 33 = 27, needs to be less than 9
 (Full rank of plus and minus basis attempts mod 12 347 are 182 and 33)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 223 - 33 - 168 = 22, needs to be less than 9
 (Full rank of plus basis attempt mod 12 347 is 182)

Entering H4Nd1indices with d = 3
 Number of indices is 347
Entering H4Nd1 routine with {d,sgn} = {3, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 223 - 179 - 33 = 11, needs to be less than 9
```

```
 (Full rank of plus and minus basis attempts mod 12 347 are 182 and 33)
Entering H4Nd1 routine with {d,sgn} = {3, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 223 - 33 - 181 = 9, needs to be less than 9
 (Full rank of plus basis attempt mod 12 347 is 182)

Entering H4Nd1indices with d = 4
 Number of indices is 378
Entering H4Nd1 routine with {d,sgn} = {4, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 223 - 182 - 33 = 8, needs to be less than 9
 (Full rank of plus and minus basis attempts mod 12 347 are 182 and 33)
H4Nd1(4) says that Jacobi restriction out to index 3 gives a weight 2 space upper bound

N = 299
{PlusBasisAttemptRank,HeuristicDimS4KNplus}: {145, 145}
{MinusBasisAttemptRank,HeuristicDimS4KNminus}: {26, 29}
dim(J_{2,N}^{cusp}) = 4
The file reports a plus basis attempt of dimensions {145, 515}
and a minus basis attempt of dimensions {26, 515}
ShortVectorFinalDet = 352
ShortVectorLength = 515

Entering H4Nd1indices with d = 1
 Number of indices is 423
Entering H4Nd1 routine with {d,sgn} = {1, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 174 - 105 - 26 = 43, needs to be less than 5
 (Full rank of plus and minus basis attempts mod 12 347 are 145 and 26)
Entering H4Nd1 routine with {d,sgn} = {1, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is
 174 - 26 - 134 = 14, needs to be less than 5
 (Full rank of plus basis attempt mod 12 347 is 145)
Entering H4Nd1 routine with {d,sgn} = {1, -1}
 Rigorous upper bound of dim(H_4(N,n,1))^- is 174 - 145 - 26 = 3, needs to be less than 4
 (Full rank of minus basis attempt mod 12 347 is 26)
H4Nd1(1,-) says that S_2(K(N))^-=0

Entering H4Nd1indices with d = 2
 Number of indices is 444
```

```
Entering H4Nd1 routine with {d,sgn} = {2, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 174 - 115 - 26 = 33, needs to be less than 5
 (Full rank of plus and minus basis attempts mod 12 347 are 145 and 26)
Entering H4Nd1 routine with {d,sgn} = {2, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 174 - 26 - 143 = 5, needs to be less than 5
 (Full rank of plus basis attempt mod 12 347 is 145)


Entering H4Nd1indices with d = 3
 Number of indices is 472
Entering H4Nd1 routine with {d,sgn} = {3, 0}
 Rigorous upper bound of dim(H_4(N,n,1)) is 174 - 121 - 26 = 27, needs to be less than 5
 (Full rank of plus and minus basis attempts mod 12 347 are 145 and 26)
Entering H4Nd1 routine with {d,sgn} = {3, 1}
 Rigorous upper bound of dim(H_4(N,n,1))^+ is 174 - 26 - 145 = 3, needs to be less than 5
 (Full rank of plus basis attempt mod 12 347 is 145)
H4Nd1(3,+) says that Jacobi restriction out to index
 2 gives a weight 2 plus space upper bound
```

Summary Report by Category

```
S_2(K(N)) = Grit(J_2^{cusp}) for these {N,dim(J2)}:
 {{62, 0}, {65, 1}, {66, 0}, {69, 0}, {70, 0}, {74, 1}, {77, 1}, {78, 0}, {82, 1}, {85, 2},
  {86, 1}, {87, 0}, {91, 2}, {93, 2}, {94, 0}, {95, 0}, {102, 1}, {105, 0}, {106, 2},
  {110, 0}, {111, 1}, {114, 1}, {115, 2}, {118, 1}, {119, 0}, {122, 2}, {123, 2}, {129, 2},
  {130, 2}, {133, 4}, {134, 2}, {138, 1}, {141, 2}, {142, 2}, {143, 1}, {145, 3}, {146, 2},
  {154, 2}, {155, 2}, {158, 3}, {159, 1}, {161, 2}, {165, 2}, {166, 2}, {170, 3}, {174, 1},
  {177, 4}, {178, 3}, {182, 2}, {183, 3}, {185, 4}, {186, 2}, {187, 5}, {190, 2}, {194, 3},
  {195, 1}, {201, 5}, {202, 4}, {203, 4}, {205, 5}, {206, 2}, {209, 3}, {210, 1}, {213, 4},
  {214, 5}, {215, 2}, {217, 6}, {218, 6}, {219, 5}, {221, 5}, {222, 1}, {226, 6}, {230, 2},
  {231, 1}, {235, 6}, {237, 5}, {238, 2}, {246, 4}, {247, 7}, {253, 8}, {254, 5}, {255, 2},
  {258, 4}, {259, 7}, {262, 5}, {265, 8}, {266, 4}, {267, 7}, {273, 6}, {274, 7}, {278, 6},
  {282, 5}, {285, 3}, {286, 3}, {287, 4}, {290, 5}, {291, 8}, {298, 8}, {299, 4}}
dim(S_2(K(N))^+) <= dim(J2)+1 and S_2(K(N))^- = 0
 for these {N,dim(J2)}: {{249, 5}, {295, 6}}
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